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SOME PROBLEMS OF MEASURE THEORY WHICH ARE RELATED TO ECONOMIC THEORY

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ABSTRACT

After a short discussion of the first application of measure theoretic tools to economics we show that it is consistent relative to the usual axioms of set theory that there exists no nonatomic probability space of power less than the continuum. This together with other results shows that Aumann's continuum-of-agents methodology provides a sound framework at least for the cooperative theory.

There are, however, other problems in economics where, without further assumptions, the continuum may be best model of large societies.

1. Introduction.

Since the origin of modern economics economists always felt that in the presence of a large number of agents the influence of a single individual on the proces in a market should be negligible. In order to show that this idea is consistent we need some model where the single individual has no weight but large coalitions have a positive weight. Such a model of an ideal economy is easily provided by a nonatomic measure space $(A,\mathfrak{a},m)$ where A denotes the set of agents, $\mathfrak{a}$ is the σ-algebra of measurable coa litions and m is a nonatomic σ-additive measure, i.e. there is no $B \in \mathcal{a}$ such that for each $C \subset B$, $C \in \mathcal{a}$, either $m(C) = 0$ or $m(B-C)=0$.[1)]

The first important result in mathematical economics where the measure space approach was applied is due to Aumann (1964) who used the unit interval endowed with the Borel sets and Lebesgue measure to model a large economy and showed that for this model two different equilibrium concepts, the core and the Walras equilibria, are equivalent. As in the absence of the continuum hypothesis the power of the continuum may be very large Aumann's choice seems to be quite arbitrary and we may ask whether there exist uncountable nonatomic probability spaces having power less than the continuum. We shall see presently that it is consistent relative to ZFC [2)] that the answer is no. This, as well as the results of Debreu and Scarf (1963), Kannai (1969) and Schmeidler (1972), shows in particular that Aumann's continuum-of-agents methodology provides a good framework for the cooperative theory. The existence of nonmeasurable sets, which means that a lot of possible coalitions must be dismissed, is a minor annoyance in this context. We also note that Aumann's result may be obtained by starting with a countable set of agents and a finitely additi-

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1) The most important measure spaces used in economics are separable metric spaces endowed with their Borel sets and a finite measure m. For such measure spaces it is easily seen that m is nonatomic iff m vanishes for all singletons. If need be we shall tacitly assume that $\mathcal{a}$ is completed by the negligible coalitions, i.e. if there are sets $B, C \in \mathcal{a}$ and $D \subset A$ such that $B \subset D \subset C$ and $m(C-B) = 0$ then $D \in \mathcal{a}$.

2) ZF denotes the usual axioms of Zermelo-Fraenkel set theory. ZFC denotes ZF together with the axiom of choice (AC).

ve nonatomic measure [1] (see Pallaschke (1976)). However, there are other economic problems where we have no good hints how to model large economies in an appropriate way. Examples to the point are the social choice problem a la Arrow (1963) and noncooperative exchange as discussed by Dubey and Shapley (1979) where serious technical and methodological difficulties occur. We shall investigate some set theoretical assumptions which may be used to overcome these difficulties. The reader who is interested in a survey of applications of measure theory to economics should consult Kirman (1981).

2. On the existence of nonatomic probability spaces.

It is well known that in the absence of the continuum hypothesis, (CH) which says $2^{\chi_o} = \chi_1$, the continuum may be very large. As some mathematicians - amongst them Godel - feel that CH is false because of its implausible consequences we should not assume it for our purposes. If we insist that the weight of the coalitions should be represented by a σ-aditive nonatomic probability measure then large economics cannot be modelled by countably infinite sets. Are there uncountably infinite nonatomic probability spaces of cardinality less than 2^{χ_o} ?. We shall see that the answer "no" is consistent relative to ZFC. Thus the continuum-of-agents methodology is a safe choice if we want the measure to be σ-additive. However, one should note that the mentioned technical and methodological difficulties with some economic problems are due to one of the following (implicit) assumptions:

(i) The use of σ-additive measures in order to characterize the influence of a coalition.

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1) If a measure is not assumed to be σ-additive we always mention this explicitely.

(ii) The unrestricted use of the axiom of choice which is responsible for the existence of nonmeasurable sets.

As neither of these assumptions can be motivated by purely economic arguments they are open for modification in order to give a more satisfactory picture of the affairs in question.

<u>Theorem 2.1.</u> It is consistent relative to ZFC that there exists no nonatomic probability space $(A,\mathfrak{a},m)$ such that $|A|<2^{\chi_o}$.

