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EXISTENCE OF A POSITIVELY HOMOGENEOUS AND TRANSLATION INVARIANT CONTINUOUS CERTAINTY EQUIVALENT

Certainty equivalent/complete preorder/utility functional/translation invariance

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ABSTRACT

Let $(\mathcal{X}, +, \cdot, \tau)$ be a *topological real vector space* of real *random variables* (i.e., measurable real functions) on a common probability space $(\Omega, \mathcal{A}, \mathcal{P})$, and consider a complete preorder (i.e., a transitive and complete binary relation) $\preccurlyeq$ on a real convex cone $\mathcal{K}$ of nonnegative real random variables in $(\mathcal{X}, +, \cdot, \tau)$, containing all the nonnegative constant random variables. Necessary and sufficient conditions are presented for the existence of a *certainty equivalence functional* $\mathcal{C} : \mathcal{K} \to \mathbb{R}_+$ for $\preccurlyeq$ (i.e., $\overline{\mathcal{C}(X)} \sim X$ for every $X \in \mathcal{K}$, and $\mathcal{C}(\overline{\lambda}) = \lambda$ for every $\lambda \in \mathbb{R}_+$, with $\overline{\lambda}$ the constant random variable equal to λ), which is *strictly monotonic, positively homogeneous, translation invariant* and continuous in the induced topology $\tau_{\mathcal{K}}$ on $\mathcal{K}$.

1. INTRODUCTION

Some authors were concerned with the existence of a linear and continuous utility functional $\mathcal{V}$ for a complete preorder $\preccurlyeq$ (i.e., a transitive and complete binary relation) on a normed space, or more generally, on a topological vector space (see e.g. Candeal and Induráin [4] and Weibull [10]). In this paper, given a *topological real vector space* $(\mathcal{X}, +, \cdot, \tau)$ of real *random variables* on a common probability space $(\Omega, \mathcal{A}, \mathcal{P})$, we consider the case of a complete preorder $\preccurlyeq$ on a real convex cone $\mathcal{K}$ in $(\mathcal{X}, +, \cdot, \tau)$, containing all the constants and such that each element of $\mathcal{K}$ is nonnegative. Such a situation is interesting in decision theory, and particularly in insurance mathematics and risk theory, because an element $X \in \mathcal{X}$ may be interpreted as a *random loss,* or a risk. We present necessary and sufficient conditions for the existence of a *certainty equivalence functional* $\mathcal{C} : \mathcal{K} \to \mathbb{R}_+$ for $\preccurlyeq$ (i.e., $\overline{\mathcal{C}(X)} \sim X$ for every $X \in \mathcal{K}$, and $\mathcal{C}(\overline{\lambda}) = \lambda$ for every $\lambda \in \mathbb{R}_+$, with $\overline{\lambda}$ the constant random variable equal to λ) which is positively homogeneous, translation invariant, strictly monotonic and continuous. The existence of such a certainty equivalent is derived from the existence of a utility functional $\mathcal{V}$ for $\preccurlyeq$, exhibiting the same properties.

We recall that in the theory of choice under uncertainty in a multiperiod context, it is usual to assume the existence of a positively homogeneous *certainty equivalent* $\mathcal{C}$ (see Epstein an Zin [7]), reflecting *constant relative risk aversion* (see Safra and Segal [8]). In insurance mathematics, positive homogeneity and translation invariance are both desiderable properties of a *premium functional* $\mathcal{H}$ (i.e., a real functional on the space of all the random losses associated with insurance contracts), which can also be thought of as a certainty equivalent (see e.g. Denneberg [5]). In particular, such properties are satisfed by any *Choquet premium functional* (see e.g. Wang, Young, and Panjer [9]).

2. DEFINITIONS

Denote by $(\mathcal{X}, +, \cdot)$ a real vector space of real *random variables* (i.e., measurable real functions) on a common probability space $(\Omega, \mathcal{A}, \mathcal{P})$.

A subset $\mathcal{K}$ of $\mathcal{X}$ is said to be

(i) a *real cone if* $\lambda X \in \mathcal{K}$ for every $X \in \mathcal{K}$, and for every $\lambda \in \mathbb{R}_+$,

(i) a *real convex cone* if it is a real cone and $X + Y \in \mathcal{K}$ for every $X, Y \in \mathcal{K}$.

