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DECISIONS TO BIAS CASE AND CASE-BASED DECISION

Biased Choice/Case-Based Decision/Discrete Choice

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ABSTRACT

The present model of individual decision-making is based on a bounded-rationality style assumption. The decision-maker's preferences over actions lead to choice probabilities that can be defined as the reflect of his behavior when facing past cases of the same choice problem, i.e. his choice memory. Actually, the actions utility is supposed to be cumulatively learnt, case after case. Satisfying a weak assumption of cognitive rationality, the decision-maker searches for *relevant* choices that increase the probability for the chosen action to be the best. Then, our main result establishes that he should design *biased* cases which are the only ones leading to relevant choices. Furthermore, we show that the bias introduced in the case must be in favor of the best past action and finally, we prove that each action that is chosen at the end of a biased case is helpful to bias next cases in order to make future choices relevant.

And don't speak too son
For the wheel's still in spin
And there's no tellin' who
That is namin'
For the loser now
Will be later to win
For the times they're a-changin.

Bob Dylan

1. INTRODUCTION

For the standard neo-classical (NC) microeconomics, choice theory can be seen as a theory of amnesia. An agent facing the same set of alternatives for the 80th time has totally forgotten what he has felt after his choice: no disappointment, no learning. His preferences remain the same as long as he is able to choose. In the very language of the theory, a choice is an *isolated* problem, context-free and time-free (see Tversky, 1972, Billot and Thisse, 1999, for further details about context-dependency of choices and its consequences in discrete choice theory).

For consistency, the NC choice behavior needs then to postulate the preferences to be only based on stable-perfectly-known-genuine-tastes. In other words, the agent would not have any disappointment coming from a past decision if he precisely remembered what he has chosen. In reality, such an assumption is hard to satisfy (see Mirrlees, 1986, for an appealing presentation of the various arguments against the individual rationality axiom and more generally McLennen, 1990, for the problem of dynamic consistency). First, preferences cannot reasonably be considered as stable in the long-run. Everyone can experience his own changes in tastes, remembering the books he read or the music he listened when he was a kid. Everyone can also observe the influence of the fashion on clothing and its implications on consumption choices while watching old pictures. Second, apart from these global preferences variations, everyone can also note some local daily distorsions. Even though, for short distances, one prefers to walk rather than taking the bus, it happens sometimes that one takes the bus for less than a mile. Even though someone generally prefers to drink whisky rather than vodka or eating meat rather than fish, he has probably experienced vodka and fish when nevertheless he faced the two options. Third, empirically, nobody can deny the fact that some of the current decisions are known to be in contradiction with the conscious long-run interest. That is the case for all addictive behaviors or, more generally, for any decision where the short-run response of the choice is a sure pleasure while the long-run response is an uncertain pain.

Hence, all these observations mean that the NC choice theory suffers from the neglect of (i) the history of the preferences changes (or some «preferences updating» as suggested by Border and Segal, 1994); (ii) the local perturbations of the preferences (or stochastic individual preferences as in Billot, 1998, for a multiself agent) and (iii) the apparent contradiction between short-run and long-run effects (or the dynamic inconsistency as defined by Strotz, 1956, for instance and discussed by McLennen, 1990).

Several authors in the last fifteen years try to theoretically emphasize the logic of the dynamics of the decision-making process by focusing onto the influence of the history and memory of the past decisions on the current choice behavior. In some way, the Regret Theory (RT) proposed by Loomes and Sugden, 1982, deals with a notion of myopic memory since a regret can be understood as the difference between the expected pleasure associated with a particular choice and its actual experience, difference which requires a (short) two-step process. More accurately, Gilboa and Schmeidler, 1995, with their Case-Based Decision Theory (CBDT), propose as basic assumption that an agent chooses the act which is best according to its past performance in similar decision problems.[1]

Hence, such theories introduce a kind of history dependency of the choice which should lead to consider the *memory*[2] —i.e. the conscious knowledge of (a part of) the past decisions viewed as a basis for information— as a central concept of the choice theory. Moreover, the principle according to which a decision problem is sometimes much more influenced by the explicit «knowledge of the past» than by theoretical choice variables can be easily and empirically observed in various situations.

When President Clinton had to decide on military intervention against Milosevic's attitude in Kosovo, two strategies were available: using only air strikes or a full-blown military intervention. His problem recalled several previous cases still alive in all American mind: World War II, the Vietnam and Gulf wars, the Bosnia-Herzegovina intervention... Clearly, Clinton's memory of the various consequences of those past cases has influenced his current decision. The well-known «Vietnamese Syndrom», i.e. the reluctance to send native soldiers beyond a reasonable distance from the national frontiers, is an example of such a memory pressure.

In the movies industry, a usual commercial strategy consists in producing sequels of previous «blockbusters» like James Bond, Rocky, Indiana Jones, Star Wars... rather than investing in new risky projects. This translates the fact that Hollywood moguls bet on the audience's memory. Besides, the observation that sequels are frequently a little less successful than the original means that this memory effect has positive and negative effects...

In many sports events, the previous performances are taken into account by the organizers when designing the competition rules. In each race of the Formula 1 Championship, the position in the starting grid is based on previous qualifying laps, the fastest drivers leading on the front row. In Tennis Championships, players are seeded according to their position in the ATP ranking that relied on their past results. In both cases, the memory is not only used to rank the contestants but also to point out the best one and then, to give him a sort of «advantage», i.e. the pole position for the drivers or the impossibility to meet the second best before the final for the tennis-players. Sports contests are not the only field where such examples of seemingly favored decisions can be observed. Experimental psychologists have shown that individuals engage in what they call «selective exposure» when choosing the best strategy, that is aligning their current decision with their previous ones (Wicklund and Brehm, 1976). Similarly, career analysts have identified the fact that organizations seem to invest the knowledge of senior managers to mentor the most promising of the junior employees (Kram, 1985). Those apparently unfair behaviors can in fact be justified by an optimal search for the best action. So, the advantage that the decision-maker seems to give to previously chosen alternatives has been interpreted in the literature as a bias in the choice process modifying his informative content.

Example 1 (Experts): Suppose that after a military agression, a political leader has to decide whether to engage in diplomatic process or military retaliation. His previous experience has created a predisposition in favor of the second option. Nevertheless, he could improve on current information while consulting one of his two favourite advisors, the first known to be a hawk and the other a dove. According to his knowledge of his preferences and those of the experts, he should in fact use the imperfect advice of the hawk, because a possible negative advice about the military option coming from this biased source would increase his confidence for the diplomatic solution. Such an optimal statistical sampling was seen as a rationale for selective exposure (see Calvert, 1985, for further details).

