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A STOCHASTIC MODEL OF CHOICE *

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ABSTRACT

An approach to choice function theory is suggested which is probabilistic and non-deterministic. In the framework of this approach fuzzy choice functions introduced and a number of necessary and sufficient conditions on a fuzzy choice function to be a fuzzy rational choice function of a certain type are established.

1. Introduction.

In general, choice function theory considers the following model (see, for example, [1]). Let A be a fixed finite set of alternatives. For each nonempty subset $X \subseteq A$ a nonempty subset $Y \subseteq X$ is chosen in accordance with some rule. In such a manner a choice function $Y = C(X)$ is given, which associates with each $X \subseteq A$ its subset $Y \subseteq X$. There are two different methods to describe the "entire choice" defined in this way. The first one points out a mechanism of choice whereby part Y is found from X. This method can be called an internal method. The second method

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indicates the set of all pairs (X,Y) and is called an external method.

Mostly, classical choice mechanisms are "pair-dominant" ones. This means that the choice of element $y \in X$ is made as a result of comparisons of this element with all $x \in X$. Some given structure on the set A is utilized to make these comparisons, for instance, a binary relation. The choice function thus arised has various attractive propertires. One of the main problems in choice theory is a description of characteristic properties of choice functions. These properties, known as "choice axioms", separate the functions which have an equivalent description in pair-dominance optimization terms.

A framework of the choice function theory just described is an algebraic or, better said, a non-probabilistic one. Moreover, it is non-deterministic in the sense that a subset C(X) chosen from X is assumed to be any subset, not necessarily a singleton.

There is another approach to choice theory which may be described as follows. Let P(x,X) be the probability of choosing an element x from a set X. It is supposed that P(x,X)>0, and

$$\sum_{x \in X} P(x,X) = 1,$$

i.e. P(x,X) defines a probability distribution on X. In practice, the choice probabilities P(x,X) are estimated by the relative frequences of choosing x. It is important to note that on each trial one and only one alternative is chosen; this means deterministic character of choice mechanism involved. A good deal of research has been done in the area of probabilistic choice theory; the reader is referred to Luce's survey [5] for further details and references.

In this paper an approach to choice theory is suggested which is probabilistic and non-deterministic. Empirically speaking, the non-deterministic character of this approach means that the relative frequencies of choosing x from X are not already estimations of a probability distribution on X. Let us consider, for example, the following data, where 1 indicates that a given alternative is chosen in a given trial, and 0 indicates that an alternative is rejected.

	Alternative				
Trial	1	2	3	4	5
1	0	0	1	1	0
2	0	1	0	1	0
3	0	0	1	1	0
Relative Frequency	0	1/3	2/3	1	0

Obviously, any data of this kind is not consistent with the general probabilistic model described above: the relative frequencies do not from a probability distribution on X. Nevertheless, we still have a nice probabilistic interpretation of these frequencies: they are estimations of probabilities $P(x \in \underline{X})$ where $\underline{X}$ is a certain random subset of X. Since there is a one-to-one correspondence between classes of random subsets and fuzzy sets (see [6] and [4] for details), one may regard relative frequencies obtained from non-deterministic choice experiments as estimations of membership function values of a fuzzy subset of X.

Generalizing these observations, we study fuzzy choice functions in this paper; these functions assign to each fuzzy set X a fuzzy subset $C_X \subseteq X$. Note again that a membership function is estimated by relative frequencies obtained by averaging experimental data in non-deterministic choice experiments (for nonfuzzy X's). Hence, fuzzy set theory is used here as a mathematical tool providing consistent representation of empirical data in a stochastic model of choice.

2. Best variant choice.

It was mentioned above that there are two alternative approaches to the algebraic theory of choice. The first one is concerned with a mechanism of choice. In this paper only mechanisms based upon binary relations are considered. Let R be a binary relation on the set A. We read xRy as "x us preferred to or indifferent with y", i.e. R is a preference relation. A choice function based upon R is defined by

$$Y = C(X) = \{x \in X: xRy \text{ for all } y \in X\}. \qquad (1)$$

This mechanism is based on comparisons in pairs of variants (alternatives). Such "pair-dominance" mechanisms can be regarded as abstract forms of classical optimization mechanisms based on scalar and vector criteria. Various types of binary relations, such as partial orderings, weak orderings, etc., define, by (1), classes of choice functions possessing specific "rational" properties.

An alternative approach to choice theory considers "characteristic properties" of choice functions and the main problem is to describe combinations of characteristic properties which separate exactly the same classes as given by pair-dominant choice mechanisms.

