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NOTAS

ON THE EFFICIENCY OF PROCEDURES FOR ESTIMATION OF PARAMETERS IN ARIMA MODELS

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The paper discusses the implementation of the Newton-Raphson iterative method of estimation of parameters in the autoregressive integrated moving average (ARIMA) models. The efficiency of this method has been compared with other well known methods of estimation.

1. INTRODUCTION

The estimation of parameters in the autoregressive integrated moving average models, usign the non-linear least squares (Marquardt, 1963) method or the sum of squares grid method (Box and Jenkins, 1976), has been widely used by various practitioners in the area of forecasting. The sum of squares grid method can be employed for estimating only two or at most three parameters. The possibility of employing the Newton-Raphson iterative method (Kawoshima, 1980) of estimation in practice, is explored in this paper, and the efficiency, with respect to the computational time, of different methods is compared.

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2.THE MODEL

The ARIMA (p, d, q) model is defined by

$$C(B)\,\nabla^d X_t = D(B) A_t \tag{2.1}$$

where $X_t (t = 1, 2, \ldots,)$ is the underlying stochastic process, d denotes the order of differencing, ∇ is the backward difference operator, and A_t is a white noise Gaussian process with zero mean and variance $\sigma^2 \cdot C(B)$ and $D(B)$ are polynomials:

$$C(B) = c_0 - c_1 B - \ldots - c_p B^p \qquad D(B) = d_0 - d_1 B - \ldots - d_q B^q$$

with $c_0 = d_0 = 1$ and B is the backshift operator. The estimation of the parameters $c_1, \ldots, c_p, d_1, \ldots, d_q$ and σ^2 is based on a set of observations at T successive time points, $x_1, \ldots, x_T$ with $T > p + q$.

The modified model for $\{x_t\}$ is (T.W. Anderson, 1977)

$$\sum_{k=0}^{p} c_k L^k x_t = \sum_{g=0}^{q} d_g L^g V_t \tag{2.2}$$

where V has the distribution $N(0, \sigma^2 I)$ and L is a matrix of order $T \times T$ given by

$$L = \begin{bmatrix} 0 & 0 \\ I & 0 \end{bmatrix}$$

where I is of order $T - 1$, L^t has a similar form in which I is of order $T - t$ $(t = 0, 1, \ldots, T - 1)$ and $L^t = 0$ $(t = T, T + 1, \ldots)$.

The parameters $c_1, \ldots, c_p, d_1, \ldots, d_q$ and σ^2 are estimated by the method of maximum likelihood. The iterative equations for estimating the parameters are

$$\begin{bmatrix} \nabla\hat{\alpha}_i & \nabla\hat{\delta}_i \\ \nabla\hat{\delta}'_i & \nabla\hat{\beta}_i \end{bmatrix} \begin{bmatrix} \hat{D}_{i+1} - \hat{D}_i \\ \hat{C}_{i+1} - \hat{C}_i \end{bmatrix} = \begin{bmatrix} \nabla\hat{g}_i \\ \nabla\hat{h}_i \end{bmatrix} \tag{2.3}$$

where

$$\hat{C}_i = [\hat{c}_1, \ldots, \hat{c}_p]'_i$$
$$\hat{D}_i = [\hat{d}_1, \ldots, \hat{d}_q]'_i$$
$$[\nabla \hat{g}_i]_g = (\hat{V}'_i L^g \nabla \hat{D}_i^{-1} V_i)/\hat{\sigma}_i^2 \tag{2.4}$$
$$[\nabla \hat{h}_i]_k = (-\hat{V}'_i L^k \nabla \hat{D}_i^{-1} \nabla X)/\hat{\alpha}_i^2 \tag{2.5}$$
$$[\nabla \hat{\delta}_i]_{gf} = (\hat{V}'_i (\nabla \hat{D}_i^{-1})' L^{g'} L^f \nabla \hat{D}_i^{-1} V_i)/\hat{\sigma}_i^2 \quad g, f = 1, 2, \ldots, q \tag{2.6}$$
$$[\nabla \hat{\delta}_i]_{gl} = (-\hat{V}'_i (\nabla \hat{D}_i^{-1})' L^{g'} L^l \nabla \hat{D}_i^{-1} \nabla X)/\hat{\sigma}_i^2 \; g, l = 1, 2, \ldots, p \tag{2.7}$$
$$[\nabla \hat{\beta}_i]_{kl} = (\nabla X' (\nabla \hat{D}_i^{-1})' L^{k'} L^l \nabla \hat{D}_i^{-1} \nabla X)/\hat{\sigma}_i^2 \; k, l = 1, 2, \ldots, p \tag{2.8}$$
$$\nabla \hat{D}_i = \sum_{j=0}^{q} d_j(i) L^j$$
$$\hat{V}_i = \hat{C}_i \nabla \hat{D}_i^{-1} \nabla X$$
$$\hat{C}_i = \sum_{j=0}^{p} c_j(i) L^j.$$