Example 2 (Mentoring): Anticipating a vacancy, a manager gathers previous information about workers in running different selection tests and finally promotes one of them after a final test. Nevertheless, designing the final test in the same way as the previous ones does not

[1] This idea to induce the decision-maker's behavior from his past decisions is close to that of «Reasoning-by-Analogy-to-Previous-Cases», often quoted in artifical intelligence research (see, e.g. Riesbeck and Schank, 1989).

[2] Rubinstein, 1998, p. 63, wrote: «*Memory is a special type of knowledge. It is what a decision-maker knows at a certain date about what he knew at a previous date*».

always increase the manager's confidence for the best worker, nor does it change the fact that the leader before the final test will be promoted. If such a test is always irrelevant, changing its informative value by introducing a bias may increase the manager's confidence in his selection. Actually, biasing consists of helping the leader, providing mentoring from a senior employee. Then, if a non-mentored worker wins the biased test, this victory is understood as a stronger signal of his ability because the quality of the contestants' field is increased. Thus, the manager becomes always more confident about the winner of the final biased test than he was about the previous leader (see Meyer, 1991, and Arai, Billot, Lanfranchi, 2000, for further details).

Example 3 (Multiself Agent): Suppose an agent who, being uncertain about himself insofar as his tastes vary from day to day, wonders what he deeply prefers between watching an old movie on TV or going to the cinema to see a new one. His current knowledge about his preferences would induce him to choose the second option. Nevertheless, if he always dines out once a week (this dinner is supposed to be a constant decision, i.e. an habit), he can bias his decision problem while forming a virtual bundle «dinner and film outside» in order to compare it with the alternative TV-movie-at-home. Consequently, two possibilities occur: first, he decides to go out and then just confirms the fact that the prefers to see true movie rather than TV-movie; second, he prefers to stay home and then learns that, in fact, he really prefers to stay home since even biased with the dinner, the decision was inconsistent with the basic choice probabilities coming from his past decisions (see Billot, 1998, for further details).

The three examples exhibit the same kind of evidence even though the setup is clearly different. Hence, in this paper, our goal is to propose a general framework which can encapsulate the notions of memory and bias fot the largest class of choice problems. Our decision-maker relies on his memory in order to revise his beliefs about the available actions and reveal his cumulative case-based utility function. Assuming the decision-maker tries to increase his relative confidence about the best action, we show that he should bias the design of the decision case (test of employee, choice of actions, projects...) in order to increase the relevance of this final experiment.

The remainder of the paper is organized as follows. Section 2 deals with the rules of the memory and the decision-maker's beliefs and behavior in terms of a cumulative utility function. Section 3 describes his way to extract supplementary information about the actions by considering expected and relevant choices. Section 4 explicits the concept of faithful memories and studies the optimal design of a biased case. Finally, it establishes the main result, namely that, for a given memory, the only relevant choice is biased and that the bias must be in favor of the best past action.

2. A GENERAL MODEL

2.1. Definitions

From the various examples above, in an intuitive way, we can say that any *choice problem* is a situation where an agent (called a *decision-maker*) —the President of the United States, a firm manager, a multiself consumer or a sport contest organizer...— must choose an *action* (a military option, a worker to promote or an alternative...) in order to answer a very basic question (i.e. to solve the problem) such as: what is the best strategy in this war? Who is the best candidate for this job? What is the true favorite alternative? Methodologically, the decision-maker first faces a problem, second considers and orders all the possibilities (the actions) according to his tastes and finally chooses the best one.

Throughout the following pages, in the extended setup, we consider a decision-maker who faces several *cases* of the same choice problem,[3] i.e. he has to choose several times the best action among the same finite set of actions: under an assumption of stability of his *rankings* (preferences for a multiself agent, performance evaluation of the employees by managers, etc...) over the actions, the best action is always the same and the decision-maker's *memory* about choices is therefore useless.[4] Nevertheless, if the rankings are allowed to *change* from case to case, the moemory naturally becomes a central concept for the choice process. Of course, the sources of these ranking variations can either be *intrinsic* (because of the decision-maker's mood or states of mind) or *extrinsic* (because of the variations of the decision-maker's information). That way, a multiself agent rather suffers from intrinsic perturbations of his own tastes (i.e. sometimes called «internal uncertainty» for the Thurstone-class of discrete choices models, as in Billot and Thisse, 1999, or Billot, 1998) while a manager's information is depending on external factors related to the quality of the contestants' performance record or that of the machines used by the candidates in contest (as Meyer, 1991, or Araï, Billot and Lanfranchi, 2000).

Formally, for each case $t \in \{1, ..., s\}$ of a constant problem, the decision-maker considers all the actions of X or any of its subsets, A, in order to point out the current best one. A *choice* $x(t, A)$ is then defined as the action x that is chosen among the actions of $A \subseteq X$ for the case t. Moreover, we assume that there exists a binary *utility response* to the choice, denoted $u(.)$ and defined from 2^A towards $\{0, 1_A\}$ where 1_A corresponds to the utility $u(A)$

[3] In CBDT's words (Gilboa and Schmeilder, 1995), the similarity level for each new case relatively to the past ones is constant and equal to 1.

[4] In this situation, it is rational to be amnesic.

of choosing *one* time *in A*, whatever the problem, with $u(\bigcup_{x\in A}x) = \Sigma_A\, u(x)$. Hence, we have for any case t:

$$u\big(x(t, A)\big) + \sum_{y \neq x} u(y) = u(A) = 1_A$$

which means, since the decision-maker chooses x, and «forgets» the others actions y, that $u\big(x(t, A)\big) = u(A) = 1_A$ while $u(y) = 0$ for all $y \neq x$.

For consistency, we suppose that, if an action is the best among a larger subset A', it is also the best among all the smallest subsets A of A' that contains it:

Choice Monotonicity: For each case $t \in \{1, ..., s\}$ of a constant problem, if $A \subset A' \subset X$ and $x(t, A') \in A$, then $x(t, A') = x(t, A)$.