Given any nonnegative real number λ, the constant random variable equal to λ will be denoted by $\overline{\lambda}$. Further, the set whose elements are all the constant random variables on $(\Omega, \mathcal{A}, \mathcal{P})$ will be denoted by $\overline{\mathcal{I}}$.

Assume that $\mathcal{X}$ is endowed with any *vector topology* τ, so that $(\chi, +, \cdot, \tau)$ is a *topological real vector space.* We recall that, given a real vector space $(\mathcal{X}, +, \cdot)$, a vector topology τ on $\mathcal{X}$ is simply a topology which makes the vector operations + and $\cdot$ continuous. Given any subset $\mathcal{K}$ of $\mathcal{X}$, denote by $\tau_{\mathcal{K}}$ the *induced topology* on $\mathcal{K}$. For example, in risk theory it is usual to consider the vector space $L_p(\Omega, \mathcal{A}, \mathcal{P})$ $(p \in \mathbb{R}_{++})$ of all the real random variables on $(\Omega, \mathcal{A}, \mathcal{P})$, for which

$$||X||_p = E^{\frac{1}{p}}\,[|X|^p] = \left[\int_{\Omega} |X|^p\, d\mathcal{P}\right]^{\frac{1}{p}} < \infty. \qquad (1)$$

Sice $||\cdot||_p$ is a *pseudonorm* on $L_p(\Omega, \mathcal{A}, \mathcal{P})$, the corresponding pseudonorm topology τ_p is a vector topology on $L_p\,(\Omega, \mathcal{A}, \mathcal{P})$.

A binary relation $\preccurlyeq$ on a subset $\mathcal{K}$ of $\mathcal{X}$ is said to be a *preorder* if $\preccurlyeq$ is *reflexive* (i.e., $[X \preccurlyeq X]$ for every $X \in \mathcal{K}$) and *transitive* (i.e., $[X \preccurlyeq Y) \wedge (Y \preccurlyeq Z) \Rightarrow X \preccurlyeq Z]$ for every $X, Y, Z \in \mathcal{K}$).

The *strict part* $\preccurlyeq$ and the *symmetric part* ~ of a given preorder on $\preccurlyeq$ $\mathcal{K}$ are defined in the usual way, namely, for every $X, Y \in \mathcal{K}$,

$$X \prec Y \Leftrightarrow (X \preccurlyeq Y) \wedge \neg (Y \preccurlyeq X), \qquad (2)$$

$$X \sim Y \Leftrightarrow (X \preccurlyeq Y) \wedge (Y \preccurlyeq X). \qquad (3)$$

A preorder $\preccurlyeq$ on $\mathcal{K}$ is *complete* if, for two elements $X, Y \in \mathcal{K}$, either $X \preccurlyeq Y$ or $Y \preccurlyeq X$. *A preorder* $\preccurlyeq$ on $\mathcal{K}$ is said to be

(i) *monotonic* if, for every $X, Y \in \mathcal{K}$,

$$(X(\omega) \leqslant Y(\omega)\ \mathcal{P} - a.s.) \Rightarrow X \preccurlyeq Y,$$

(ii) *strictly monotonic* if it is monotonic and, for every $X, Y \in \mathcal{K}$,

$$(X(\omega) \leqslant Y(\omega)\ \mathcal{P} - a.s.) \wedge (\mathcal{P}\{\omega \in \Omega : X(\omega) < Y(\omega)\} > 0) \Rightarrow X \prec Y.$$

A preorder $\preccurlyeq$ on a real cone $\mathcal{K}$ in a vector space $\mathcal{X}$ is said to satisfy *constant relative risk aversion* (see e.g. Safra and Segal [8]) if, for every $X, Y \in \mathcal{K}$, and for every $\lambda \in \mathbb{R}_{++}$,

$$X \preccurlyeq Y \Leftrightarrow \lambda X \preccurlyeq \lambda Y. \qquad (4)$$

In economic literature, a preorder satisfying condition (4) is said to be *homothetic* (see e.g. Bosi, Candeal and Induráin [3]).