In order to prove this theorem we show that Martin's Axiom (MA) implies the nonexistence of the probability spaces in question. As Solovay and Tennenbaum (see Kunen (1980)) proved the consistency of ZFC + MA + $2^{\chi_o}>\chi_1$ our theorem immediately follows.

Let us now state a topological version of Martin's Axiom. For a proof of the equivalence to the original form see e.g. Kunen (1980).

For any $\kappa>\chi_o$ MA(κ) is the following assertion:

If X is any compact Hausdorff space which has the countable chain condition (c.c.c) [1] and G are dense open sets for $\alpha<\kappa$, then $\bigcap_{\alpha<\kappa} G \neq \emptyset$.

Martin's axiom MA states that MA(κ) is true for every $\kappa< 2^{\chi_o}$.

We should like to note that MA (χ_o) is a theorem of ZFC and that MA(κ) implies $\kappa< 2^{\chi_o}$.

For the Lebesgue measure Martin and Solovay (see Kunen (1980)) proved the following

<u>Theorem 2.2.</u> Assume MA(κ). Let $A_\alpha,\alpha<\kappa$, be subsets of $\mathbb{R}$, each of Lebesgue measure 0. Then $\bigcup_{\alpha<\kappa} A_\alpha$ has Lebesgue measure 0.

The proof of theorem 2.2. immediately generalizes to regular measures on separable metric spaces. Thus, assuming MA, no sepa-

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1) The topological space X has the c.c.c. iff there is no uncountable family of pairwise disjoint nonempty open subsets of X.

rable metric space of cardinality less than $2^{\aleph_0}$ can carry a nonatomic regular Borel measure. Using a result of Marczewski and Sikorski (1948) this statement even generalizes to nonseparable metric spaces.

Lemma 2.3. Assume MA(κ). Let X be a locally compact Hausdorff space and let m be a regular Borel measure on X. If $F_\alpha, \alpha<\kappa$, are closed subsets of X such that $m(F_\alpha)=0$ for all $\alpha<\kappa$, then either $m(\bigcup_{\alpha<\kappa} F_\alpha) = 0$ or $\bigcup_{\alpha<\kappa} F$ is nonmeasurable.

Proof: To get a contradiction we assume that there exist closed subsets F_α, $\alpha<\kappa$, such that $m(F_\alpha)=0$ for all $\alpha<\kappa$ and $m(\bigcup_{\alpha<\kappa} F_\alpha)>0$.

By the inner regularity of m there exists a compact $C \subset \bigcup_{\alpha<\kappa} F$ such that $m(C)>0$. Let m_C denote the restriction of m to C and observe that m_C is regular. Hence the support C' of m_C exists, i.e. there is a closed (compact) subset $C' \subset C$ such that $m_C(C-C')=0$ and $m_C(C' \cap G)>0$ for every open subset $G \subset X$ for which $C' \cap G \neq \emptyset$.

Our assumption $m(F_\alpha)=0$, for all $\alpha<\kappa$, immediately implies that $m(C' \cap F_\alpha^c)=m(C')$ for all $\alpha<\kappa$. Let us now assume that there is an $\alpha<\kappa$ such that $(C' \cap F_\alpha^c)$ is not dense in C'. Then there exists a nonempty open $C'' \subset C'$ such that $C'' \cap (C' \cap F_\alpha^c)=\emptyset$. By the support property of C' we have $m(C'')>0$ and hence $m(C'')+m(C' \cap F_\alpha^c) > m(C')$. But this is impossible as $C'' \cup (C' \cap F_\alpha^c) \subset C'$.

In order to apply MA(κ) it remains to show that C' has the c.c.c. This is done by adopting an argument of Marczewski-Sikorski (1948). As $m(C')<\infty$ (m is Borel) each family of disjoint subsets of C' having positive measure is at most countable. By the definition of a support each open subset C' must have positive measure. Hence C' has the c.c.c.

We are now ready to apply MA(κ) and get $\bigcap_{\alpha<\kappa} (C' \cap F_\alpha^c) = C' \cap (\bigcap_{\alpha<\kappa} F_\alpha^c) \neq \emptyset$. This gives us the required contradiction as by our assumption $C' \subset C \subset \bigcup_{\alpha<\kappa} F_\alpha$.

