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# SAMPLE SIZE AND TOLERANCE LIMITS

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**SUMMARY**

Several new criteria are proposed for the determination of suitable sample size for assessing the statistical tolerance limits. The application of the criteria is illustrated on the solution of some problems from the theory of errors and theory of reliability.

**RESUMEN**

En este trabajo se proponen algunos nuevos criterios que tratan de fijar el tamaño de la muestra necesario para establecer los límites estadísticos de tolerancia. Como aplicación de estos criterios se resuelven algunos problemas de la teoría de los errores y de la teoría de la confiabilidad.

## 1. Introduction

Fourty years ago Wilks' treatise "Determination of sample sizes for setting tolerance limits" (Wilks 1941) provided an impetus for the development of the theory of statistical tolerance regions; since then one of the basic problems has been the sample size necessary for the determination of the regions.

The main problem in the case of Wilks' distribution-free tolerance regions is the determination of a sample size fulfilling a basic requirement, viz. that an appropriate probability statement holds for the tolerance regions. Wilks' solution was complemented by approximative formulas (Tukey and Scheffé 1944) and its application is facilitated by numerous tables (e.g. Somerville 1958, Jílek and Líkař 1960, Owen 1962, Harman 1967, Müller, Neumann and Storm 1973, Likeš and Laga 1978), graphs and graphical methods (Murphy 1948, Birnbaum and Zuckerman 1949).

For families of probability distributions with a known distribution form (e.g. when a random variable is known to be distributed normally with one or two parameters unknown), the statistical tolerance regions may be constructed usually even with very small samples; it should be borne in mind, however, that with small samples the constructed tolerance regions are too "large" and thus unsuitable for practical for normal distribution with both parameters unknown (see, e.g., Owen 1962, Odeh 1978, Likeš and Laga 1978).

The selection of a sample size for the determination of tolerance regions in the case of a known distribution form (with some unknown parameters) has so far been studied only sparsely (Albert and Johnson 1951, Faulkenberry and Weeks 1968, Faulkenberry and Daly 1970, Guenther 1972, Passi and Williams 1978) and the results are not widely known; statistical textbooks and handbooks usually make no mention of these problems.

We propose several new criteria for the determination of sample size with regard to the purpose to which the tolerance regions are to serve. We shall limit our study to one-dimensional continuous distributions; the tolerance regions will then be replaced by tolerance intervals bounded by tolerance limits. If the tolerance interval is bounded by tolerance limits from above as well as from below, we speak of two-sided tolerance limits while if it is bounded from one side only we speak of a lower or upper one-sided tolerance limit.

The extension of these considerations to multidimensional random variables is easy.

The use of the criteria is illustrated on two examples.

## 2. Tolerance regions

Let $\Theta$ be a non-empty set (called parameter space), let $\mathbf{F} = \{F(.; \theta), \theta \in \Theta\}$ be a family of probability distribution functions on $R^1$, and let $(X_1, ..., X_n)$ be a random sample from any distribution $F \in \mathbf{F}$, where $n$ is called a sample size.

Then a statistic $T$ defined over $R^n$ and taking values in the $\sigma$-algebra of subsets of $R^1$ will be called a (statistical) tolerance region; particularly, if $T(x_1, ..., x_n)$ is a (finite or infinite) interval (or empty set) for every $(x_1, ..., x_n) \in R^n$, then $T$ is called a tolerance interval, and statistics $L$ or/and $U$ bounding the tolerance interval from below or/and from above are called lower or/and upper tolerance limits, respectively. The statistic $W_T$ defined by

$$W_T(X_1, ..., X_n) = \int_{T(X_1, ..., X_n)} dF(x; \theta)$$

will be called a (probability) covering of the tolerance region $T$.

Let $\beta$ and $\gamma$ be two real numbers $(0 < \beta, \gamma < 1)$; (A) if

$$Pr\ \{(X_1, ..., X_n) : W_T(X_1, ..., X_n) \geqslant \beta | \theta\} = \gamma \tag{1}$$

and if it does not depend on $\theta$, then $T$ is called a $\beta$-content tolerance region at confidence level $\gamma$; (B) if

$$E\ \{W_T(X_1, ..., X_n)\} = \beta \tag{2}$$

(where $E$ denotes the expectation operator with respect to $(X_1, ..., X_n)$), and if it does not depend on $\theta$, then we call the statistic $T$ a $\beta$-expectation tolerance region.

