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ON THE SURJECTIVITY OF THE RIBENBOIM REPRESENTATION OF A LATTICE ORDERED GROUP

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Let G be a lattice group, and $G \longrightarrow \prod_{x\in X} G_x$ a separated and completely regular representation [1]. It is obvious that if $G = G_1 \times \ldots \times G_n$, G is its own representation, which is also surjective. Furthermore the converse property also holds. To see this, we define a topology in X by considering the set $C \subset X$ closed either if it is X or it can be expressed as $C = \bigcap_{i\in I} \sup(f_i)$. (In considering sup(f), it can always be assumed $f \geqslant 0$, simply by using $|f|$ instead of f when necessary).

The separation property of the representation shows that the topology is T_1 since when $y \neq x$ they will exist some $g \in G$ such that $y \notin \sup(g)$, $x \in \sup(g)$. Thus $\{x\} = \bigcap \sup(f_i)$ with $x \in \sup(f)$ and $\{x\}$ is closed.

In general the above mentioned topology is not T_2 as can be seen taking as group G the collection of real functions on R with bounded support. The inclusion $G \subset R^R$ affords a completely regular and separating representation. The closed sets in R are, in this case, the whole of R and the intersection of bounded subsets of R, which are also bounded sets of R. Obviously, a singleton is closed, but two open sets will have a non void intersection being, as they are, complements of bounded sets.

<u>Proposition</u>. Let $G \longrightarrow \prod_{x\in X} G_x$ be the Ribenboim representation. Then

the topology defined above is quasi-compact.

Proof: Let C_i, $i \in I$, an arbitrary family of closed sets with the finite intersection property, each C_i is of the form $\bigcap_j \sup(f_{ij})$, and then it will suffice to prove the result for basic closed sets given by $C_i = \sup(f_i)$, $i \in I$. Let F be the filter of carriers [2] generated by the elements f_i. If F is the totality of all the carriers we would have $\hat{0} = \hat{f}_{i_1} \wedge \ldots \wedge \hat{f}_{i_s}$ or

$0 = f_{i_1} \wedge \ldots \wedge f_{i_s}$ together with

$$\sup(f_{i_1} \wedge \ldots \wedge f_{i_s}) = \sup(f_{i_1}) \wedge \ldots \wedge \sup(f_{i_s}) = \phi,$$

which is absurd. Consequently, F is contained in some ultrafilter $\overline{F}$ and $\hat{f}_i \in \overline{F}$, for all $i \in I$. Therefore, for any $\hat{x} \in \overline{F}$ we have $\hat{f}_i \wedge \hat{x} \neq \hat{0}$; namely, $\overline{F} \in \sup(f_i)$ and $F \in \bigcap_{i \in I} \sup(f_i)$. Hence, we get the following

<u>Proposition</u>. Let G be a lattice group for which the Ribenboim representation $G \longrightarrow \prod_{x \in X} G_x$ is surjective. Then X is finite.

The result is obvious since a surjective representation will imply a discrete topology on X.

References

[1] RIBENBOIM, P., "Théorie des groupes ordonnés", Univ. Nacional del Sur, Bahía Blanca, 1963.

[2] JAFFARD, P., "Les systèmes d'ideaux", Dunod, 1960.

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