Using Maharam's (1942) decomposition theorem we get:

Corollary 2.4. Assume $MA(\kappa)$. Let $(A,\mathfrak{a},m)$ be any nonatomic (complete) probability space, then $m(B) = 0$ for every $B \subset A, |B| < \kappa$.

Thus we have proved theorem 2.1.

3. The measurable utility theorem.

By using v. Neumann's selection theorem Aumann (1967) proved the following

Theorem 3.1. Let $(A,\mathfrak{a},m)$ be any probability space (the space of agents). For each $a \epsilon A$ let $\varphi(a)$ be a connected subset of R^n and let $\geqslant_a$ be a continuous preference order on $\varphi(a)$. Assume that the set $\{(x,y,a): x\epsilon\varphi(a),\ x \geqslant_a y\} \subset R^n \times R^n \times A$ is measurable in the product structure, when R^n is endowed with its Borel structure. Then almost every continuous preference order $\geqslant_a$ can be represented by a function u_a in such a way that $u_a(x)$ is simultaneously measurable in a and x.

As $\varphi(a)$ may be interpreted as the set of all bundles of goods considered by agent a one would like to get rid of the restrictive assumption of connectedness. Aumann argued that theorem 3.1. would follow without the connectedness assumption if every Σ_2^1-or PCA-set (= projection of a coanalytic set) of reals is Lebesgue measurable. In view of Gödel's (1938) proof that the existence of non-Lebesgue measurable PCA-set is consistent with ZFC the best we can expect is an independence proof. This however is easily established by

Theorem 3.2. Assume $MA(\chi_1)$. Then every PCA-set of reals is Lebesgue measurable.

Proof. It is well known that every PCA-set can be represented as union of $\aleph_1$ Borel sets. As theorem 2.2. immediately implies that the union of $\leq \aleph_1$ Lebesgue measurable sets is Lebesgue measurable we are done.

Theorem 3.2. has been observed by Martin and Solovay (1970) and implies that we may consistently assume the existence of an appropriate utility function in the disconnected case.

The answer to the second question of Aumann, whether we also get an independence result if the continuum hypothesis is added to ZFC is not so satisfactory as it depends on a large cardinal assumption. Solovay (1970) proved the following

Theorem 3.3. Suppose that "ZFC + I" is consistent (I denotes the statement: "There is an inaccessible cardinal number"). Then "ZFC + the generalized continuum hypothesis (GCH) + every set of reals definable from a countable sequence of ordinals is Lebesgue measurable" is consistent.

As every projective set and in particular every PCA-set is definable from a countable sequence of ordinals theorem 3.3. answers the second question of Aumann.

It is well known that in ZFC we cannot prove the existence of an inaccessible cardinal. Thus I is an extra set theoretical assumption which may well be false. However, up to now no contradiction occured when assuming I and so many mathematicians believe in I. It is an open problem whether the consistency of "ZFC + GCH + every PCA-set of reals is Lebesgue measurable" can be proved without any large cardinal assumption.

4. Social choice - the measure space approach.

We have already mentioned that there are a lot of hints that the continuum-of-agents methodology is adequate for studying large exchange economics and some extensions e.g. economics with production (see e.g. Hildenbrand (1974)). The situation is radically different with the social choice problem as different assumptions lead to qualitatively different answers and we have no hint which assumption is the "right" one.

We shall sketch the problem briefly and refer the interested reader to Arrow (1963) and Skala (1981). Let A denote the set of agents, X the set of alternatives and P the set of preference orderings on X. Let F be the set of all possible profiles $f:A \to P$. We look for a social preference rule $\sigma:F \to P$. Arrow proved that for finite societies certain rationality assumptions imply that only dictatorial rules are left. I.e. there exists an $a \in A$ such that, no matter what the preferences of the other agents are, if a prefers x to y, then the society prefers x to y. In the usual notation this may be written as $x\ f\ (a)\ y$ implies $x\ \sigma\ (f) y$. As dictatorial rules are socially undesirable we ask whether for large (infinite) societies this result still holds. Let us say that a set of agents $B \subset A$ is decisive if, when all members of B prefer x to y, then so does the whole society. We very well know the structure of the decisive sets. It simply forms an ultrafilter on A. As for finite A every ultrafilter on A simply consists of all supersets of $\{a\}$ for some $a \in A$, Arrow's impossibility theorem is easily established. However for infinite A one easily proves in ZFC that there exist ultrafilters which are not generated by a singleton. This shows the possibility of nondictatorial rules for infinite societies. However, if the set of alternatives involved is also infinite, then an appropriate strengthening of one of Arrow's axioms implies that nondictatorial rules exist iff there exists a measurable cardinal [1)] (see Skala (1981)). As we cannot expect to prove in ZFC that a measurable cardinal exist one may safely assume that for infinite A and X only dictatorial rules exist.