We say that a preorder $\preccurlyeq$ on a real convex cone $\mathcal{K}$ containing the constant $\bar{1}$ (or equivalently $\bar{\mathcal{I}}$) is *translation invariant* if, for every $X, Y \in \mathcal{K}$, and for every $\lambda \in \mathbb{R}_+$,

$$X \preccurlyeq Y \Leftrightarrow X + \bar{\lambda} \preccurlyeq Y + \bar{\lambda}. \qquad (5)$$

Let $\preccurlyeq$ be a preorder on a subset $\mathcal{K}$ of $\mathcal{X}$. A real number λ_X such that $\bar{\lambda}_X \in \mathcal{K}$ is said to be the *certainty equivalent* of X $(\in \mathcal{K})$ if $X \sim \bar{\lambda}_X$. Further, we say that a real functional $\mathcal{C}$ on $\mathcal{K}$ is a *certainty equivalence functional for* $\preccurlyeq$ if the following conditions are satisfied:

(i) *$\mathcal{C}(X)$ is the certainty equivalent of X for every $X \in \mathcal{K}$,*

(6)

(ii) *$\mathcal{C}(\bar{\lambda}) = \lambda$ for every real number λ with $\bar{\lambda} \in \mathcal{K}$.*

Given a topological space (χ, τ), and any subset $\mathcal{K}$ of $\mathcal{X}$, a preorder $\preccurlyeq$ on $\mathcal{K}$ is said to be *continuous if* $\{Y : Y \preccurlyeq X\}$ and $\{Y : X \preccurlyeq Y\}$ are closed sets in the induced topology $\tau_{\mathcal{K}}$ on $\mathcal{K}$ for every $X \in \mathcal{K}$.

A real functional $\mathcal{V}$ on $\mathcal{K}$ is said to be a *utility functional* for a complete preorder $\preccurlyeq$ on $\mathcal{K}$ if, for every $X, Y \in \mathcal{K}$,

$$X \preccurlyeq Y \Leftrightarrow \mathcal{V}(X) \leqslant \mathcal{V}(Y). \qquad (7)$$

A real functional $\mathcal{V}$ on real cone $\mathcal{K}$ in a real vector space $(\mathcal{X}, +, \cdot)$ is said to be *positively homogeneous* if, for every $X \in \mathcal{K}$, and for every $\lambda \in \mathbb{R}_{++}$,

$$\mathcal{V}(\lambda X) = \lambda \mathcal{V}(X). \qquad (8)$$

A real functional $\mathcal{V}$ on real convex cone $\mathcal{K}$ in real vector space $(\mathcal{X}, +, \cdot)$, containing the constant $\bar{1}$, is said to be *translation invariant* if, for every $X \in \mathcal{K}$, and for every $\lambda \in \mathbb{R}_{++}$,

$$\mathcal{V}(X + \bar{\lambda}) = \mathcal{V}(X) + \lambda. \qquad (9)$$

A real functional $\mathcal{V}$ on any subset $\mathcal{K}$ of $\mathcal{X}$ is said to be

(i) *monotonic* if, for every $X, Y \in \mathcal{K}$,

$$(X(\omega) \leqslant Y(\omega) \quad \mathcal{P} - a.s.) \Rightarrow \mathcal{V}(X) \leqslant \mathcal{V}(Y),$$

(ii) *strictly monotonic* if it is monotonic and, for every $X, Y \in \mathcal{K}$,

$$(X(\omega) \leqslant Y(\omega) \quad \mathcal{P} - a.s.) \wedge (\mathcal{P}\{\omega \in \Omega : X(\omega) < Y(\omega)\} > 0) \Rightarrow \mathcal{V}(X) < \mathcal{V}(Y).$$

It is clear that, if there exists a positively homogeneous, translation invariant and strictly monotonic utility functional $\mathcal{V}$ for a complete preorder $\preccurlyeq$ on a real convex cone $\mathcal{K}$ in real vector space $(\mathcal{X}, +, \cdot)$, containing the constant $\bar{1}$, then $\preccurlyeq$ satisfies constant relative risk aversion, and it is translation invariant and strictly monotonic.

3. EXISTENCE OF A CONTINUOUS CERTAINTY EQUIVALENT WHICH IS POSITIVELY HOMOGENEOUS AND TRANSLATION INVARIANT

There is a strict connection between the existence of a positively homogeneous certainty equivalence functional and the existence of a positively homogeneous utility functional for a complete preorder $\preccurlyeq$ on a real cone $\mathcal{K}$ in real vector space $(\mathcal{X}, +, \cdot)$. The elementary proof of the following lemma is left to the reader.