3. RESULTS

The data consisted of 45 daily observations of clay values for a factory claybody in a ceramic factory. The problem was to develop a control system for the claybody by taking account of the daily change in its clay value that occurred as a result of source clay variability. The model identified was **ARIMA (1, 1, 1), (Chandra and Sinha, 1978):**

$$(1 - c_1 B)\nabla X_t = (1 - d_1 B)A_t.$$

The autocorrelations of the differenced series, ∇X_t, for lags 1-24 are given in the following table.

Table 3.1

Lags	1	2	3	4	5	6	7	8
1-8	0.56	0.62	0.39	0.32	0.19	0.07	0.02	−0.05
9-16	0.007	−0.15	−0.11	−0.16	−0.12	−0.21	−0.17	−0.21
17-24	−0.10	−0.09	0.04	0.04	0.04	0.26	0.25	0.28

The initial estimates for the parameters c_1 and d_1 were found by substituting the estimated autocorrelations at lag 1 and 2 for p_1 and p_2 in the following expressions:

$$p_1 = (1 - \hat{d}_1\hat{c}_1)(\hat{c}_1 - \hat{d}_1)/(1 - \hat{d}_1^2 - 2\hat{c}_1\hat{d}_1)$$
$$p_2 = p_1\hat{c}_1.$$

The initial estimates were $\hat{c}_1 = -0.62431$ and $\hat{d}_1 = 0.06717$.

The final estimates were found by three methods, namely the sum of squares grid method, Marquardt's algorithm and the Newton-Raphson iterative method of estimation by the method of maximum likelihood. The sum of squares grid is shown in Table 3.2, where the minimum sum of squares region is marked. The final estimates were −0.63 and 0.06 for c_1 and d_1, respectively, and the processing time was 0.44 seconds.

Table 3.2

c_1 \ d_1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1,0
−1,0	1583	1646	1735	1859	2031	2268	2600	3078	3790	4828	6138
−0.9	1452	1504	1582	1691	1844	2057	2356	2786	3427	4364	5554
−0.8	1368	1405	1467	1559	1691	1877	2142	2526	3099	3937	5013
−0.7	1333	1349	1390	1461	1570	1730	1960	2297	2804	3548	4514
−0.6	1344	1334	1351	1399	1483	1613	1808	2100	2542	3196	4056
−0.5	1403	1361	1350	1371	1428	1528	1688	1934	2314	2882	3640
−0.4	1509	1430	1387	1378	1405	1475	1599	1800	2120	2605	3267
−0.3	1663	1542	1463	1421	1416	1453	1540	1697	1959	2366	2935
−0.2	1864	1695	1576	1498	1460	1462	1513	1627	1833	2163	2645

Given the initial estimates, the final estimates, using de Marquardt algorithm were −0.65645 and 0.03104, respectively, and the convergence took place on the 3rd step itself. The values obtained at the different steps being

	c_1	d_1
Step 1 (initial values)	−0.62431	0.06717
Step 2	−0.65645	0.03104
Step 3	−0.65645	0.03104

In this case the central processing unit (CPU) took 0.50 seconds to process the job.

The Newton-Raphson method gave the estimates as −0.66175 and 0.15016. The values converged at the 5th iteration.

Iteration 1 (initial values)	−0.62431	0.06717
Iteration 2	−0.66056	0.13125
Iteration 3	−0.66168	0.14029
Iteration 4	−0.66175	0.15016
Iteration 5	−0.66175	0.15016

The processing time taken to find the estimates by the Newton-Raphson method was 18.43 seconds, even though only two parameters had to be estimated. Furthermore, there were only 45 observations. The cost of processing will become much higher when the number of observations is large and when more parameters have to be estimated. Due to the high cost involved in using this method, it is preferable to use the sum of squares grid method (in the case of estimation of two parameters) or the Marquardt algorithm (for the estimation of any number of parameters).

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