Following [1] we define main characteristic properties as follows:

1. <u>Heritage</u> (H): if $X' \subseteq X$, the
   $C(X') \supseteq C(X)\ X'$.
2. <u>Strict heritage</u> (K): if $X' \subseteq X$, and $X' \cap C(X) \neq, \emptyset$,
   then $C(X') = c(X) \cap X'$.
3. <u>Concordance</u> (C): if $X = X' \cup X''$, then
   $C(X) \supseteq C(X') \cap C(X'')$.

4. Independence (I): if $C(X) \subseteq X' \subseteq X$, then then $C(X') = C(X)$.

Most works concerned with choice theory unanimously declare these properties as clearly representing the idea of what is "better".

The following proposition represents main classical statements on correspondence between pair-dominant mechanisms and choice functions.

Proposition 1. [1] For a choice function to be generated by choice mechanism (I) of i) an arbitrary binary relation, ii) a weak ordering, and iii) a quasi-transitive binary relation it is necessary and sufficient that it satisfies a respective condition i) H&C, ii) K, and iii) H&C&O.

This proposition is extended on fuzzy choice function theory in this paper.

3. Fuzzy preferences.

Recall that a fuzzy binary relation R on a set A is a fuzzy set with universe AxA and is defined by its membership function $R(x,y)$.

Definition 1. A fuzzy binary relation R is said to be

reflexive if $R(x,x) = 1$ for all $x \in A$;

antireflexive if $R(x,x) = 0$ for all $x \in A$;

symmetric if $R(x,y) = R(y,x)$ for all $x,y \in A$;

antisymmetric if $R(x,y)>0$ implies $R(y,x) = 0$ for all $x \neq y$;

complete if $R(x,y) = 0$ implies $R(y,x)>0$ for all $x,y \in A$;

acyclic if $R(x_i,x_{i+1})>0$ for $i=1,\dots,k-1$, implies $R(x_k,x_1)=0$ for any sequence $x_1,\dots,x_k$.

<u>transitive</u> if $R(x,y)>0$ and $R(y,z)>0$ imply $R(x,z)>0$ for all $x,y,z \in A$.

It should be noted that our definition of transitivity is different from standard ones (see [9]). Fuzzy set theory admits various definitions of transitivity [3]. Transitivity as defined above may be regarded as a weak one.

The notion of a strict preference plays a significant role in choice theory.

<u>Definition 2.</u> Let R be a fuzzy binary relation (a preference). A fuzzy binary relation P_R defined by

$$P_R(x,y) = \begin{cases} R(x,y), & \text{if } R(y,x) = 0. \\ 0, & \text{otherwise} \end{cases}$$

is said to be a <u>strict preference.</u>

In choice theory aP_Rb if and only if <u>not</u> bRa, i.e. $P_R = R \cap \overline{R^{-1}}$ where $\overline{R^{-1}}$ is a complement of the converse relation R^{-1} considered as a subset of AxA. Definition 2 is based upon the intuitionistic negation [8] for which we have

$$P_R = R \cap \overline{R^{-1}} .$$

<u>Definition 3.</u> A fuzzy binary relation is said to be

1) a <u>partiel ordering</u> if it is reflexive, antisymmetric, and transitive;
2) a <u>chain</u> if it is a complete partial ordering;
3) an <u>ordering</u> if it is reflexive, complete, and transitive;
4) a <u>quasi-transitive relation</u> if it is reflexive and complete, and P_R is a transitive relation.

The following structural properties of fuzzy binary relations are used in the paper:

Proposition 2. Let R be an ordering. Then

1) R is a quasi-transitive relation, and

2) $P_R(x,y) > 0$ and $R(y,z) > 0$ imply $P_R(x,z) > 0$, and
$R(x,y) > 0$ and $P_R(y,z) > 0$ imply $P_R(x,z) > 0$.

We omit proofs of these statements which are similar to the crisp ones.

4. Fuzzy pair-dominant choice functions.

The following definition is an immediate extension of definition (1) and is in accordande with a general approach to a fuzzy decision-making developed by Bellman and Zadeh in [2].

Definition 4. A pair-dominant choice function based on a fuzzy binary relation R is a mapping which, to each fuzzy set X, assigns a fuzzy subset with a membership function

$$C_X^R(x) = \bigwedge_{y \in X} R(x,y) \wedge X(x) \tag{2}$$

(We write $y \in X$ iff $X(y) > 0$.)