(For more detailed discussion of theoretical background of statistical tolerance regions see, e.g., Fraser and Guttman (1956), or Guttman (1970); a survey of many results of practical importance was given by Guenther (1972).)

## 3. Sample size for determination of tolerance limits

As stated in Introduction, tolerance limits can be often determined even from very small samples but in these cases they may be worthless for practical use. We thus have to consider what to expect from tolerance limits apart from their meeting the basic demands (1) or (2). These additional requirements should be intuitively clearly justified (they should concern, e.g., the variability of tolerance limits or variability of covering, length of tolerance interval, etc.).

The two solutions published so far are based on the variability of probability covering of the tolerance interval:

1) Albert and Johnson (1951) proposed the following interesting idea: $\beta$-expectation tolerance limits give no guarantee of how widely will the values of the probability coveri rg in individual cases differ from the required value of $\beta$. However, we may set a complementary requirement, viz. that for suitably selected numbers $d_1$, $d_2$ and $\gamma$ ($0 \leqslant d_1 \leqslant \beta$, $0 \leqslant d_2 \leqslant 1 - \beta$, $0 < \gamma < 1$) it should hold that

$$Pr\,\{\beta - d_1 \leqslant W_T \leqslant \beta + d_2\} \geqslant \gamma \tag{3}$$

With simultaneous requirements (2) and (3) the original task is modified and the tolerance interval sought is subject simultaneously to requirements $A$ and $B$ (formula (3) is obviosly only a slight modification of formula (1)). The simultaneous fulfilment of (2) and (3) may be guaranteed only for sample sizes larger than a certain value. Albert and Johnson (1951) showed this on the example of two-sided tolerance limits of a normal distribution with both parameters unknown and they presented a short table of minimal $n$ for given $\beta$, $\gamma$, $d_1$ and $d_2$.

2) A similar notion may be used in the construction of $\beta$-content tolerance limits (Faulkenberry and Weeks 1968, Faulkenberry and Daly 1970, Guenther 1972): Let us select four real numbers $\beta$, $\beta_1$, $\gamma$, $\gamma_1$ such that $0 < \beta, \beta_1, \gamma, \gamma_1 < 1$, $\beta < \beta_1$ and let us further ask that (1) and simultaneosly

$$Pr\,\{W_T \geqslant \beta_1\} \leqslant \gamma_1 \tag{4}$$

should hold.

The meaning of this additional assumption is intuitively clear: (1) expresses the basic requirement for $\beta$-content tolerance limits at level $\gamma$ whereas (4) ensures that the tolerance intervals should not be excessively "large". In this case we can also look for the least $n$ for which both requirements will be fulfilled. A short table for both one-sided and two-sided tolerance limits for normal distribution with both parameters unknown is given by Faulkenberry and Daly (1970).

## 4. Some new criteria

The above criteria proposed by Albert and Johnson (1951) and others have a sound theoretical basis, are elegant and lead mostly to tasks that are readily solved (cf. Guenther 1972). However, in some cases they do not correspond exactly to the intuitive requirements of the users; e.g. when using statistical tolerance limits for solving some problems associated with precision of measurement methods or instruments (Jílek 1980, 1981), we shall want to know how wide the variability of the tolerance limit may be, etc.

For this reason we propose here some new criteria based on the variability of tolerance limits and length of tolerance intervals.

We shall use the following notation:

TL ............. tolerance limit

TI ............. tolerance interval

$\mathcal{L}$(TI) ........... length of the tolerance interval (i.e. its Lebesgue measure).

asTL, asTI ..... asymptotic tolerance limit, asymptotic tolerance interval.

