Machines and machines aggregates' stability problems theoretical and experimental investigations. I. Dynamic machining systems stability

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Resumen

En este trabajo se exponen modelos matemáticos referentes a problemas lineales y no lineales de Mecánica de Máquinas y se investiga la estabilidad global de los mismos.

Abstract

In a series of papers the authors investigated, single or in collaboration with some of their co-workers, stability problem of a selfcontrolled system generalizing mathematical model of a bifilar lifting dynamics [6]; the *global stability* of cutting machine-tools spindles; the stability of nonlinear and multivariable time-lags dynamic machining systems [7], [8], a.o.

In the present work certain mathematical models concerning linear and nonlinear dynamic machining systems, DMS, and their stability are investigated and the influence of the overlap factor on the absolute stability charts is established, while the comprhensive experimental results related with as well as the set of the corresponding figures and algorithmic methods of studying nonlinearities, as for instance the harmonic linearization method, will be exposed in some of the forthcoming papers.

To the founders of Engineering Mechanics
IN MEMORIAM

1. INTRODUCTION

In the framework of the categorizing mechanical systems regarding stability and the use of various analytical approaches for studying stability of nowadays vehicles, machines and their aggregates, and on the ground of

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various machines' details stability researches, as for instance, if we restrict ourselves to mention here just some very recent one:

a) Dynamic stability of Timoshenko beams on an elastic foundation [1]; *b*) Stability and vibration of thick-wave-ilnduced instability of structures [3]; *d*) Geometrically nonlinear problems of structural dynamic stability and accuracy [4], the authors, single or joint with some of their co-workers, did investigated, among others, in a set of their already published papers, favourably reviewed in the international bibliographical periodicasl [5]; *e*) Stability problem of a selfcontrolled system generalizing mathematical model of the bifilar lifting dynamics [6]; *f*) The *Global stability* of cutting machine tools spidles [7]; *g*) The stability of nonlinear and multivariable tyme-lags dynamic machining systems [8], a.o.

In the present work certain mathematical models concerning linear and nonlinear dynamic machining systems, DMS, and their stability are investigated and the influence of the overlap factor on the absolute stability charts is established, while the comprehensive experimental results related with as well as the set of the corresponding figures, worked out under one of the authors direction by Dr. Eng. I. Balcu in the Machines' Dynamics Laboratory of the University of Brasov and their quite a good coincidence with the established mathematical models will be exposed in some of the forthcoming papers.

2. DYNAMIC MACHINING SYSTEM

In the first approach every DMS can be considered as an elastic structure subject to certain cutting forces. Such a structure with n normal modes of vibrations can be described by a system of differential equations of the form

$$m_i \frac{d^2 v_i}{dt^2} + c_i \frac{dv_i}{dt} + k_i v_i = \beta_i F \quad , \quad (i = 1, 2, \ldots, n) \qquad [1]$$

where m_i, c_i, k_i are the equivalent masses (Kg), the damping coefficients of the oscillatory modes (N·s/mm), and the spring coefficients (N/mm) of the structure, respectively. Further β_i's are the angles (rad) of oscillatory modes of the structure related with the resultant cutting force F (N) which is assumed to be normal to the machined surface, v_i's are displacements (mm) in the directions of the oscillatory modes and t (sec) is the time variable.

The dynamic deviation of uncut chip thickness x (mm) can be expressed as a function of u or y in the form

$$x = u_0 - u = y(t) - \mu y(t - T) \qquad [2]$$

where the u_0 (mm) is the nominal uncut chip thickness, u (mm) is the instantaneous uncut xhip thickness, $y(t)$ (mm) is the relative displacement between tool and workpiece normal to the machined surface, T (sec) is the delay time between the tip and face of the too and μ is the overlap factor. For the time lag T we have the following experimental formulae: $T = 1/N$ for lathe, $T = 1/2N$ for drill, and $T = 1/3N$ for milling machine, where N (rot/min) is spidle speed.

It has been shown in [9] that

$$y = \sum_{i=1}^{n} \alpha_i v_i \qquad [3]$$

where the α_i's (rad) are the angles of the oscillatory modes of the structure relative to a line normal to the machined surface.

Using symbolic calculus with one variable, Eqs. [1]-[3] yield

$$y = G(p)F \quad ; \quad x = H(p)y.$$

and

$$G(p) = \sum_{i=1}^{n} \frac{\alpha_i \beta_i}{m_i p^2 + c_i p + k_i} \quad ; \quad H(p) = 1 - e^{-pT} \qquad [5]$$

where $p = d/dt$.

In the general the variation $\Delta F(N)$ of the resultant cutting force F is a function of x and $x = dx/dt = px$ [10]. Therefore

$$\Delta F = \Delta F(x, px) = F_0 - F \qquad [6]$$

where $F_0(N)$ is the nominal value of the resultant cutting force F.