One can also compare (2) with the definition of a fuzzy upper bound due to Zadeh [9]. Note that if R and X are crisp sets then (2) is equivalent to (1).

Lemma 1-3 establish general properties of fuzzy pair-dominant mechanisms. We define the carrier carX of X by carX = $\{x \in A\colon X(x) > 0\}$.

Lemma 1. $C_X^R \subseteq C_{CarX}^R$.

Proof follows immediately from (2).

Lemma 2. C_X^R satisfies the heritage property (H):

$$X' \subseteq X \text{ implies } C_{X'}^R \supseteq C_X^R \cap X'.$$

Proof. $C_{X'}^R(x) = \bigwedge_{y \in X'} R(x,y) \wedge X(x) \geq$

$$\bigwedge_{y \in X} R(x,y) \wedge X'(x) \wedge X(x) = C_X^R(x) \wedge X'(x).$$

Lemma 3. C_X^R satisfies the concordance property (C):

$$C_{X \cup Y}^R \supseteq C_X^R \cap C_Y^R.$$

Proof. $C_X^R(x) \wedge C_Y^R(x) =$

$$[\bigwedge_{u \in X} R(x,u) \wedge X(x)] \wedge [\bigwedge_{v \in Y} R(x,v) \wedge Y(x)] =$$

$$\bigwedge_{y \in X \cup Y} R(x,y) \wedge X(x) \wedge Y(x) \leqslant$$

$$\bigwedge_{y \in X \cup Y} R(x,y) \wedge (X(x) \vee Y(x)) = C_{X \cup Y}^R(x).$$

Two last lemmas show that a fuzzy pair-dominant choice function satisfies the same properties as a crisp one (cf. Proposition 1). In addition to these properties, it satisfies very important property (3) which has no crisp analog. The role of this property will be clarified in the last section of the paper. We only note here that (3) has a quite clear interpretation: if x is chosen from a fuzzy set X then it should be chosen from the carrier of X and the degree of its belongness to C_X does not exceed that to C_{carX}.

Lemma 4 gives some properties of fuzzy pair-dominant choice mechanisms following from main properties of fuzzy preferences.

Lemma 4. 1) Let R be a reflexive relation. Then

$$C^R_{\{x\}} = \{x\}. \tag{4}$$

2) Let R be a reflexive and complete relation. Then

$$C^R_{\{x,y\}}(u) = \begin{cases} R(x,y), & \text{if } u = x, \\ R(y,x), & \text{if } u = y, \\ 0, & \text{otherwise.} \end{cases}$$

3) Let R be reflexive, complete relation and P_R an acyclic relation. Then

$$C^R_X \neq \emptyset \text{ for any } X \neq \emptyset. \tag{5}$$

Proof. 1) and 2) follow immediately from (2). 3) The proof is quite similar to the crisp one.

Corollary. $C^R_X \neq \emptyset$ for aby nonempty X if R is an ordering or quasi-transitive relation.

The main properties of fuzzy pair-dominant choice functions based on particular types of fuzzy binary relations are given below.

Lemma 5. Let R be an ordering. Then $X' \subseteq X$ and $C^R_X \cap X' \neq \emptyset$ imply together

$$\text{car}C^R_{X'} = \text{car}C^R_X \cap \text{car}X'. \tag{6}$$

Proof. By lemma 2 it is sufficient to prove that

$$\text{car}C^R_{X'} \subseteq \text{car}C^R_X \cap \text{car}X'.$$

Let $x \in \text{car}C^R_{X'}$ and $x \notin \text{car}C^R_X \cap \text{car}X'$, i.e. $x \notin \text{car}C^R_X$.
Then there is y such that $R(x,y)=0$, i.e. $P_R(y,x)>0$. On the other hand, x belongs to $\text{car}C^R_{X'}$, which implies $y \in \text{car}X \setminus \text{car}X'$. Since

$C_X^R \cap X' \neq \emptyset$ there is z such that $z \in \text{car}X'$ and $z \in \text{car}C_X^R$. Hence, $R(x,z) > 0$. By Proposition 2, we have $P_R(y,z) > 0$, which implies $R(z,y) = 0$. But $z \in \text{car}C_X^R$ which implies $R(z,y) > 0$. This contradiction completes the proof.