1) One of the possible requirements of tolerance intervals is a limited variability of the tolerance interval length (in the case of two-sided tolerance limits) or limited variability of tolerance limits (in the case of one-and two-sided tolerance limits): Let $\delta > 0$, and let us require that, in addition to (1) or (2), it should hold that:

$$\frac{\{\mathrm{Var}\,(\mathcal{L}(\mathrm{TI}))\}^{1/2}}{E\,(\mathcal{L}(\mathrm{TI}))} \leqslant \delta \qquad (5)$$

or

$$\frac{\{\mathrm{Var}\,(\mathrm{TL})\}^{1/2}}{E\,(\mathrm{TL})} \leqslant \delta \tag{6}$$

respectively.

2) In other cases we can require that the tolerance interval length variability or tolerance limit variability should be small as compared with the variability of the observed random variable: Let $\delta > 0$, and let us require that

$$\{\mathrm{Var}\,(\mathcal{L}\,(\mathrm{TI}))/\mathrm{Var}\,(X)\}^{1/2} \leqslant \delta \tag{7}$$

or

$$\{\mathrm{Var}\,(\mathrm{TL})/\mathrm{Var}\,(X)\}^{1/2} \leqslant \delta \tag{8}$$

respectively.

3) Tolerance limits tend in probability to appropriate quantiles of the probability distribution of a random variable $X$ when $n$ tends to infinity. We may thus require that the tolerance limits should be in a sense close enough to these asymptotic values, or that the length of the tolerance intervals should not differ too much from the length of the asymptotic tolerance interval, e.g. that for given $\delta > 0$ it should hold that

$$E\left\{\left|\frac{\mathcal{L}\,(\mathrm{TI}) - \mathcal{L}\,(\mathrm{asTI})}{\mathcal{L}\,(\mathrm{asTI})}\right|\right\} \leqslant \delta \tag{9}$$

or

$$E\left\{\left|\frac{\mathrm{TL} - \mathrm{asTL}}{\mathrm{asTL}}\right|\right\} \leqslant \delta \tag{10}$$

or that for given $\delta > 0$ and given $\epsilon$ $(0 < \epsilon < 1)$ it should hold that

$$Pr\left\{\left|\frac{\mathcal{L}(\mathrm{TI})-\mathcal{L}(\mathrm{asTI})}{\mathcal{L}(\mathrm{asTI})}\right|\leqslant\delta\right\}\geqslant\epsilon \tag{11}$$

or

$$Pr\left\{\left|\frac{\mathrm{TL}-\mathrm{asTL}}{\mathrm{asTL}}\right|\leqslant\delta\right\}\geqslant\epsilon \tag{12}$$

Note 1. The proposed criteria (9) - (12) may be replaced by their one-sided variants if we do not consider the absolute value of the ratios ($\mathcal{L}$(TI) – $\mathcal{L}$(asTI))/$\mathcal{L}$(TI) and (TL – asTL)/asTL; cf. Example 2.

Note 2. It may happen that for some distribution family some of the criteria will not be useful: thus criteria (5) and (6) are not suitable for normal distributions with one or both parameters unknown (except the case $\mu = 0$) since they are dependent on these parameters. These criteria are applicable, however, for gamma distributions.

## 5. Examples

*Example 1. Measurement precision*

The theory of measurement usually assumes that the results of the measurement $X$'s are independent and are normally distributed with mean value $\mu$ and variance $\sigma^2$ (one or both parameters are unknown). Thus the absolute errors $\Delta$'s,

$$\Delta = X - \mu\,,$$

are normally distributed with zero mean and variance $\sigma^2$.

For assessing the measurement precision, the upper two-sided $\beta$-content tolerance limit at confidence level $\gamma$ for absolute error $\Delta$ has been proposed (Jílek 1980, 1981). This tolerance limit is equal to $ks$, where

$$k = u\,((1+\beta)/2)\left\{\frac{n-1}{\chi^2_{n-1}\,(1-\gamma)}\right\}^{1/2}$$

$$s = \left\{ \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right\}^{1/2}$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

and $n$, the size of the random sample which served as a basis for the calculation of the tolerance limit, is greater than 1; $u(p)$ (where $0 < p < 1$) denotes the $p$-quantile of the distribution $N(0, 1)$, and $\chi_f^2(p)$ is the $p$-quantile of the $\chi^2$-distribution with $f$ degree of freedom.