1) A cardinal κ is measurable if there exists a nontrivial $\{0,1\}$-valued σ-additive measure which is defined on all subsets of κ.

What result is typical for large real societies? It is the author's opinion that in some sense nondictatorial rules should exist for large real societies. As up to now our results give us hint how to model large societies in the social choice context (the results are highly sensitive with respect to various set theoretical assumptions (see Skala (1981)) it seems safe to retain formal finiteness and investigate *-finite societies in the sense of nonstandard analysis. We shall not do this here but restrict ourselves to present some results of the measure spaces approach to the social choice problem.

Let $(A,\mathcal{a},m)$ be a nonatomic probability space. Intuitively we want that "small" coalitions should not be decisive. If only single agents should be nondecisive than obviously nondictatorial rules exist. However, in the measure space context it makes not much sense to discuss single agents.

The next notion of smallness which comes into mind is the one of measure zero. [1] As the complements of sets of measure zero have the finite intersection property they can be extended to a free ultrafilter and we have again nondictatorial rules.

A still stronger notion of smallness was used by Kirman and Sondermann (1972). They proved that any social rule has the following property:

> For any given $\varepsilon > 0$ there exists a decisive set B with $m(B)<\varepsilon$.

Hence under the strong notion of smallness used by them there exists no nondictatorial rule.

However, if we model our society by a nonatomic σ-finite but infinite measure space, then even under the strong notion of

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(1) Note that sets of measure zero may be quite large from another point of view. For example we may split the unit interval into a set of Lebesgue measure zero and a meager set.

smallness nondicatorial rules are possible (see Schmitz (1977)).

The reader should note that in any case there occur decisive sets which are nonmeasurable. One way to avoid this annoyance is to work in Solovay's (1970) model of set theory or to assume that a real-valued measurable cardinal exists. As there is no purely economic argument to insist in σ-additive measures we propose to model large societies by a countable set endowed with a nonatomic finitely additive probability measure and get the following

Theorem 4.1. Let A be a countable set. There exist nonatomic finitely additive probability measures on the power set of A allowing nondictatorial social choice rules even in the sense of Kirman and Sondermann.

We would like to stress that the axiom of choice is essentially used in order to define a finitely additive probability measure on the power set of a countable set which vanishes for singletons. Thus theorem 4.1. is not available e.g. in Solovey's model of set theory.

5. Independent decisionmaking in large societies.

In a recent paper Dubey and Shapley (1979) discussed noncooperative game models of price formation and trade. They proved, for example, that in a certain model one can establish an equivalence between the noncooperative (or strategic) Nash equilibria and the classical competitive ones. We shall not go into details but only remark that the ideal of strictly independent individual decisionmaking causes a serious technical and methodological difficulty.

As Shapley and Dubey remark: "It goes to the heart of a basic distinction that divides the game-theoretical methodology and the more familiar behavioristic approach: Is the economic agent a free decisionmaker or an automaton?" After shortly reviewing Shapley's procedure we shall discuss a few aspects of the measure extension approach to Shapley's problem. For more informations and a discussion of other procedures see Skala (1982).

Let $(A,\mathcal{A},m)$ be a nonatomic probability space (usually the unit interval with the Borel sets and Lebesgue measure) and assume for simplicity that the agents' strategy spaces are all the same, say $S_a = [0,1]$ for all $a \in A$. As the agents are assumed to act completely independent any function $g : A \to [0,1]$ may occur as strategy selection. The difficulty arises when the agents' declared intentions as represented by g are to be implemented into the market as a whole. This requires to pass from g to a set function $G: \mathcal{A} \to R$. The most obvious way to do so is by direct integration:

$$G(B) = \int_B g\, dm.$$

This, however, leaves us with the non-measurable functions which will be the typical outcome of noncooperative actions.

Shapley's original idea was to trap g between measurable functions and to define

$$G_*(B) = \max \{\int_B f\, dm : f \leqslant g \quad \text{a.e., } f \text{ m-measurable}\},$$

$$G^*(B) = \min \{\int_B f\, dm : f \geqslant g \quad \text{a.e., } f \text{ m-measurable}\}.$$

Obviously, one may choose any G between G_* and G^* and leave the dicision to the modeller. For example the modeller may choose any $\alpha \in [0,1]$ and define $G = \alpha\, G_* + (1-\alpha)\, G^*$.