<u>Lemma 6.</u> Let R be a quasi-transitive relation. Then

$$C_X \subseteq X' \subseteq X \text{ implies } \text{car}C_{X'}^R = \text{var}C_X^R. \qquad (7)$$

<u>Proof.</u> By lemma 2 it is sufficient to prove that $\text{car}C_{X'}^R \subseteq \text{car}C_X^R$. Let us suppose that $x \in \text{car}C_{X'}^R$ and $x \notin \text{car}C_X^R$. Then there is y_1 in X such that $P_R(y_1,x) > 0$. We have $y_1 \notin \text{car}C_{X'}^R$ implying $R(x,y) > 0$ for all $y \in X'$. Since $y_1 \in \text{car}C_X^R$ there is y_2 such that $P_R(y_2,y_1) > 0$, which implies $P_R(y_2,x) > 0$, by transitivity of P_R. If y_2 does not belong to $C_{X'}^R$, then there is y_3 different from y_1 and y_2 such that $P(y_3,x) > 0$ and so on. By finiteness of A we find y such that $P_R(y,x) > 0$ and $y \in \text{car}C_X^R$. But $x \in \text{car}C_{X'}^R$ and $y \in \text{car}X'$, i.e. $R(x,y) > 0$ which contradicts $P_R(y,x) > 0$.

Note that any fuzzy pair-dominant choice function fulfills properties (H) and (C). On the other hand, properties (6) and (7) are weaker than (K) and (O), respectively, although for crisp sets and preferences they coincide. Simple examples demonstrate that there are fuzzy orderings and quasi-transitive relations which do not satisfy (K) and (O). It is mentioned in [7], that this fact is stipulated by pair-dominance choice function structure (2), "since this function considers not only the ties between alternatives but also their 'power'. Having excluded certain alternatives from consideration, we have naturally increased the degree of membership to the fuzzy set C_X for other alternatives."

5. Characterizations of fuzzy choice functions.

In this section characteristic properties of fuzzy choice functions are introduced. Various conjunctions of these properties define classes of choice functions based upon pair-dominant choice mechanisms.

Definition 5. The following properties of fuzzy choice functions are said to be characteristic properties:

1) Boundedness (B): $C_X \subseteq C_{carX}$;

2) Heritage (H): $X' \subseteq X$ implies $C_{X'} \supseteq C_X \cap X'$;

3) Concordance (C): $X = X' \cup X''$ implies $C_X \supseteq C_{X'} \cap C_{X''}$;

4) Fuzzy strict heritage (FK): if $X' \subseteq X$ and $C_X \cap X' \neq \emptyset$, then $carC_{X'} = carC_X \cap carX'$;

5) Fuzzy independence (FO): $C_X \subseteq X' \subseteq X$ implies $carC_{X'} = carC_X$;

6) Singleton law (S): $C_{\{x\}} = \{x\}$;

7) Nonvoidness (N): $X \neq \emptyset$ implies $C_X \neq \emptyset$.

Note that properties (B), (K), (O), (S), and (N) are the same as (3), (6), (7), (4), and (5), respectively.

Lemma 7. Conjunction (B) & (H) implies

$$C_X = C_{carX} \cap X \tag{8}$$

Proof. We have $C_X \supseteq C_{carX} \cap X$, by (H), and $C_X \subseteq C_{carX} \cap X$, by (B

Theorem 1. A fuzzy choice function C_X is a fuzzy pair-dominant choice function for some fuzzy prefrence R iff C_X satisfies properties (B), (H) and (C).

Proof. Necessity follows from lemmas 1-3. To prove sufficiency let us define R by $R(x,y) = C_{\{x,y\}}(x)$.
By (H) and (8), $X \cap \{x,y\} \subseteq X$ implies

$$C_X \cap X \cap \{x,y\} \subseteq C_{X \cap \{x,y\}} = C_{\{x,y\}} \cap X \cap \{x,y\}$$

or $C_X \cap \{x,y\} \subseteq C_{\{x,y\}} \cap X$.
Hence,

$$C_X(x) < C_{\{x,y\}}(x) \wedge X(x),$$

which implies

$$C_X(x) \leqslant \bigwedge_{y \in X} C_{\{x,y\}}(x) \wedge X(x) = \bigwedge_{y \in X} R(x,y) \wedge X(x) = C^R_X(x).$$

On the other hand,

$$X = \bigcup_{y \in X} [X \cap \{x,y\}],$$

which implies, by (8) and (C),

$$C_X \supseteq \bigcap_{y \in X} C_{X \cap \{x,y\}} = \bigcap_{y \in X} [C_{\{x,y\}} \cap \{x,y\} \cap X]$$

or

$$C_X(x) \geqslant \bigwedge_{y \in X} [C_{\{x,y\}}(x) \wedge X(x)] = C_X(x).$$

One can compare the statement of this theorem with statement (1) of proposition 1.