Determination, or more precisely estimate, of the measure of measurement precision is very important and should thus be performed with the highest possible precision. The tolerance limits of absolute errors are random variables, have their own variability, and we can thus naturally require that this variability should not be too great. A sound demand of the precision measure is that it should not differ too much from the asymptotic value which is in this case $u((1 + \beta)/2)\,\sigma$. A suitable criterion to be used is thus (10) or (12).

The assumption of normal distribution of absolute errors implies that (1) and (10) hold simultaneously only for those $n$ (Jílek and Burianová 1982) for which the following inequality holds

$$R_{n-1} \left\{ \frac{n-1}{\chi_{n-1}^2 (1-\gamma)} \right\}^{1/2} [1 - G_n (\chi_{n-1}^2 (1-\gamma))] \leqslant \delta + 2\gamma - 1 \qquad (13)$$

where

$$R_f = (2/f)^{1/2}\, \Gamma\,[(f+1)/2] \,/\, \Gamma\,(f/2)$$

and $G_f$ denotes the distribution function of the $\chi^2$-distribution with $f$ degrees of freedom. This criterion is obviosly independent on $\beta$. For conventionally used values of $\gamma = 0.90$, 0.95 and 0.99 and for $\delta = 0.10$ and 0.20, the minimum $n$ fulfilling (13) are shown in Tab. 1.

If (1) is to hold simultaneously with (12), we may easily see (Jílek and Burianová 1982) that the following condition has to be fulfilled:

$$G_{n-1}\left((1+\delta)^2 \chi^2_{n-1}(1-\gamma)\right) - G_{n-1}\left((1-\delta)^2 \chi^2_{n-1}(1-\gamma)\right) \geqslant \epsilon. \quad (14)$$

(This inequality is also independent on $\beta$). This can be achieved in many ways; a sound approach seems to be to select a number $\epsilon_0$ such that $0 < \epsilon_0 < 1 - \epsilon$ and to require that the following inequalities should hold simultaneously

$$G_{n-1}\left((1+\delta)^2 \chi^2_{n-1}(1-\gamma)\right) \geqslant \epsilon + \epsilon_0,$$

$$G_{n-1}\left((1-\delta)^2 \chi^2_{n-1}(1-\gamma)\right) \leqslant \epsilon_0;$$

this yields an inequality

$$[\chi^2_{n-1}(\epsilon + \epsilon_0) - \chi^2_{n-1}(\epsilon_0)] / \chi^2_{n-1}(1-\gamma) \leqslant 4\,\delta.$$

Minimal $n$ satisfying this inequality are given in Tab. 2 for $\gamma$ and $\delta$ the same as in Tab. 1, for $\epsilon = 0.90$ and $0.95$, and for $\epsilon_0 = (1-\epsilon)/2$.

**TABLE 1**
**Minimum $n$ fulfilling (13)**

| $\delta$ | 0.10 | | | 0.20 | | |
|---|---|---|---|---|---|---|
| $\gamma$ | 0.90 | 0.95 | 0.99 | 0.90 | 0.95 | 0.99 |
| $n$ | 113 | 170 | 322 | 34 | 51 | 95 |

**TABLE 2**
**Minimum $n$ fulfilling (15)**

| $\delta$ | 0.10 | | | 0.20 | | |
|---|---|---|---|---|---|---|
| $\epsilon$ \ $\gamma$ | 0.90 | 0.95 | 0.99 | 0.90 | 0.95 | 0.99 |
| 0.90 | 181 | 194 | 218 | 58 | 65 | 78 |
| 0.95 | 244 | 261 | 289 | 77 | 84 | 100 |

*Example 2. Reliability*

Studies concerned with the reliability of products or technological devices pose often the question of the lenght of time for which the product or device will serve satisfactorily their purpose. When working with laboratory animals, we are interested in their life-span and very often we have to know how long they survive under certain conditions. The life-span or durability of an individual item is determined by the action of a large complex of various factors, for the most part uncontrollable, and has therefore a stochastic character; its mathematical description is often done with the aid of gamma distribution (Mann, Schafer and Singpurwalla 1974, Gross and Clark 1975, etc.) with the probability density

$$f(x) = \begin{cases} [\theta^{r/2}\,\Gamma(r/2)]^{-1}\, x^{r/2-1} \exp(-x/\theta), & x > 0, \\ 0 \text{ otherwise,} \end{cases} \tag{16}$$

where $\theta > 0$, $r > 0$ ($r$ is usually taken to be known).

