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PART DETERIORATION IN STRESS VS. STRENGTH PROBLEM

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Abstract

The paper deals with the stress vs. strength problem incorporating part deterioration. The model developed has been illustrated assuming that the distribution fo strength follows Rayleigh distribution and that of stress follows normal distribution. Part deterioration has been assumed to follow exponential distribution. A particular case of stress distribution viz. stress being an impulse has been derived. Numerical example has also been added at the end.

Introduction

Many reliability models[1] to [6]have been developed for stress vs. strength problem. But only Shooman[6] has considered the effect of part deterioration in his models. However, he has not taken into account the variation in stress while considereing the part deterioration. He considers the case whese stress is an impulse and strength follows Rayleigh distribution.

Keeping this fact in view the author, in this paper, has developed a realibility model for stress vs. strength problem taking into account the variation in stress. Deterioration in part's strength has been assumed to follow exponential distribution and that of strength follows Rayleigh distribution.

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Mathematical Formulation and Solution

2.1. *Definition*

In order to develop the reliability model define:

$Y\ (0 < y < \infty)$ = a continuous random variable representing strength of a component.

$X\ (0 < x < \infty)$ = a continuous random variable representing stress magnitude affecting the operation of the component.

$F\ (y)$ = the probability that the strength of a randomly chosen component is less than y.

$E\ (x)$ = the probability that the magnitude of stress occurred is less than x.

$f\,(y) = \frac{d}{dy}\, F\,(y)$, the probability density function (p.d.f.) of Y.

$e\,(x) = \frac{d}{dx}\, E\,(x)$, the p.d.f. of X.

R = the reliability of a component, i.e. the probability that the component will not fail.

2.2. *Development of Model*

Assume that (a) the random variables X and Y are measured in the same unit, (b) the failure in Y occurs only due to stress and (c) the probability of occurrence of stress X, when the component is under operation, is unity.

Under assumption (c) the conditional probability that the magnitude of stress occurred is x_i, is given by $e\,(x_i)\,dx$. Obviously the probability that the magnitude of stress is less than x_i is given by

$$E\,(x_i) = \int_0^{x_i} e\,(x)\,dx \tag{1}$$

Similarly the probability that a randomly chsen component has strength greater than x_i is given by

$$1 - F\,(x_i) = \int_{x_i}^{\infty} f\,(y)\,dy \tag{2}$$

The product of relations (1) & (2) gives the success probability (R) of the component under consideration. Thus

$$R = \int_0^{x_i} e(x) \int_{x_i}^{\infty} f(y)\, dx\ dy \tag{3}$$

Now allowing x_i to vary over the whole range of x we have

$$R = \int_0^{\infty} e(x) \int_x^{\infty} f(y)\, dx\ dy \tag{4}$$

Model (4) gives the reliability of the component when part deterioration is not present. However, in situation where component's strength is subjected to deterioration due to storage or some other reasons, the above model does not fit well.

Since the decrease in component's strength reduces the mean strength, more and more area under the curve $f(y)$ slides down towards the left and thus affects the realibility of the component. In such cases, following Shooman[6], the strength parameters can be represented as a function of time. Thus, if $K(t)$ stands for strength parameters, expression (4) can be rewritten as

$$R = \int_0^{\infty} e(x) \int_x^{\infty} f(y\, K(t))\ dx\ dy \tag{5}$$

The evaluation of model (4) & (5) depends upon (a) the distribution of stress, (b) the distribution of strength and (c) the type of growth of $K(t)$. If we assume that (a) stess (x) follows normal distribution with mean μ and variance σ^2, (b) strength (y) follows Rayleigh distribution with parameter $K(t)$ and (c) $K(t)$ has an exponential growth given by, $K(t) = k_1\, c^{k_2 t}$, then expression (5) can be written as

$$R = \int_0^{\infty} \frac{1}{\sqrt{(2\pi)}\,\sigma} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\} \int_x^{\infty} K_1 e^{K_2 t} y \ \exp \cdot$$

$$\cdot \left\{-\frac{1}{2} K_1 e^{K_2 t} y^2\right\} dx\, dy \quad k_1 \geqslant 0,\ k_2 \geqslant 0$$

$$= \exp\left\{-0.5\, k_1 e^{k_2 t} \mu^2 / A_1(t)\right\} A_2(t) \left\{1 - \phi\,(A_3(t))\right\}, \tag{6}$$

Where,

$$A_1(t) = 1 + K_1 e^{K_2 t} \sigma^2, \quad A_2(t) = \{A_1(t)\}^{-1/2}, \quad A_3(t) = -\frac{\mu}{\sigma} A_2(t)$$

and

$$\phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{(2\pi)}} e^{-1/2\, y^2}\, dy$$

Usually, in practice, $Pr(-\infty < x < 0)$ is negligible. However, if it is not so, then expression (6) must be divided by $\phi(\mu/\sigma)$ to ensure the total area under normal distribution to be unity.

2.3. *Particular Case*

Assuming stress to be an impulse with magnitude x_1, Shooman[6] obtained

$$R = \exp\left(-0.5\, k_1 e^{k_2 t} x_1^2\right) \tag{7}$$

This result can be obtained by setting $\mu = x_1$ and $\sigma = 0$ in (6).

2.4. *Numerical Example*

Model (6) has been illustrated numerically assigning hypothetical values to the constants involved. The values of R, thus obtained, have been tabulated in table 1.

Table 1

Values of R for different values of k_1, k_2 and t when $\mu=3$ & $\sigma=0.5$

$k \rightarrow$	$k_1=0.01$,	$k_2=0.015$			$k_1=k_2=0.015$			
Time $t \rightarrow$	0	4	8	12	0	4	8	12
$K(t)$	0.010	0.0106	0.0113	0.0120	0.015	0.0159	0.0169	0.0180
R	0.9549	0.9522	0.9502	0.9472	0.9332	0.9293	0.9263	0.9219

It is clear from table 1 that reliability decreases with increase in time.

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