By theorem 1 the mapping $F: R \to C^R_X$ is a surjection of the set of all fuzzy binary relations onto the set of all fuzzy choice functions satisfying properties (B), (H) and (C). This mapping is not a bijection since there are binary relations with the same image under F. Let us define $R_1 \sim R_2$ is and only if $C^{R_1}_X = C^{R_2}_X$. Then $\sim$ is an equivalence relation on the set of all fuzzy preferences. Each fuzzy choice function, satisfying (B), (H) and (C), is an image of some class of the relation $\sim$ under the mapping F. Theorem 2 describes all fuzzy preferences $R \in F^{-1}(C_X)$ for a given C_X.

Theorem 2. Let us define R_c for any given R by

$$R_c(x,y) = R(x,y) \wedge R(x,x).$$

Then $R \sim R_c$, and $R' \sim R''$ iff $R_c' = R_c''$.

Proof. We have

$$C_X^{R_c}(x) = \bigwedge_{y \in X} [R_c(x,y) \wedge X(x)] =$$

$$\bigwedge_{y \in X} [R(x,x) \wedge R(x,y) \wedge X(x)] = \bigwedge_{y \in X} [R(x,y) \wedge X(x)] = C_X^R(x),$$

i.e $R \sim R_c$. Let $R_c' = R_c''$; then $R' \sim R_c' = R_c'' \sim R$ which implies $R' \sim R$, by transitivity of $\sim$. Let $R' \sim R''$, i.e. $C_X^{R'} = C_X^{R''}$. Then

$$R_c'(x,y) = R'(x,y) \wedge R'(x,x) = C_{\{x,y\}}^{R'}(x) =$$

$$C_{\{x,y\}}^{R''}(x) = R''(x,y) \wedge R''(x,x) = R_c''(x,y).$$

One can consider R_c as a "canonical representative" in the class of $\sim$ containing R. These canonical representative are completely characterized by the property

$$R_c(x,y) \leqslant R_c(x,x).$$

Note that this condition is always satisfied for reflexive binary relations. We have the following

Corollary. The mapping F is a bijection of the set of all reflexive fuzzy binary relations onto the set of all fuzzy choice functions satisfying properties (B), (H), (C) and (S).

In general, it is possible for C_X^R to be an empty set for some nonempty X. It was mentioned already that acyclicity implies nonvoidness of a choice from nonempty sets. The converse is also true.

Theorem 3. Let R be a reflexive and complete fuzzy binary relation. Then C_X^R is a nonempty fuzzy set for all nonempty X iff P_R is an acyclic fuzzy binary relation.

Proof. The necessity follows from lemma 4. Let $C_X^R \neq \emptyset$. Suppose that P_R is not an acycle relation. Then there is a sequence $x_1,\dots,x_n$ such that

$$P_R(x_i,x_{i+1})>0, \text{ for } i = 1,\dots,n-1, \text{ and } P_R(x_n,x_1)\ 0. \qquad (9)$$

By definition 4 we have

$$C^R_{\{x_1,\dots,x_n\}}(x) = \bigwedge_{i=1}^{n} R(x,x_i)$$

for $x \in \{x_1,\dots,x_n\}$.

We have $R(x_{i+1},x_i)=0$, for $i=1,\dots,n-1$, and $R(x_1,x_n)=0$, by (9). Hence, $C^R_{\{x_1,\dots,x_n\}}= \emptyset$. This contradiction completes the proof.

We will now study conditions determining the class of fuzzy choice functions having an equivalent description in terms of a pair-dominant choice mechanism based on a quasi-transitive fuzzy binary relation.

Theorem 4. A fuzzy function C_X is a pair-dominant choice function C based on a fuzzy quasi-transitive relation R iff it satisfies conditions (B), (H), (C), (FO), (N) and (S).

Proof. The necessity follows from lemmas 1-4 and 6. Let C_X satisfy the conditions listed in the theorem. By theorem 1 we have

$C_X = C_X^R$ for some R. By (S) and (N) R is a reflexive complete fuzzy binary relation. It is sufficient now to prove that P_R is a transitive relation. Let $X = \{x,y,z\}$. Then

$C_X^R(t) = R(t,x) \wedge R(t,y) \wedge R(t,z)$ for $t \in X$. Hence,

$C_X^R(x) = R(x,y) \wedge R(y,z)$,

$C_X^R(y) = 0$, if $P_R(x,y)>0$, and

$C_X^R(z) = 0$, if $P_R(y,z)>0$.