Eliminating F and y from [4] and [6] whe obtain

$$x + G(p)H(p)\Delta F(x, \mathrm{px}) = G(p)H(p)F_0 \qquad [7]$$

In the following we briefly discuss the stability of certain linear and nonlinear DMS described by Eq [7].

3. LINEAR DYNAMIC MACHINING SYSTEM

If ΔF is assumed to be linear in x, i.e.

$$\Delta F = k_u x \quad ; \quad k_u = \text{const.}$$

where k_u (N/mm) is the static cutting stiffness, one can consider:

(i) *Structures with constant parameters*

In this case whe have m_i = const., c_i = const., k_i = const., and Eqs [7] and [8] yield

$$W(p) = \frac{x(p)}{F_0(p)} = \frac{G(p)H(p)}{1 + k_u G(p)H(p)}$$

where $W(p)$ is the transfer function for the DMS.

The stability of the DMS depends on the location of the roots of the characteristic equation

$$1 + k_u G(p) H(p) = 0 \qquad [10]$$

in the complex plane. Such a stability analysis can be easily carried out employing the method of Merrit [11] introducing the equation

$$k_u G(j\omega) = -1/(1 - \mathrm{e}^{-j\omega T})$$
$$p = j\omega \quad , \quad (j = \sqrt{-1}) \qquad [11]$$

where ω (frad/sec) is the angular frequency.

Clearly, we can use other well known methods in the stability analysis [12]-[15].

In any case, no matter which method is used, the stability of the DMS is of asymptotic type. A necessary and sufficient condition to assure such a state is

$$\int_0^{\infty} |w(t)|\, dt < \infty \qquad [12]$$

where $w(t)$ is the weighting function of the closed loop system [16].

It was shown in [17] that asymptotic stability type criteria equivalent to [12] play only a moderate role in the machine synthesis or in machine design.

In [17] it was shown that the global stability criterion

$$\int_0^{\infty} |w(t)|\, dt \leqslant \frac{M_2}{M_1} = \varepsilon^{-1}$$

makes possible for a given DMS to find parametric domain where the amplitudes of the variations does not surpass certain given limits and the angular frquency remains in a prescribed range, while the *global instability* estimation for the DMS is expressed by

$$\int_0^{\infty} |w(t)|\, dt > \varepsilon^{-1} \qquad [14]$$

where

$$M_1 = F_{0\,\text{máx}}(N) \quad ; \quad M_2 = \delta_1 = \lambda\delta = x_{0\,\text{máx}}$$
$$\lambda > 1 \qquad [15]$$

δ (mm) is the tolerance size and δ_1 (mm) is the part of the tolerance size which compensates the errors resulting from the combined action of random factors.

The above global criteria have been successfully applied to certain problems involving adaptive controls of machine-tools, as well as to boring (or drilling) bars and to the optimization of cutting process parameters [18].

(ii) *Structures with slowly varying parameters*

In this case $m_i = m_i(t)$, $c_i = c_i(t)$ and $k_i = k_i(t)$ are slowly varying functions of time.

Ilt has been shown experimentally and theoretically [19] that the asymptotic stability criterion for structures with slowly varying parameters is of the form

$$\int_0^\infty |w(t - \xi, \xi)| \, d\xi < \infty \quad , \quad (0 < t < t_0) \qquad [16]$$

where t_0 (sec) is the machining time and $w(t - \xi, \xi)$ is the associated weighting function obtained from the generalized weighting function $w(t - \xi, \xi)$ ($\xi \neq$ const., $t \neq$ const.) by setting $t =$ const.

The «global stability» and the «global instability» criteria are

$$\int_0^\infty |w(t - \xi, \xi)| \, d\xi \leqslant \varepsilon^{-1} \quad , \quad (0 < t < t_0) \qquad [17]$$

and

$$\int_0^\infty |w(t - \xi, \xi)| \, d\xi > \varepsilon^{-1} \quad , \quad (0 < t < t_0) \qquad [18]$$

respectively.

Note that in Eqs [13] and [17] the equality sign corresponds to the bound of stability for the DMS.

(iii) Structures with general time varying parameters $\mathrm{m}_i = \mathrm{m}_i(\mathrm{t})$, $\mathrm{c}_i = \mathrm{c}_i(\mathrm{t})$ *and* $\mathrm{k}_i = \mathrm{k}_i(\mathrm{t})$

For this general case we only mention that in [19] the stability of machine-tools spindle and the asymptotic stability of a work-piece system during a turning process have been analysed [20].

4. NONLINEAR DYNAMIC MACHINING SYSTEMS

Clearly, in realistic structures of machine-tools dynamics the most appropriate mathematical models can be described by a system of differential equations of the form

$$m_i \frac{d^2 v_i}{dt^2} + c_i \frac{dv_i}{dt} + k_i [v_i + \varphi_i(v_i)] = F \cos \beta_i \qquad [19]$$

where

$$\varphi_i(v_i) = a_i v_i^2 + b_i v_i^3 \quad , \quad (a_i = \text{const.}, b_i = \text{const.})$$

and the resultant cutting forces F and F_0 are nonlinear. The actual determination of the dynamic variation ΔF of F as a function of as a function of x and px, $\Delta F = \Delta F(x, px)$, is a very difficult problem.