The s choices are now assumed to be memorized: the decision-maker always remembers what action was chosen for the case t.[5] A *choice memory*, denoted $M_A[1, s]$, is then defined as a finite sequence of s repeated choices $x(t, A)$ beginning on $t = 1$: $M_A[1, s] = [x(t, A)]_{t=1}^{s}$.[6]

Consequently, for any choice memory $M_A[1, s]$, we can set up a *cumulative utility* function (case-based) as a mapping $u^s(.) \geqslant 0$, defined from 2^A towards $\mathbb{R}^+$ and depending on the repeated choices of the decision-maker: the utility of an action is assumed to increase with the number of cases it has «solved», i.e. the number of times the decision-maker has chosen this action. More precisely, for all $s > 0$ and $t \leqslant s$ and all $x \in A$, $A \subset X$:

$$u^t(x) = u^{t-1}(x) + u(x)$$

with

$$u^s(\textstyle\bigcup_{x\in A}x) = \displaystyle\sum_{x\in A} u^s(x) = s \times 1_A \qquad (1)$$

This way to rule each case in adding the response of one more choice to the best action's utility (and nothing to the others') clearly recalls the basic features of discrete choice theory in the sense where the response of the model is binary. It translates the fact that either the action is the best to solve the problem, either it is fully rejected and then does not gain anything (see Anderson, de Palma and Thisse, 1992, chap. 2).

The four following assumptions constrain the cumulative utilities:

Assumption (a): If $A = \varnothing$, then $1_\varnothing = 0$ while $A \neq \varnothing$ implies $1_A > 0$.

Assumption (b): If $A \subset A' \subset X$, then $1_A \leqslant 1_{A'} \leqslant 1_X$.

Assumption (c): For any given memory $M_A[1, s]$, the cumulative utilities are not uniform, i.e.: (x_*) such that $0 < u^s(x_*) < u^s(x^*)$.

Assumption (d): For any $x \in A \subseteq X$, $u^a(x) = 1_A$ where a denotes the cardinal of A.

Assumption (a) is for normalization and consistency. Assumption (b) is a natural requirement of monotonicity which translates the intuition that choosing in a large swet of actions is more satisfying than choosing in any of its subsets. More precisely, the value 1_A must be seen as a unit measure (as a centimeter) while another value $1_{A'}$ would be another unit measure (a meter) with: 1_A is proportional to $1_{A'}$ such that any value expressed on A can be expressed on A' by means of a constant ratio. For $1_x = 1$, we have the natural identity between the number of cases where an action was chosen to solve a problem and its cumulative utility: $u^s(X) = \Sigma_{x\in X}\, u^s(x) = s$. Assumption (*c*) means that it is always possible to identify a $u^s(.)$-maximal element (i.e. an action with the highest cumulative utility that we called the *best* action x^*) different from a $u^s(.)$-minimal element (i.e. an action with the lowest cumulative utility that we called the *worst* action x_*). Assumption (d) means that the function $u^a(.)$, i.e. the *a priori* utility, is uniform. It corresponds to a spontaneous decision-maker's total ignorance about the actions. In the standard situation, i.e. when there is no change in the rankings associated to the different cases, $u^s(x^*) = (s + 1 - a) \times 1_A$ and $u^s(x) = 1_A$, for any $x \neq x^*$. Of course, by construction, we assume $s > a$.

2.2. The Decision-Maker's Beliefs

Since we consider that the decision-maker is globally ignorant about the actions before his very first choice, this means that he knows the set X but nothing about the actions' potential ability to solve the constant problem he faces. According to Assumption (d), the *prior probability of any action x to be the best in any subset $A \subset X$*, denoted $P_A^0(x)$, is then given by the uniform distribution, i.e. $\frac{1}{a}$, for all $x \in A$ and $a = A$. These probabilities $P_A^0(.)$ can be interpreted *as if* they were inferred from a number a of preliminary cases where each action x of $A \subset X$ has just been chosen once, i.e. for all $x \in A$:

$$P_A^0(x) \equiv P_A^a(x) = \frac{u^a(x)}{u^a(A)} = \frac{1_A}{a \times 1_A} \qquad (2)$$

By convenience, we therefore consider a priori that for all $A \subset X$ and all $x \in A$ and $t < a$:

$$M_A[0] \equiv M_A[1, 1] \equiv \cdots \equiv M_A[1, a - t] \equiv \cdots$$
$$\equiv M_A[1, a - 1] \equiv M_A[1, a] \equiv [A]$$

which means also:

$$P_A^0(x) \equiv P_A^1(x) \equiv \cdots = P_A^{a-t}(x) \equiv \cdots \equiv P_A^{a-1}(x) \equiv P_A^a(x) = \frac{1}{a}$$

and

$$P_A^a(x) = \frac{1}{a} \text{ while } \begin{cases} P_A^{a+1}(x) = \dfrac{1}{a+1} & \text{if } x \neq x(a+1, A) \\ P_A^{a+1}(x) = \dfrac{2}{a+1} & \text{if } x = x(a+1, A). \end{cases}$$

By dynamically extending (2), the decision-maker is assumed to set up a probability distribution according to his memory $M_A[1, s]$ as follows:

Axiom 1: For any given memory $M_A[1, s]$, all $A \subset X$, and all $x \in A$,

$$P_A^s(x) = \frac{u^s(x)}{u^s(A)} = \frac{u^s(x)}{S \times 1_A} \quad (3)$$

where $P_A^s(x)$ can be viewed as the agent's belief for the event «x is the best action after s repeated cases in A».

In the standard situation, $P_A^s(x^*) = \frac{s+1-a}{s}\cdot$ This means:

$$P_A^s(x^*) \underset{s \longrightarrow \infty}{\longrightarrow} 1$$

which is quite natural since the best action being always the same, its probability to be the best increases up to the evidence. Besides, when $A = X$, (3) becomes:

$$P_A^s(x) = \frac{u^s(x)}{s}\cdot$$

Hence, the probability given by Axiom 1 is one of the simplest *belief* that can be rationally constructed from the decision-maker's information after s repeated cases. This is consistent with most of the probabilistic choice models, and especially with Luce, 1959. Actually, our model can be interpreted as a discontinuous sequence of s Luce models. The famous Luce Theorem expressed in our terms is the following:

Theorem 1. *For any given memory $M_A[1, t]$ and for all $x \in A$ such that $P_A^t(x) \neq 0, 1$,*

$$P_A^t(x) = P_A^t(A) \times P_A^t(x)$$
$$D_A^t(x) = D_A^t(A') \times D_{A'}^t(x)$$

for all subset $A' \subset A \subset X$, if and only if there exists a positive real-valued function $u(.)$ defined on A such that Axiom 1 holds.