By (N), $C_X^R(x)>0$, which implies $R(x,z)>0$, and $\mathrm{car}C_X^R = \{x\}$. Let $X' = \{x,z\}$. Then $C_{X'}^R(t) = R(t,x) \wedge R(t,z)$, for $t \in \{x,z\}$. Hence, $C_{X'}^R(t) = R(x,z)$ and $C_{X'}^R(z) = R(z,x)$. Now, by (F0), $C_X^R \subseteq X' \subseteq X$ implies $\mathrm{car}C_{X'}^R = \mathrm{car}C_X^R = \{x\}$, which implies $R(z,x) = 0$. Hence, $P_R(x,z) > 0$.

As it follows from proposition 1 in the crisp case the property (K) is a very strong one. The power of this property shows itself very clearly in the fuzzy case, too. Let C_X fulfill conditions (K) and (S). Let X be a crisp set and $x \in C_X$. Then $\{x\} \subseteq X$ and $C_X \cap x \neq \emptyset$.

By (K) we have $C_{\{x\}} = C_X \cap \{x\}$, or $C_X(x) = 1$, by (S), i.e. C_X is a crisp set. From (K) it also follows immediately that

$$C_X = C_{\mathrm{car}X} \cap X. \qquad (10)$$

Hence, C_X is, essentially, a crisp choice function which coincides with C_X for some crisp ordering R, by proposition 1 and (10)

On the other hand it was proved in lemma 5 that fuzzy orderings satisfy the condition (FK) which coincides with (K) for crisp sets and orderings.

We complete this section by the following

Theorem 5. A fuzzy choice function C_X is a pair-dominant choice function based on a fuzzy ordering iff it satisfies conditions (B), (H), (C), (FK), (N), and (S).

Proof. The necessity follows from lemmas 1-5. By (B), (H), (C), (N), and (S) we have $C_X = C_X^R$ where R is fuzzy reflexive complete relation. Let us prove that R is a transitive relation, i.e.. that $R(x,y)>0$ and $R(y,z)>0$ imply $R(x,z)>0$. Let $X = \{x,y,z\}$. Then

$C_X^R(x) = R(x,y) \wedge R(x,z)$, $C_X^R(y) = R(y,x) \wedge R(y,z)$, and

$C_X^R(z) = R(z,x) \wedge R(z,y)$.

Let now $X' = \{x,y\}$. Then $C_{X'}^R(x) = R(x,y)$ and $C_{X'}^R(y) = R(y,x)$. If $C_X^R \cap X' = \emptyset$, i.e. $\operatorname{car} C_X^R = \{z\}$, then $R(x,z) = R(y,x) = 0$. By (N), $C_X^R(z) > 0$, which implies $R(z,x) > 0$ and $R(z,y) > 0$. Let $X'' = \{y,z\}$. We have $C_{X''}^R(y) = R(y,z) > 0$, and $C_{X''}^R(z)=R(z,y)>0$. Since $C_X^R \cap X'' \neq \emptyset$ then, by (FK), $\operatorname{car} C_{X''}^R = \operatorname{car} C_X^R \cap \{y,z\}$. But $\operatorname{car} C_{X''}^R = X''$, which contradicts $\operatorname{car} C_X^R = \{z\}$. Hence, $C_X^R \cap X' \neq \emptyset$. Then, by (FK), $\operatorname{car} C_{X'}^R = \operatorname{car} C_X^R \cap X'$. We have $x \in \operatorname{car} C_{X'}$, since $R(x,y)>0$. Hence, $x \in \operatorname{car} C_X^R$ which implies $R(x,z)>0$.

6. Conclusion.

The fuzzy choice theory described above has some characteristic features which distinguishes it from the crisp one. The difference is mainly stipulated by the purely fuzzy property (B). For instance, there are many "pathological" fuzzy choice functions which satisfy (H) and (C) and do not satisfy (B).

From the fuzzy set theory point of view there is a significant difference between classical properties (H), (C), (O) and (K). The properties (H) and (C) play the same role in both fuzzy and crisp cases. It seems that various transitivity properties which play a great role in classical choice theory are not so important in general fuzzy choice theory.

Only pair-dominant mechanisms based on fuzzy preferences are considered in this paper. It is an interesting problem to study different mechanisms of choice based, for example, upon fuzzy utility functions. We leave this study for further publications.

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