Lemma 1. *Let $\preccurlyeq$ be a complete preorder on a real cone $\mathcal{K}$ of nonnegative real random variables in a real vector space $(\mathcal{X}, +, \cdot)$, containing the constant $\bar{1}$. Then the following assertions hold:*

(i) *If $\mathcal{C}$ is a positively homogeneous utility functional for $\preccurlyeq$, then $\mathcal{C}$ is a certainty equivalence functional for $\preccurlyeq$ if and only if $\mathcal{C}(\bar{1}) = 1$;*

(ii) *If the restriction of $\preccurlyeq$ to $\mathcal{T}$ is strictly monotonic, then a certainty equivalence functional for $\preccurlyeq$ is also a utility functional for $\preccurlyeq$.* □

Remark 1. It is clear that, if there exists a strictly monotonic certainty equivalence functional $\mathcal{C}$ for a complete preorder $\preccurlyeq$ on a real cone $\mathcal{K}$ of nonnegative real random variables in a real vector space $(\mathcal{X}, +, \cdot)$, containing the constant $\bar{1}$, then the restriction of $\preccurlyeq$ to $\mathcal{T}$ is strictly monotonic. □

In the sequel, if $(\mathcal{X}, +, \cdot, \tau)$ is a topological real vector space, $\mathcal{K}$ is any subset of χ, and $\preccurlyeq$ is any complete preorder on $\mathcal{K}$ which is continuous in the induced topology $\tau_{\mathcal{K}}$ on $\mathcal{K}$, then we shall simply say that $\preccurlyeq$ is continuous.

Lemma 2. *Let $\preccurlyeq$ be a complete preorder on a real convex cone $\mathcal{K}$ of nonnegative real random variables in a topological real vector space $(\mathcal{X}, +, \cdot, \tau)$, containing the constant $\bar{1}$. If $\preccurlyeq$ is strictly monotonic and continuous, then, for every $X, Y \in \mathcal{K}$,*

$$X \prec Y \Rightarrow \exists \lambda \in \mathbb{R}_{++} : X \prec X + \bar{\lambda} \prec Y.$$

Proof. Since $\preccurlyeq$ is strictly monotonic, it is clear that $X \prec X + \bar{\lambda}$ for every $X \in \mathcal{K}$, and for every $\lambda \in \mathbb{R}_{++}$. So, it suffices to show that, given any two real random variables $X, Y \in \mathcal{K}$ with $X \prec Y$, there exists $\lambda \in \mathbb{R}_{++}$ such that $X + \bar{\lambda} \prec Y$. Assume that $Y \preccurlyeq X + \bar{\lambda}$ for every $\lambda \in \mathbb{R}_{++}$, and consider any real sequence $\{\lambda_n\}$ of positive real numbers converging to 0. Since the vector operation + is continuous, it is obvious that $X + \bar{\lambda}_n$ converges to X. Hence, from continuity of $\preccurlyeq$, it is also $Y \preccurlyeq X$. □

In the following theorem we characterize the existence of a positively homogeneous, translation invariant, continuous and strictly monotonic certainty equivalence functional $\mathcal{C}$ for a complete preorder $\preccurlyeq$ on a real convex cone K of nonnegative real random variables in a topological vector space $(\mathcal{X}, +, \cdot, \tau)$.

Theorem 1. *Let $\preccurlyeq$ be a complete preorder on a real convex cone $\mathcal{K}$ of nonnegative real random variables in a topological real vector space $(\mathcal{X}, +, \cdot, \tau)$, containing the constant $\bar{1}$. Then there exists a positively homogeneous, translation invariant, continuous and strictly monotonic certainty equivalence functional $\mathcal{C}$ for $\preccurlyeq$ if and only if the following conditions hold:*