The proof is exactly that of Luce, 1959. Then, it is sufficient to identify the function $u(.)$ with the cumulative utility $u^t(.)$ associated to the given memory $M_A[1, t]$ and the two approaches are strictly equivalent for each particular case. Since the theorem above is based on an «if and only if» condition, our presentation just consists to reverse the two parts of the condition in defining first the utility function and second, to build up the associated choice probabilities[7] in such a way that the so-called Choice Axiom (in fact a Bayesian updating rule) is here replaced by an axiom of definition about the probabilities. This presentation of the Luce model reveals incidently that Luce had the intuition of the memory as a determinant concept for choices. But, he did not express it explicitly as we do since he did not need to dynamically link the different cases in order to study the relevance of the decision-maker's behavior in terms of beliefs.

Throughout this paper, we will follow an example of a particular decision-maker's memory illustrating the functioning of the model.

Example (a): Consider $A = \{x, y\}$ and suppose that after $s = 4$ cases in A, the choice memories of the decision-maker are described as follows:

$$\begin{cases} M_A[0] = M_A[1,1] = M_A[1,2] = [x, y], \\ M_A[1,3] = [M_A[1,2],\ M_A[3,1]] = [x, y, x], \\ M_A[1,4] = [M_A[1,2],\ M_A[3,1],\ M_A[4,1]] = [x, y, x, x]. \end{cases}$$

Then, by (1), the cumulative utilities corresponding to the choice memories are respectively given by:

$$\begin{cases} u^2(x) = 1 \times 1_A & ; \quad u^2(y) = 1_A, \\ u^3(x) = 2 \times 1_A & ; \quad u^3(y) = 1_A, \\ u^4(x) = 3 \times 1_A & ; \quad u^4(y) = 1_A \end{cases}$$

Then, for x and y, we have in terms of probability:

$$P_A^0(x) = P_A^a(x) = P_A^2(x) = \frac{u^2(x)}{u^2(A)} = \frac{1_A}{2 \times 1_A} = \frac{1}{2},$$

$$P_A^3(x) = \frac{u^3(x)}{u^3(A)} = \frac{2 \times 1_A}{3 \times 1_A} = \frac{2}{3} \quad \text{and} \quad P_A^3(y) = \frac{1}{3}\cdot$$

Then, $u^4(x) = 3 \times 1_A$ and $u^4(y) = 1_A$ means that x was actually chosen 2 times and y never. Hence, the probabili-

[7] As done in Billot and Thisse, 1999, for nonadditive utilities.

ties for the two actions to be the best after 4 cases finally correspond to:

$$P_A^4(x) = \frac{u^4(x)}{u^4(A)} = \frac{3 \times 1_A}{4 \times 1_A} = 3/4$$

and

$$P_A^4(x) = \frac{u^4(x)}{u^4(A)} = \frac{1_A}{4 \times 1_A} = 1/4.$$

3. EXPECTATIONS AND RATIONALITY

3.1. Expected Choices

For any given choice memory $M_A[1, s]$, the decision-maker can extract some information about the $(s + 1)$th case he will face in computing the expected utility of each action after $(s + 1)$ cases. Before choosing $x(s + 1, A)$, each action's cumulative utility can take *a priori* the two following values where $x(s + 1, A)$ corresponds to the chosen action:

$$\begin{cases} u^{s+1}(x) = u^s(x) + 1_A \text{ if } x = x(s+1, A), \\ u^{s+1}(x) = u^s(x) \text{ otherwise.} \end{cases}$$

If, by convenience, we denote $u^{s+1}(x \mid x(s + 1, A))$ the conditional cumulative utility of x when $x(s + 1, A)$ is chosen, then $u^{s+1}(x \mid x) = u^s(x) + 1_A$ and $u^{s+1}(x \mid y) = u^s(x)$ for $y \neq x$. Besides, the decision-maker knows the probabilities $P_A^s(.)$ and then can reasonably consider that he will choose for the next case any action x with its associated probability $P_A^s(x)$ to be the best while choosing any other action with a probability $P_A^s(A - x)$. Then, by denoting the *expected cumulative utility* o f x after $(s + 1)$ cases as $u^{s+1}(x)$, we have:

$$u^{s+1}(x) = u^{s+1}(x \mid x) \times P_A^s(x) + u^{s+1}(x \mid y) \times P_A^s(A - x).$$

Then, by Axiom 1:

$$u^{s+1}(x) = \left(u^s(x) + 1_A\right) \times P_A^s(x) + u^s(x) \times \left(1 - P_A^s(x)\right)$$

$$= \frac{[u^s(x)]^2 - [u^s(x)]^2 + u^s(x) \times s \times 1_A + u^s(x) \times 1_A}{s \times 1_A}$$

$$u^{s+1}(x) = \frac{(s+1)}{s} u^s(x). \qquad (4)$$

This result can be extented from case to case. Hence, for a given memory $M_A[1, s]$, the expected cumulative utility of an action x after $s' > s$ cases can be more generally defined as follows:

$$u^{s'}(x) = \frac{s'}{s} u^s(x) \qquad (5)$$

Proposition 1: *Let $M_A[1, s]$ be a given memory. Then, for all $A \subset X$, all $x \in A$ and all $s' > s$, $P_A^{s'}(x) = P_A^s(x)$.*

Proof. From (4), we have for all $x \in X$:

$$u^{s'}(x) = \frac{s'}{s} u^s(x),$$

i.e. by Axiom 1, for all $x \in A$, $P_A^{s'}(x) = P_A^s(x)$. □

In that sense, for a given memory, without any new actual case, the beliefs can be replicated by means of expected choices but the infromation conveyed by s actual cases is definitely invariant to any replication by expectation. The «learning process» is here purely evolutive and not eductive.

Remark: The probability $P_A^{s'}(x)$ can be viewed (and computed) as the expected probability for x to be the best after s' cases for a memory $M_A[1, s]$. For illustration, take $s' = s + 1$ and define $P_A^{s+1}(x \mid x(s + 1, A))$ as the probability for x to be the best action in A when $x(s + 1, A)$ is chosen:

$$P_A^{s+1}(x) = P_A^{s+1}(x \mid x) \times P_A^s(x) + P_A^{s+1}(x \mid y) \times \left(1 - P_A^s(x)\right)$$

$$= \frac{u^s(x) + 1_A}{(s+1) \times 1_A} \times P_A^s(x) + \frac{u^s(x)}{(s+1) \times 1_A} \times \left(1 - P_A^s(x)\right)$$

$$= \frac{[u^s(x)]^2 - [u^s(x)]^2 + u^s(x) \times 1_A + u^s(x) \times s \times 1_A}{(s+1) \times s \times 1_A}$$

$$= \frac{u^s(x)}{s \times 1_A} \times \frac{(s+1)}{(s+1)}$$

$$= P_A^s(x).$$

3.2. Bounded Rationality and Choice Relevance

After any given sequence of s real repeated cases, the decision-maker faces the choice problem for a $(s + 1)$th time. He is here assumed to be *bounded rational* for the two following reasons:

(i) The choice problem, even constant, is associated with an assumption of variations of the decision-maker's rankings. This corresponds to a given imperfection of the information conveyed by the memory. This imperfection comes not only from the uncertainty about the actions' ability to solve the problem but also from the decision-maker's *impossibility to cardinally measure or totally rank* the actions. Since a choice problem may include a number of actions possibly greater than 2 but the decision-maker only identifies the best, then his ability to rank is obviously *bounded.*

(ii) The decision-maker only tries to *increase* (and not to maximize) his confidence after the choice. Hence, we assume his cognitive rationality device to be entirely described by his *willingness to increase the confidence regarding the best action.*

In various situations, this criterion readily corresponds to a bounded-rationality-style-constraint leading to increase the expected value of standar tools such as profit for firm, utility for consumers and so forth...

Hence, we define a choice as *relevant* (for the confidence) when it satisfies this constraint of increasing confidence. In other words, the decision-maker is *rational* when he makes a choice that increases the maximal probability for the actions to be the best.

Definition 1: *For a given memory $M_A[1, s]$, a $(s + 1)$th choice in any $A \subset X$ is* relevant (for the confidence) *if and only if, for any $x(s + 1, A)$:*

$$\max_x P_A^{s+1}\big(x \mid x(s + 1, A)\big) > \max_x P_A^s(x) \qquad (6)$$

The following example will give us some intuition about the situation of choice irrelevance.

Example (b): Consider the same memory than in the previous example, where after 4 previous cases in $A = \{x, y\}$, the cumulative utilities are given by:

$$u^4(x) = 3 \times 1_A \quad \text{and} \quad u^4(y) = 1_A.$$

Hence, the probabilities correspond to:

$$P_A^4(x) = 3/4 \quad \text{and} \quad P_A^4(y) = 1/4.$$

Therefore, a 5th choice over A can only lead to the following results:

(i) $x(5, A) = y$: $P_A^5(x \mid y) = 3/5 < 3/4$ and $P_A^5(y \mid y) = 2/5 < 3/4$ which confirms that x is the best action even if the choice of y decreases the agent's confidence regarding x as the best or

(ii) $x(5, A) = x$: $P_A^5(x \mid x) = 4/5$ and $P_A^5(y \mid x) = 1/5$ which also confirms that x is the best action.

Then, after running this last case, one phenomena could appear, namely the irrelevance in terms of confidence since the probability attached to the previous best action x can become inferior after this last choice: a 5th choice in A would be relevant if and only if $P_A^5\big(x \mid x(5, A)\big) > 3/4$. Clearly, it is not true if $x(5, A) = y$ since in this case $P_A^5(x \mid y) = 3/5$.

More generally, the following result expresses the impossibility to run a relevant $(s + 1)$th choice in A.

Proposition 2: *For any given memory $M_A[1, s]$, any choice $x(s + 1, A)$ is irrelevant.*

Proof: Suppose the $(s + 1)$th choice in A to be relevant. Then, by (6), we must have: $\max_x P_A^{s+1}\big(x \mid (s + 1, A)\big) > P_A^s(x^*)$ for any $x(s + 1) \in A$, which means:

$$\max_x u^{s+1}\big(x \mid x(s + 1, A)\big) > \frac{(s + 1)}{s} u^s(x^*).$$

Now, consider that $x(s + 1, A) = x_*$. Then, $u^{s+1}(x \mid x_*) = u^s(x)$ for any $x \neq x_*$ with $u^s(x) \gtrless u^{s+1}(x_*)$ by (1) and Assumption (c). Hence, we obtain in this case: $\max_x u^{s+1}(x \mid x_*) = \max_x u^s(x) > \dfrac{(s + 1)}{s} u^s(x^*)$, i.e. $u^s(x^*) > \dfrac{(s + 1)}{s} u^s(x^*)$ which means $u^s(x^*) > u^s(x^*)$, i.e. a contradiction. □

This proposition means that any new choice $x(s + 1, A)$ is not sufficient for the decision-maker to increase his confidence in the best action he faces. This impossibility is clearly related to the set of actions A since the problem is constant. Thus, intuitively, a relevant choice can only be obtained from a case involving a different subset of actions.

4. BIASED CASES

4.1. Faithful Memories

The following definition allows to focus onto particular choice memories which can be considered as *faithful.*

Definition 2: *Two memories $M_A[t, s]$, $M_{A'}[t, s']$ are* faithful *when exists a pair (A, A') of subsets of X such that:*

$$u^s(A) = u^{s'}(A') \qquad (7)$$

i.e. by (1): $s \times 1_A = s' \times 1_{A'}$.

Of course, by monotonicity of the unit 1_A and $1_{A'}$, we have necessarily $s < s'$ if $A' \subset A$ and symetrically $s' < s$ if $A \subset A'$. More generally (7) means that choosing a few times in a great set of actions can be interpreted as faithfully informative as choosing a lot of times in a smaller set of actions. For a given memory $M_A[1, s]$, by Proposition 2, we know each supplementary choice to be irrelevant in A. Then, in order to avoid such a situation, we consider that the decision-maker needs to modify the $(s + 1)$th case in considering a pair of subsets (A, A') such that $A' \supset A$ implies that running one more case in A' can be faithfully informative as running $(n + 1)$ more choices in A, i.e.:

$$u^{n+1}(A) = (n + 1) \times 1_A = 1_{A'} = u(A').$$

This means that the decision-maker can *really* choose one time in A' instead of choosing for the $(s+1)$th time in A such that the choice memory $M_{A'}[s, 1]$ becomes faithful to $M_A[s, n+1]$. Then, in choosing one time in A' instead of A, we distinguish one actual choice and n expected ones in such a way that this single choice in A' is informatively equivalent to $(n+1)$ more choices in A.

Consider the problem to know how to precisely modify the set A. From the definition of two faithful memories, we immediatly infer that A' must be larger than A. However, we have to evaluate the relation between the size of the subset $(A' - A)$ and the corresponding number n of expected choices.