As, without any further motivation, this is not a very satisfactory procedure Shaply proposes a heuristically acceptable method by demanding some sort of cooperation for small (in the sense of the measure m) coalitions. We shall not go into details but only remark that, in order to produce a measurable function, the demand on the cooperative bahaviour is quite strong. To assume that only coalitions of measure zero correlate their strategies is not enough. This is an immediate consequence of a theorem due to Solovay (see Prikry (1977)):

Theorem 4.1. Let $\{B_i : i \in I\}$ be a partition of the space of agents $[0,1]$ into pairwise disjoint sets with Lebesgue measure zero. Then there exists an $I' \subset I$ such that $\bigcup_{i \in I'} B_i$ is not Lebesgue measurable.

Let us now assume that the agents are restricted to choose independently only from a preassigned countable set. Then we get

Theorem 4.2. Let $(A,\mathfrak{a},m)$ be a nonatomic probability space and let the strategy selection g only assume countably many different values. There exists an $\mathfrak{a}$-measurable function f and a measure extension $(A,\bar{\mathfrak{a}},\bar{m})$ such that $\bar{m}\{a \in A : f(a) \neq g(a)\} = 0$.

Proof: A result of Saks and Sierpinski (1972) readily generalizes to arbitrary nonatomic probability spaces. We then apply a measure extension theorem due to Łoś and Marczewski (1949).

Without restricting the strategy spaces we obtain from theorem 4.2. the following approximation result:

Corollary 4.3. Let $(A,\mathfrak{a},m)$ be a nonatomic probability space. For any strategy selection g and every $\varepsilon > 0$ there exists a measurable function f and a measure extension $(A,\bar{\mathfrak{a}},\bar{m})$ such that $\bar{m}\{a \in A : |f(a) - g(a)| < \varepsilon\} = 1$.

Bierlein (1978, 1981) approached Shapley's problem by looking for conditions which allow to make a given function measurable. We shortly sketch a proof of one of his theorems.

Lemma 4.4. (Sainte-Beuve (1974)) Let X and Y be Polish spaces and let A be an analytic subset of X. To every Borel-measurable function $f : A \to Y$ there exists a function $s : f(A) \to A$ such that

(i) $f \circ s\,(y) = y$ for all $y \in f(A)$,

(ii) $s^{-1}(B) \in \hat{\mathcal{B}}(f(A))$ for all $B \in \mathcal{B}(A)$.

($\mathcal{B}(A)$ denotes the Borel sets and $\hat{\mathcal{B}}(A)$ the universally measurable sets of A).

Using Lemma 4.4. it is not hard to prove

Theorem 4.5. (Landers and Rogge (1974)). Let $(A,\mathfrak{a})$ be a Suslin space and let $\mathcal{C}$ be a countably generated sub-σ-algebra of $\mathfrak{a}$. Then every probability measure on $\mathcal{C}$ can be extended to $\mathfrak{a}$.

An image measure argument results in Bierlein's

Theorem 4.6. Let $(A,\mathfrak{a},m)$ be a probability space and let $g: A \to [0,1]$ be a function such that

(i) $\mathfrak{a} \subset g^{-1}(\mathcal{B}[0,1])$,

(ii) $\mathfrak{a}$ is countably generated,

(iii) $g(A)$ is an analytic subset of $[0,1]$, then every probability measure can be extended to $g^{-1}(\mathcal{B}[0,1])$.

In the above mentioned proof von Neumann's selection theorem is essentially involved. As there is a more general selection theorem available we may like to generalize the stated result of Landers and Rogge.

Lemma 4.5. (Kondô (1939)). Let X and Y be Polish spaces (e.g. $X = Y = R$) and let $Q \subset X \times Y$ be a coanalytic (PCA) set. In ZF it can be proved that there is a selector of Q such that the graph of it is coanalytic (PCA).

A single generalization of theorem 3.2. together with lemma 4.5. and lemma 4.4. results in

Theorem 4.6. Assume $MA(\aleph_1)$. Let A be a PCA subset of a Polish space and let $\mathfrak{a}$ be a countably generated sub-σ-algebra of $B(A)$. Then every probability measure on $\mathfrak{a}$ can be extended to $B(A)$.

There are several set theoretical assumptions which allow to generalize theorem 4.6. to sets higher up in the projective hierarchy (see Skala (1982)).

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