(i) *$\preccurlyeq$ satisfies constant relative risk aversion;*

(ii) *$\preccurlyeq$ is translation invariant;*

(iii) *$\preccurlyeq$ is continuous;*

(iv) *$\preccurlyeq$ is strictly monotonic.*

Proof. The necessity part is obvious, since a strictly monotonic certainty equivalence functional $\mathcal{C}$ for the complete preorder $\preccurlyeq$ is also a utility functional for $\preccurlyeq$ (see lemma 1 and remark 1). So let us prove the sufficiency part. From conditions (i) and (iii), there exists a positively homogeneous continuous utility functional $\mathcal{C}'$ for $\preccurlyeq$ (see the corollary in Bosi, Candeal and Induráin [3]). Observe that $\mathcal{C}'$ may be chosen so that $\mathcal{C}'(\bar{1}) = 1$. In this case, $\mathcal{C}'$ is a certainty equivalence functional for $\preccurlyeq$ (see lemma 1). Define a real functional $\mathcal{V}$ on K by

$$\mathcal{C}(X) = inf\{\mathcal{C}'(Z) + \lambda : X \prec Z + \bar{\lambda}, \lambda \in \mathbb{R}_{++}, Z \in \mathcal{K}\} \quad (X \in \mathcal{K}).$$

Since $\preccurlyeq$ is strictly monotonic, $\mathcal{C}$ is well defined. Further, it is easily seen that $\mathcal{C}'(X) \leq \mathcal{C}(X)$ for every $X \in \mathcal{K}$.

Let us first show that $\mathcal{C}$ is a utility functional for $\preccurlyeq$. Consider $X, Y \in \mathcal{K}$ such that $X \preccurlyeq Y$. Since $Y \prec Z + \bar{\lambda}$ entails $X \prec Z + \bar{\lambda}$, it is clear that $\mathcal{C}(X) \leq \mathcal{C}(Y)$. If $Y \ll X$, then from lemma 2 there exists $\lambda \in \mathbb{R}_{++}$ such that $Y \prec Y + \bar{\lambda} \prec X$, $\mathcal{C}'(X) < \mathcal{C}'(Y + \bar{\lambda}) < \mathcal{C}'(Y)$. Since there exists $\mu \in \mathbb{R}_{++}$ such that $\mathcal{C}'(Y + \bar{\lambda}) + \mu < \mathcal{C}'(X)$, it is $\mathcal{C}(Y) < \mathcal{C}(X)$ from the definition of $\mathcal{C}$.

Now, let us show that $\mathcal{C}$ is positively homogeneous. Assume that there exists $X \in K$, and $\lambda' \in \mathbb{R}_{++}$ such that $\mathcal{C}(\lambda' X) < \lambda' \mathcal{C}(X)$. From the definition of $\mathcal{C}$, there exists $\lambda'' \in \mathbb{R}_{++}$, and $Z \in \mathcal{K}$, with $\mathcal{C}(\lambda' X) < \mathcal{C}'(Z) + \lambda'' < \lambda' \mathcal{C}(X)$, $\lambda' X \prec Z + \overline{\lambda''}$. Since $\preccurlyeq$ satisfies constant relative risk aversion, it is $X \prec \frac{1}{\lambda'} Z + \frac{\overline{\lambda''}}{\lambda'}$, and therefore, using the fact that $\mathcal{C}'$ is positively homogeneous, we arrive at the contradiction $\mathcal{C}(X) < \frac{1}{\lambda'} \mathcal{C}'(Z) + \frac{\lambda''}{\lambda'}$. Analogously, it can be shown that for no $X \in K$, and for no $\lambda' \in \mathbb{R}_{++}$ it is $\lambda' \mathcal{C}(X) < \mathcal{C}(\lambda' X)$.