Proposition 3: *For any memory* $M_{A'}[s, 1]$, *if there exists a faithful memory* $M_A[s, n+1]$, *then the utility of* $(A' - A)$ *is defined as:*

$$u(A' - A) = 1_{A'} - 1_A = n \times 1_A. \quad (8)$$

Proof: The cumulative utility of $(A' - A)$ after s cases, i.e. according to the memory $M_{A'}[1, s]$, is given by: $u^s(A' - A) = u^s(A') - u^s(A)$. Hence, $u^s(A' - A) = s \times (1_{A'} - 1_A)$. Now, we have by (4): $u^1(A' - A) = u(A' - A) = = \frac{1}{s}u^s(A' - A) = 1_{A'} - 1_A$. Now, focus on the special family of faithful memories $M_{A'}[s, 1]$, $M_A[s, n+1]$, i.e. such that $1_{A'} = (n+1) \times 1_A$. Then, we have: $u(A' - A) = = n \times 1_A$. □

In enlarging the subset A towards A', the decision-maker defines a set union between the previous actions set A and a subset $(A' - A)$ of other actions belonging to X such that their cumulative utility modifies the information conveyed by the $(s+1)$th case. Note also that $u(A' - A)$ does not take its values on $\{0, 1_A\}$ but on $\{0, 1_{A'}\} \equiv \{0, (n+1) \times 1_A\}$ since $A' \supset A$.

4.2. Bias and Relevance

The following proposition shows that the *bias* $B = (A' - A)$ cannot be chosen in A' if $M_{A'}[s, 1]$ and $M_A[s, n+1]$ are faithful.

Proposition 4: *Consider two faithful memories* $M_{A'}[s, 1]$, $M_A[s, n+1]$. *Then,* $x(s+1, A') \neq B$.

Proof: If $M_A[s, n+1]$ is a faithful memory of $M_{A'}[s, 1]$ then: $1_{A'} = (n+1) \times 1_A$. Now, suppose that B is chosen for the case in A', then: $u(B) = 1_{A'} = (n+1) \times 1_A$. Hence, since $u(B) = n \times 1_A$ by Proposition 3, for $n \geqslant 1$, it implies $(n+1) \times 1_A = n \times 1_A$, i.e. $1_A = 0$ which is wrong for all $A \neq \varnothing$. □

Then, the bias B is not a possible choice even though the decision-maker must choose in $A' = A \cup B$. Hence, B must be joined with any action in A:

$$A' = (A - x) \cup \{x \cup B\}.$$

Denote simplier $x_B = \{x \cup B\}$, the union between the action x and the actions B such that the utility of the action x_B corresponds to a «*joint utility*», that is a utility for which the decision-maker cannot distinguish x's contribution and B's inside the global utility associated to x_B. For instance, a candidate can be helped by a supervisor (i.e. a mentor with an employee, a professor with a student...), or an alternative can be associated with a constant act (i.e. a positive habit).

This leads to consider that if x_B is chosen for the $(s+1)$th case, the decision-maker is not allowed to interpret x as the *true choice*: because x is not chosen *itself* in A' but jointly with B, the decision-maker does not credit x with the whole utility $1_{A'}$ of choosing in A', neither with the $(n+1) \times 1_A$ of choosing in A. Besides, if x_B is not chosen, then the chosen action, y, which is not joined with B, is chosen *alone*. Consequently, in that case, $u^{n+1}(y) = 1_{A'}$. Then, because $M_{A'}[s, 1]$ is faithful to $M_A[s, n+1]$, we have $u^{n+1}(y) = (n+1) \times 1_A$ which means that y benefits from the whole utility of its victory in A'.

Hence, for two faithful memories $M_{A'}[s, 1]$, $M_A[s, n+1]$, we always have:

$$\begin{cases} u^{n+1}(x) < 1_{A'} \text{ when } x = x(s+1, A') \text{ with } x = x_B \\ u^{n+1}(x) = 1_{A'} \text{ when } x = x(s+1, A') \text{ with } x \neq x_B \end{cases}, \quad (9)$$

This also means, since

$$P_A^{s+n+1}(x) = \frac{s \times P_A^s(x) + (n+1) \times P_A^{n+1}(x)}{s+n+1},$$

that:

$$P_A^{s+n+1}(x) = \frac{s \times P_A^s(x) + (n+1)}{s+n+1} \quad \text{when } x = x(s+1, A') \text{ with } x \neq x_B \quad (10)$$

Definition 3: *For any given memory* $M_A[1, s]$, *the* $(s+1)$*th case is* biased *if there exists a memory* $M_A[s, n+1]$ *faithful to* $M_{A'}[s, 1]$ *and an action* $x \in A$ *such that* x *is joined with* $B = (A' - A)$ *whose utility is defined as* $u(B) = n \times 1_A$.

Consequently, a *biased choice* is relevant if and only if, for any $x(s+1, A')$:

$$\max_x P_A^{s+n+1}\left(x \mid x(s+1, A')\right) > \max_x P_A^s(x). \quad (11)$$

Any biased choice, i.e. designed in A', must be reinterpreted in A. The following proposition traslates the intuitive idea that whenever the decision-maker bias a case, then his final beliefs are different from the previous ones.

Proposition 5: *For any given memory $M_A[1, s]$, consider two faithful memories $M_{A'}[s, 1]$, $M_A[s, n+1]$. Then, for all $x \in A$, $P_A^{s+s'}(x) = P_A^s(x)$, for all $s' \neq n$.*

Proof: If $M_{A'}[s, 1]$ is faithful to $M_A[s, n+1]$, it means by Proposition 3 that there exists a bias $B = (A' - A)$ which has a utility defined by $u(B) = n \times 1_A$. Hence, for any $s' \neq n$, the subset A' being given in order for $M_{A'}[s, 1]$ to be faithful to $M_A[s, n+1]$, the memory $M_A[s, s'+1]$ is different of $M_A[s, n+1]$ and consequently is not faithful to $M_{A'}[s, 1]$. Then, the $(s+1)$th case cannot be *biased* with the particular subset $B = (A' - A)$ whose utility is $u(B) = n \times 1_A$. Hence, in this case, the s' new cases just correspond to a uniform replication of the previous beliefs as shown by Proposition 1. □

More generally, we have:

Proposition 6: *Consider two faithful memories $M_{A'}[s, 1]$, $M_A[s, n+1]$. Then, the possible posterior cumulative utilities after the $(s+1)$th biased case are:*

$$\begin{cases} u^{s+n+1}(x) = u^s(x) + (n+1) \times 1_A \ \text{if } x = x(s+1, A') \\ \qquad \text{but } x \neq x_B \\ u^{s+n+1}(y) = u^s(y) \ \text{otherwise,} \\ \text{or} \\ u^{s+n+1}(x) = \frac{s+n}{s} u^s(x) + 1_A \ \text{if } x = x(s+1, A') \\ \qquad \text{with } x = x_B \\ u^{s+n+1}(y) = \frac{s+n}{s} u^s(y) \ \text{otherwise.} \end{cases}$$

Proof: Two cases occur. First, $x = x(s+1, A')$ but $x \neq x_B$. Then, by (10), it is straightforward that:

$$\tfrac{s+n}{s} u^s(x) = u^s(x) + (n+1) \times 1_A.$$

The other actions y of A remain unchanged since $u^{n+1}(A' - y) = 0$ when $y \neq x(s+1, A')$. It corresponds to the situation where x can claim all the benefit of the $(s+1)$th choice.