When enquiring about the lower life-span limit, we usually select a suitable real $\beta$ $(0 < \beta < 1)$ and search for a value above which the life-span of the majority of items (at least $100\,\beta$ %) will lie.

If we know $\theta$, the answer to our query is the $(1 - \beta)$-quantile of distribution (16), $\theta\,\chi^2_r(1 - \beta)/2$.

However, in most cases $\theta$ is unknown and it is then sensible to try to determine, on the basis of experimental data, such a limit above which the life-span of most (at least $100\,\beta$ %) of items will lie with a high confidence (with probability $\gamma$). The solution is the lower one-sided $\beta$-content tolerance limit at confidence level $\gamma$, which is equal to $k\bar{x}$, where (Guenther 1971).

$$k = n\,\chi^2_r\,(1 - \beta)\,/\,\chi^2_{rn}(\gamma) \tag{17}$$

(for optimality of this tolerance limit cf. Guenther 1971).

This formula indicates a strong dependence of the tolerance limit on the sample size $n$, especially for small $r$. This is confirmed, e.g., also by consulting the table of tolerance factors $k$ for exponential distribution, i.e. in the case of $r = 2$ (Epstein 1960).

We naturaly want the tolerance limit to be as high as possible, i.e. we want a garantee (with given probability guarantees $\beta$ and $\gamma$) that the life-span will be sufficiently long.

If we use the requirement of a simultaneous validity of (1) and (4), then we see (Guenther 1972, fórmula (4.4)) that this requirement is fulfilled for those $n$ for which the following inequality holds

$$\chi^2_{rn}(\gamma)\,\chi^2_r(1-\gamma_1) \leqslant \chi^2_{rn}(\gamma_1)\,\chi^2_r(1-\beta)\,. \tag{18}$$

(If we chose, in the case of an exponential distribution, i.e. for $r = 2$, the following values: $\beta = \gamma = 0.95$, $\beta_1 = 0.975$ and $\gamma_1 = 0.10$, then we could see easily from the tables of quantiles of the $\chi^2$-distribution that inequality (18) is fulfilled for $n \geqslant 18$.)

Relatively small samples thus suffice for the simultaneous satisfaction of requirements (1) and (4) at the above probability guarantees. However, a question arises if the criterion satisfies in this case the needs of users of tolerance limits. The users usually wish that the tolerance limits should be fairly close to the asymptotic values, i.e. to the corresponding quantile of the original distribution. However, the mean value of tolerance limits for small samples may in fact be appreciably remote from the asymptotic value – the difference is $100\,(1 - r\,n/\chi^2_{rn}(\gamma))$ % of the asymptotic value, in our case 37.65%. Individual tolerance limits can obviously be still much smaller.

A much more natural approach seems to be to set a suitable number $\delta$ $(0 < \delta < 1)$ and to require that the tolerance limit $k\bar{x}$ should not differ from the asymptotic value by more than $100\,\delta$ % of this value, either in its mean value,

$$E\left\{\frac{\text{asTL} - \text{TL}}{\text{asTL}}\right\} \leqslant \delta\,, \tag{19}$$

or with a probability equal to $\epsilon$,

$$Pr\left\{\frac{\text{asTL} - \text{TL}}{\text{asTL}} \leqslant \delta\right\} \geqslant \epsilon \tag{20}$$

(these are obviously one-sided variants of criteria (10) and (12) from the preceding paragraph).