Observe that $\mathcal{C}$ is continuous, since it is a positively homogeneuos utility functional for the complete continuous preorder $\preccurlyeq$ (see Bosi [2, Lemma 1]). Further, since $\mathcal{C}$ is a utility functional for the strictly monotonic complete preorder $\preccurlyeq$, it is clear that $\mathcal{C}$ is is strictly monotonic. In order to prove that $\mathcal{C}$is translation invariant, assume that there exists $X \in K$, and $\lambda' \in \mathbb{R}_{++}$ such that $\mathcal{C}(X + \overline{\lambda'}) < \mathcal{C}(X) + \lambda'$. From the definition of $\mathcal{C}$, there exists $\lambda'' \in \mathbb{R}_{++}$, and $Z \in \mathcal{K}$ such that $\mathcal{C}(X + \overline{\lambda'}) < \mathcal{C}'(Z) + \lambda'' < \mathcal{C}(X) + \lambda'$, $X + \overline{\lambda'} \prec Z + \overline{\lambda''}$. If $\lambda'' \geqslant \lambda'$, the $X \prec Z + \overline{\lambda'' - \lambda'}$ since $\preccurlyeq$ is translation invariant. Hence, $\mathcal{C}(X) < \mathcal{C}'(Z) + \lambda'' - \lambda'$ from the definitión of $\mathcal{C}$, and this is contradictory. If $\lambda'' < \lambda'$, then $\mathcal{C}'(Z) < \mathcal{C}(X) + \lambda' - \lambda''$, and $X + \overline{\lambda' - \lambda''} \prec Z$ from translation invariance of $\preccurlyeq$. Hecen, there exists $\mu \in \mathbb{R}_{++}$ such that $\mathcal{C}'(Z) + \mu - \lambda' + \lambda'' < \mathcal{C}(X)$. If $\mu \geqslant \lambda' - \lambda''$, then $Z + \overline{\mu - \lambda' + \lambda''} \preccurlyeq X$ from the definition of $\mathcal{C}$, and therefore $Z \preccurlyeq X$. If $\mu < \lambda' - \lambda''$, then $\mathcal{C}'(Z) + \mu < \mathcal{C}(X) + \lambda' - \lambda''$ entails $Z + \bar{\mu} \preccurlyeq X + \overline{\lambda' - \lambda''}$. So, in both cases $X + \overline{\lambda' - \lambda''} \prec Z$ is cointradictory by strict monotonicity of $\preccurlyeq$. Analogously, it can be shown that for no $X \in K$ and for no $\lambda' \in \mathbb{R}_{++}$ it is $\mathcal{C}(X) + \lambda' < \mathcal{C}(X + \overline{\lambda'})$.

It remains to show that $\mathcal{C}$ is a certainty equivalence functional for $\preccurlyeq$. By lemma 1, it suffices to show that $\mathcal{C}(\overline{1}) = 1$. Observe that $\mathcal{C}(\overline{1}) \geqslant 1 = \mathcal{C}'(\overline{1})$. Assume that $1 < \mathcal{C}(\overline{1})$, and consider any real number $\lambda \in \mathbb{R}_{++}$ such that $1 < \lambda < \mathcal{C}(\overline{1})$. Then $1 < \lambda$ entails $\overline{1} \prec \overline{\lambda}$ since $\preccurlyeq$ is strictly monotonic. On the other hand, it is easily seen from the definition of $\mathcal{C}$ that $\lambda < \mathcal{C}(\overline{1})$ entails $\overline{\lambda} \preccurlyeq \overline{1}$. So the proof is complete. □

Finally, let us consider the following example of a continuous, strictly monotonic, positively homogeneous and translation invariant certainty equivalence functional $\mathcal{C}$ which is not additive.

Example 1. [Choquet integral] Consider a real convex cone $\mathcal{K}$ of nonnegative real random variables in $L_p(\Omega, \mathcal{A}, \mathcal{P})$, endowed with the *$L_p$-(pseudo)norm topology* (p is any positive real number). Let $g : [0, 1] \to [0, 1]$ be a increasing and concave continuous function such that $g(0) = 0$, $g(1) = 1$. It is well known that the *Choquet integral* of $X \in \mathcal{K}$ with respect to the *distorted probability* $g \circ \mathcal{P}$ is defined as follows:

$$\mathcal{C}(X) = \int_{\Omega} X \, dg \circ \mathcal{P} = \int_0^{\infty} g \circ \mathcal{P}\{\omega \in \Omega : X(\omega) > u\} du.$$

$\mathcal{C}$ is a L_p-norm continuous (see e.g. Denneberg [6, Proposition 9.4]), strictly monotonic, positively homogeneous and translation invariant real functional on $\mathcal{K}$. Let $\preccurlyeq$ be the complete preorder on $\mathcal{K}$ defined by $X \preccurlyeq Y \Leftrightarrow \mathcal{C}(X) \leqslant \mathcal{C}(Y)$. Since it is clear that $\mathcal{C}(\overline{1}) = 1$, $\mathcal{C}$ is a certainty equivalence functional for $\preccurlyeq$ by lemma 1. Nevertheless, $\mathcal{C}$ is not additive, in general. □

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