Reffering to the above equations we observe that the DMS is a time-lag nonlinear system. The influence of the overlap factor on the stability charts may be investigated by the method of harmonic linearization of the nonlinearites due to cutting force and elastic structure.

Taking into consideration the dynamic component of cutting force F [6], where F_0 is the steady-state value of cutting force,

$$x = \sum_{i=1}^{n} v_i \cos \alpha_i \qquad [21]$$

and α_i (rad), just as it was previously underlined, is the angle of oscillatory mode v_i relative with a line normal to machined surface, let us expose what follows.

5. CHARACTERISTIC EQUATIONS OF DYNAMIC MACHINING SYSTEM

The transfer matrice of the cutting process can be written in the form

$$H(p) = \frac{\Delta F(p)}{x(p)} = \frac{\Delta F_0}{x_0} [\cos \varphi_0 - (p/\omega) \sin \varphi_0](1 - \mu \, e^{-pT})$$

where ΔF_0 is the amplitude of the force ΔF; φ_0 is the phase difference between x and ΔF; $\mu \in (0, 1]$ is the overlap factor, and T is the delay time between tip and tool face.

Applying the method of harmonic linearization to the nonlinearities [20] we find

$$\varphi_i(v_i) = 0.75 \, b_i v_{i0}^2 v_i$$

where v_{i0} are constants.

Reffering to Eqs [19], [6] (in which F_0), [22] and [23], the characteristic equation is obtained as follows

$$1 + H(p) \sum_{i=1}^{n} \frac{\cos \alpha_i \cos \beta_i}{m_i p^2 + c_i p + k_i(1 + 0.75 \, b_i v_{i0}^2)} = 0$$

6. ABSOLUTE STABILITY CHARTS

By substituting Eq. [22] in [24] and putting $p = j\omega$ the following equations are obtained

$$1 + \frac{\Delta F_0}{x_0}[\lambda_1(\varepsilon)R_e(\omega) - \lambda_2(\varepsilon)I_m(\omega)] = 0$$
$$\lambda_1(\varepsilon)I_m(\omega) + \lambda_2(\omega)R_e(\omega) = 0 \qquad [25]$$

where

$$\varepsilon = \omega T - 2\pi l \quad , \quad (l = 0, 1, 2, \ldots) \qquad [26]$$

$$\lambda_1(\varepsilon) = \cos\varphi_0 - \mu\cos(\varepsilon + \varphi_0) \quad ; \quad \lambda_2(\varepsilon) = \sin\varphi_0 - \mu\sin(\varepsilon + \varphi_0) \qquad [27]$$

$$R_e(\omega) = \sum_{i=1}^{n} \frac{[-m_i\omega^2 + k_i(1 + 0.75\, b_i v_{i0}^2)]\cos\alpha_i \cos\beta_i}{[-m_i\omega^2 + k_i(1 + 0.75\, b_i v_{i0}^2]^2 + c_i^2\omega^2} \qquad [28]$$

$$I_m(\omega) = \sum_{i=1}^{n} \frac{c_i\omega\cos\alpha_i\cos\beta_i}{[-m_i\omega^2 + k_i(1 + 0.75\, b_i v_{i0}^2]^2 + c_i^2\omega^2} \qquad [29]$$

The amplitude ΔF_0 and v_{i0} are not independent. The relationship between ΔF_0 and v_{i0} has the form

$$\frac{9}{16}b_i^2 k_i^2 v_{i0}^2 + 1.5\, b_i k_i (k_i - m_i\omega^2) v_{i0}^2 + [(k_i - m_i\omega^2)^2 + c_i^2\omega^2] v_{i0}^2 -$$
$$\Delta F_0^2[\lambda_1^2(\varepsilon) + \lambda_2^2(\varepsilon)]\cos^2\beta_i = 0 \qquad [30]$$

Using Eqs. [25]-[30] a computer program has been elaborated to find the unknown values ΔF_0 (or x_0) and $n = 60/T$ and some stability charts to be given in one of our forthcoming paper have been plotted.

Remarks. *a*) In one of the forthcoming papers we shall present the graphs and the numerical results for a more general DMS stability problem. We shall take into consideration the regulating uncut chip thickness, the regulating value of the resultant cutting force, the deviation of the machined surface dimension, as well as various other perturbating factors.

b) It iw worthy to underline the domain of applications of one of the authors' «global stability» and «global instability» criteria is extendible to various other forms of the transfer functions [21].

c) It is of interest to find out practical methods to apply the «global stability» criteria for dynamic machining systems with more complicated structures and more than two degrees of freedom.

d) The harmonic linearization method permits to indicate the influence of the different parameters on the absolute stability charts of the DMS.

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