Second, $x = x(s+1, A')$ and $x = x_B$. Then, we know: $u^{n+1}(x) < 1_{A'}$ by (9). By Proposition 5, the probabilities for $M_A[1, s+n]$ are the same than that associated to $M_A[1, s]$ because $M_A[1, s+n]$ is not faithful to $M_A[1, s]$. Hence, according to (5), we can write:

$$u^{s+n}(x) = \frac{s+n}{s} u^s(x).$$

Morever, by choice monotonicity, if $x = x(s+1, A')$ then $x = x(s+1, A)$ since $A' \supset A$, i.e. it actually gains 1_A. Then:

$$u^{s+n+1}(x) = \frac{s+n}{s} u^s(x) + 1_A.$$

In order to satisfy $u^{s+n+1}(A) = (s+n+1) \times 1_A$, when x_B wins, we have naturally for all $y \neq x_B$:

$$u^{s+n+1}(y) = \frac{s+n}{s} u^s(y). \qquad \square$$

When the action x_B is chosen, it cannot be considered as fully responsible of the choice. In that case, the decision-maker affects the value of the bias to all the actions proportionally («expectedly») to their actual cumulative utility after s choices, thanks to equation (5) and increases the cumulative utility of x with just one choice.

4.3. The Theorem of Biased Cases

We have now to search for the best case «implementation», i.e. to find the right action to join with B. In Example (b), it is straightforward that if the decision-maker wants the 5th choice to be relevant in the sense of (11), it must be the best action after 4 cases. Actually, in any case where this best action is joined with B and chosen, the decision-maker learns that it remains the best action but with a higher confidence. On the contrary, if another action y is chosen, then he can conclude that y is the action whose probability of being the best is the highest if the biased choice is relevant. This way, a choice of any of the actions which are not joined with a bias is a sufficient signal that it is more likely to be the best.

For a given memory $M_A[1, s]$, we call *maximal expected advantage of the best action* the quantity du_x^* measuring the spread between the expected utility $u^{s+n+1}(x^*)$ of the best action and the current utility of the worst one $u^s(x_*)$ which actually corresponds to its conditional utility $u^{s+n+1}(x_* \mid y)$ when the chosen action y of the biased case is not x_* nor x^*: $du_x^* = u^{s+n+1}(x^*) - u^s(x_*)$. The below theorem establishes that the right action to join with B is the best one x^* as soon as $u(B) \geqslant du_x^*$:

Theorem 2. *Consider two faithful memories $M_{A'}[s, 1]$, $M_A[s, n+1]$, such that $u(B) \geqslant d_x^*$. Then, the $(s+1)$th choice is relevant if and only if $x_B = \{x^* \cup B\}$.*

Proof. (i) Assume any other action $y \neq x^*$ to be joined with B. Then, according to (11), the $(s+1)$th choice is relevant iff:

$$\max_A [P_A^{s+n+1}(x) \mid x^* \neq y_B; x(s+1, A')] > P_A^s(x^*).$$

Now, consider that $x(s+1, A') = y_B \neq x^*$, i.e.:

$$\max_A [P_A^{s+n+1}(x) \mid x^* \neq y_B;\ x(s+1, A') = y_B] = \frac{\max[\frac{s+n}{s} u^s(x^*),\ \frac{s+n}{s} u^s(y) + 1_A]}{(s+n+1) \times 1_A}.$$

Then, for the relevance, we need:

$$\frac{\max[\frac{s+n}{s} u^s(x^*),\ \frac{s+n}{s} u^s(y) + 1_A]}{(s+n+1) \times 1_A} > \frac{u^s(x^*)}{s \times 1_A}$$

which is true if and only if, for all $y \in A - \{x^*\}$:

$$\frac{s+n}{s} u^s(x^*) < \frac{s+n}{s} u^s(y) + 1_A$$

i.e.

$$u^s(x^*) - u^s(y) < \frac{s}{s+n} 1_A \leqslant 1_A \text{ for any } n \geqslant 0.$$

Then, since $u^s(x^*) - u^s(y) \geqslant 1_A$ by construction, it is inconsistent with $u^s(x^*) - u^s(y) < 1_A$ and the choice is then irrelevant.

(ii) Assume the best action x^* to be joined with B. Then, the $(s+1)$th choice is relevant iff:

$$\max_A [P_A^{s+n+1}(x) \mid x^* = x_B;\ x(s+1, A')] > P_A^s(x^*).$$

Two cases occur: (1) The best action $x^* = x_B = x(s + + 1, A')$, i.e., by Proposition 6:

$$\max_A [P_A^{s+n+1}(x) \mid x^* = x_B;\ x(s+1, A') = x^*] = \frac{\frac{s+n}{s} u^s(x^*) + 1_A}{(s+n+1) \times 1_A}.$$

Then, we always have:

$$\frac{\frac{s+n}{s} u^s(x^*) + 1_A}{(s+n+1) \times 1_A} > \frac{u^s(x^*)}{s \times 1_A}$$

since $u^s(x^*) < s \times 1_A$ by Assumption (d) and the choice is always relevant.

(2) Another action $x \neq x^*$ is chosen, i.e.:

$$\max_A [P_A^{s+n+1}(x) \mid x^* = x_B;\ x(s+1, A') = x] = \frac{\max[u^s(x^*),\ u^s(x) + (n+1) \times 1_A]}{(s+n+1) \times 1_A}. \quad (12)$$

Then, for the relevance, we must have:

$$\frac{\max[u^s(x^*),\ u^s(x) + (n+1) \times 1_A]}{(s+n+1) \times 1_A} > \frac{u^s(x^*)}{s \times 1_A}$$

which is true if and only if for all $x \in A$ and then for the worst x_* with the lowest reputation:

$$\frac{s+n+1}{s} u^s(x^*) < u^s(x_*) + (n+1) \times 1_A$$

i.e.

$$du_x^* = u^{s+n+1}(x^*) - u^s(x_*) \leqslant u(B) = n \times 1_A.$$

Hence, the $(s+1)$th choice is relevant if and only if the best action x^* is joined with B such that $u(B) \geqslant du_x^*$. □

The following example highlights the way a biased case leads to relevance.