As we can easily see, the ratio

$$\frac{\text{asTL} - \text{TL}}{\text{asTL}} = 1 - 2\,n\,\bar{x}\,/\,[\theta\, \chi^2_{rn}(\gamma)]$$

i.e. criteria (19) and (20) do not depend on $\beta$.

On using criterion (19) we obtain that

$$E\,\{1 - 2\,n\,\bar{x}\,/\,[\theta\,\chi^2_{rn}(\gamma)]\} = 1 - r\,n\,/\,\chi^2_{rn}(\gamma)\,;$$

hence, for given $\delta$ we look for the least natural $n$ such that

$$r\,n\,/\,\chi^2_{rn}(\gamma) \geqslant 1 - \delta\,. \tag{21}$$

For $\delta = 0.10$ and 0.20, for conventionally used confidence levels $\gamma = 0.90$, 0.95 and 0.95, and for $r = 2(2)10$ we find values of the least $n$ satisfying (21) in Tab. 3.

For criterion (20) we obtain

$$Pr\,\{1 - 2\,n\,\bar{x}\,/\,[\theta\,\chi^2_{rn}(\gamma)] \leqslant \delta\} = 1 - G_{rn}((1 - \delta)\,\chi^2_{rn}(\gamma))\,.$$

**TABLE 3**
**Minimum $n$ fulfilling (21)**

| $\delta$ | 0.10 | | | 0.20 | | |
|---|---|---|---|---|---|---|
| $r$ \ $\gamma$ | 0.90 | 0.95 | 0.99 | 0.90 | 0.95 | 0.99 |
| 2 | 137 | 230 | 465 | 28 | 48 | 98 |
| 4 | 69 | 115 | 233 | 14 | 24 | 49 |
| 6 | 46 | 77 | 155 | 10 | 16 | 33 |
| 8 | 35 | 58 | 117 | 7 | 12 | 25 |
| 10 | 28 | 46 | 93 | 6 | 10 | 20 |

Since we require that the last expression should at least equal $\epsilon$, we obtain finally that criterion (20) is satisfied by $n$ fulfilling the inequality

$$\chi^2_{rn}(1-\epsilon)\,/\,\chi^2_{rn}(\gamma) \geqslant 1-\delta\,. \tag{22}$$

For $\gamma$, $\delta$ and $r$ as in Tab. 3 and for $\epsilon = 0.90$ and 0.95 we find values of least $n$ satisfying (22) in Tab. 4.

Note. The solution of inequalities (21) and (22) is aided by using some tables of quantiles of $\chi^2$-distribution (e.g. Harter 1964, Likeš and Laga 1978) or good approximations of these quantiles. Tables 3 and 4 were set up using the Cornish - Fisher approximation (Goldberg and Levine 1945, Zar 1978) and Kelley's tables of quantiles of normal distribution (Kelley 1948). When using (21) we can also use existing tables of the $\chi^2_f(\gamma)/f$ ratios (Janko 1958) or $(f/\chi^2_f(\gamma))^{1/2}$ (Weissberg and Beatty 1960).

**TABLE 4**

**Minimum $n$ fulfilling (22)**

| $\delta$ | 0.10 | | | 0.20 | | |
|---|---|---|---|---|---|---|
| $r$ \ $\gamma$ | 0.90 | 0.95 | 0.99 | 0.90 | 0.95 | 0.99 |
| | | | $\epsilon$=0.90 | | | |
| 2 | 593 | 769 | 1162 | 133 | 171 | 257 |
| 4 | 297 | 385 | 581 | 67 | 86 | 129 |
| 6 | 198 | 257 | 388 | 45 | 57 | 86 |
| 8 | 149 | 193 | 291 | 34 | 43 | 65 |
| 10 | 119 | 154 | 233 | 27 | 35 | 52 |
| | | | $\epsilon$=0.95 | | | |
| 2 | 776 | 926 | 1413 | 174 | 218 | 314 |
| 4 | 388 | 463 | 707 | 87 | 109 | 159 |
| 6 | 259 | 326 | 471 | 58 | 73 | 105 |
| 8 | 194 | 232 | 354 | 44 | 55 | 79 |
| 10 | 156 | 196 | 283 | 35 | 44 | 63 |

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