Example (c): Consider as, in Examples (a) and (b), a memory where after 4 previous cases over $A = \{x, y\}$, the cumulative utilities are given by:

$$u^4(x) = 3 \times 1_A \text{ and } u^4(y) = 1_A.$$

Hence, the probabilities for the two actions to be the best correspond to:

$$P_A^4(x) = 3/4 \text{ and } P_A^4(y) = 1/4.$$

We know any 5th choice to be irrelevant. Hence, introduce a bias B such that $u(B) = n \times 1_A$ with $du^* \leqslant (n \times 1_A)$, since $x = x^*$ and $y = x_*$. We can choose $n = 11$ for instance, i.e. the necessary minimum to bias since the expected advantage constraint yields:

$$\left(\frac{(5+n)}{4} 3 - 1\right) \leqslant n.$$

Then, we can consider the memory $M_A[4, 12]$ as faithfully informative to the memory $M_{A'}[4, 1]$ where $u(A') = 12 \times 1_A$. Now, by Theorem 2, $x = x_B$ and according to Proposition 6, two possibilities occur:

(i) $x_B = x(s+1, A')$. Then, $u^{4+11+1}(x \mid x_B) = \frac{4+12}{4} u^4(x) + 1_A = (4 \times 3 + 1) \times 1_A = 13 \times 1_A$. Hence, $P_A^{16}(x \mid x_B) = \frac{13}{16} \times 1_A > 3/4$, while $P_A^{16}(y \mid x_B) = \frac{3}{16}$.

(ii) $y = x(s+1, A')$. Then, $u^{16}(y \mid y) = u^4(y) + 12 \times 1_A = 13 \times 1_A$. Hence, $P_A^{16}(y \mid y) > 3/4$ while $P_A^{16}(x \mid y) = \frac{3}{16}$.

Such a result could seem to be linked with what we can call an *arbitrary choice,* namely the decision to design a case such that a particular action is declared ex-abrupto as the best because of the decision-maker's willingness to bias in favor of it (fashion, social influence, favoritism, mimetism... are potential sources of explanation for arbitrary choices). However, our decision-maker's behavior cannot be mistaken with such an arbitrary behavior for two reasons. (i) At the beginning of the cases history, the decision-maker has no prior preference for any ot the actions. The repeated cases reveal the relative desirability of the actions and then, the decision-maker finally decides to bias the next case to satisfy the relevance criterion. In other words, biased cases help to exhibit the best action with more confidence, whatever it is, while arbitrary choices would be irrelevant and then, in our terminology, irrational. (ii) Analyzing carefully the biased case leads to consider that the best action for the memory $M_A[1, s]$, cannot be viewed as really favored. Actually, if the $(s + 1)$th case is still designed as the s previous ones, that is with no bias, the best action x^* is sure to be also considered as the best for the memory $M_A[1, s + 1]$ (see Example (b)). However, even though biasing in favor of x^* must reasonably increase its 'relative performance' for the $(s + 1)$th case, it does not fully protect x^* against the risk to be finally outdone by another action.

4.4. Minor Results

(1) A relevant biased choice has always the property to finally offer to all of the actions a last chance to become the best action for $M_A[1, s + n + 1]$:

Corollary 1: *Consider two faithful memories $M_{A'}[s, 1]$, $M_A[s, n + 1]$, such that $u(B) \geqslant du_x^*$. Then, a relevant $(s + 1)$th choice allows the worst action x_* to become the best for the memory $M_A[1, s + n + 1]$.*

Proof. Straightforward since by relevance, we have:

$$du_x^* < (n + 1) \times 1_A$$

i.e.

$$u^s(x_*) + (n + 1) \times 1_A > \frac{s + n + 1}{s} u^s(x^*).$$

Now, if the worst action x_* is chosen for the biased case, it follows from Proposition 6:

$$u^{s+n+1}(x_*) = u^s(x_*) + (n+1) \times 1_A > \frac{s+n+1}{s} u^s(x^*) > u^s(x^*)$$

and the result holds. □

(2) Actually, we know the decision-maker to consider as the best the action with the highest cumulative utility, that is with the highest probability for a given choice memory. Then, we show that each biased choice $x(s + 1, A')$ can be used as a bias for any other choice memory $M_{A_t}[1, t]$, for all t and all $A_t \subset X$, where the expected advantage of the best action after the t cases is inferior to the final $x(s + 1, A')$'s cumulative utility.

Corollary 2. *Consider two faithful memories $M_A[s, 1]$, $M_A[s, n + 1]$, such that $u(B) \geqslant du_x^*$. Then, the chosen action $x(s + 1, A')$ of the $(s + 1)$th choice can be used to bias any $(t + 1)$th choice if $du_x^* \geqslant u^{t+n+1}(z^*) - u^t(z_*)$ where z^*, z_* are respectively the best and the worst actions for the choice memory $M_{A_t}[1, t]$, for all t and all $A_t \subset X$.*

Proof. It is sufficient to prove that $u^{s+n+1}(x(s + 1, A')) \geqslant u^{t+n+1}(z^*) - u^t(z_*)$ for all t.

By Theorem 2, a biased $(s + 1)$th case is such that $x_B = \{x^* \cup B\}$. Then, two possibilities occur: (i) An action $x(s + 1, A') \neq x^*$ is chosen. In this case, Proposition 6 establishes that:

$$u^{s+n+1}(x) = u^s(x) + (n + 1) \times 1_A > du_x^*.$$

(ii) The best action x^* is chosen, i.e. $x(s + 1, A') = x^*$:

$$u^{s+n+1}(x) = \frac{s + n}{s} u^s(x^*) + 1_A > du_x^*.$$

Thus, we know that the final chosen action's cumulative utility is always greater than du_x^*, whatever it is. Now, consider a memory $M_{A_t}[1, t]$ such that $du_x^* \geqslant u^{t+n+1}(z^*) - u^t(z_*)$. Then, it is always true that:

$$u^{s+n+1}(x) \geqslant u^{t+n+1}(z^*) - u^t(z_*). \qquad \square$$

In the case of agents selection within an organization, this result shows that each winner of a biased contest can bias afterwards any other contest where the advantage of the leader is inferior to his final reputation (see, Araï, Billot and Lanfranchi, 2000, Theorem 2.). It allows us to identify the mentor as a previously promoted worker.

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