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On a reaction-diffusion system involving the critical exponent.

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Abstract

In this paper we study the existence and multiplycity of the nontrivial solutions for the following elliptic system with Dirichlet boundary conditions and critical nonlinearity

$$\begin{cases} -\Delta u = \lambda u + W(x)u\,|u|^{2^*-2} - kv & \text{in } \Omega \\ -\Delta v = \delta u - \gamma v & \text{in } \Omega \\ u = v = 0 & \text{on } \partial\Omega \end{cases},$$

where $\Omega \subset \mathbf{R}^N (N \geq 3)$ is a bounded regular domain, $W(\cdot) \in L^\infty(\Omega)$ with the property that there exists $\eta > 0$ such that $W(\cdot) \geq \eta$ a.e. in Ω and λ, δ, γ are real parameters. We show that the number of nontrivial solutions, in a left neighbourhood of each $\widehat{\lambda_j}$, $j = 1, 2, \ldots$, is at least twice the multiplicity of $\widehat{\lambda_j}$, where the set $\left\{\widehat{\lambda_j}\right\}_{j\in\mathbf{N}^*}$ represents the spectrum of a certain integro-differential operator.

1 Introduction

Rothe in [R] considered the system of reaction diffusion equations

$$\begin{cases} \partial u \partial t = \mu \Delta u + f(u) - v \\ \\ \varepsilon \partial v \partial t = \Delta v + u - v \end{cases}, \tag{1}$$

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for $(t,x) \in (0,\infty) \times \Omega$. Here u, v are real functions of $(t,x) \in [0,\infty) \times \overline{\Omega}$, where $\Omega \subset \mathbf{R}^N$ $(N \geq 1)$ is open, bounded and connected. As explained in [RM], u and v, which are called the activator and inhibitor respectively, can be interpreted as relative concentrations of substances known as morphogens. The system (1) is supplemented by Dirichlet boundary conditions

$$u = v = 0, \text{ for } (t,x) \in (0,\infty) \times \partial\Omega$$

and the initial conditions

$$u(0,x) = u_0(x),\ v(0,x) = v_0(x), \text{ for all } x.$$

As shown in [RM], the existence of equilibrium solutions in (1) is determined by the problem with $\varepsilon = 0$ and the equilibrium states are solutions of the elliptic system

$$\begin{cases} \mu\Delta u + f(u) - v = 0 & \text{in } \Omega \\ \Delta v + u - v = 0 & \text{in } \Omega \end{cases}$$

subject to Dirichlet boundary conditions

$$u = v = 0 \text{ on } \partial\Omega.$$

It will be convenient to split the function f, which models autocatalytic and saturation effects, into the linear and higher order terms

$$f(u) = \lambda u + g(u).$$

Notation. In the rest of the paper we make use of the following notation

$L^p(\Omega)$, $1 \leq p \leq \infty$, denote Lebesgue spaces; the norm in L^p is denoted by $\|\cdot\|_p$;

$W^{k,p}(\Omega)$ denote Sobolev spaces;

$H_0^1(\Omega)$ denotes $W_0^{1,2}(\Omega)$, endowed with the norm $\|u\|^2 =_\Omega |\nabla u|^2 dx$;

$H^{-1}(\Omega)$ denotes the topological dual of $H_0^1(\Omega)$; the norm in this space is denoted by $\|\cdot\|_{H^{-1}}$.

We consider below the problem of finding nontrivial solutions of the slightly more general elliptic system with Dirichlet boundary conditions and critical nonlinearity

$$\text{(P)} \qquad \begin{cases} -\Delta u = \lambda u + W(x) u |u|^{2^*-2} - kv & \text{in } \Omega \\ -\Delta v = \delta u - \gamma v & \text{in } \Omega \\ u = v = 0 & \text{on } \partial\Omega \end{cases},$$

where $\Omega \subset \mathbf{R}^N (N \geq 3)$ is a bounded regular domain, δ, γ and k are constants such that $k\delta > 0$ and $\gamma > -\lambda_1(\Omega)$, where $\lambda_1(\Omega)$ is the first eigenvalue of the Dirichlet Laplacian on Ω, and $W(\cdot) \in L^\infty(\Omega)$ with the property that there exists $\eta > 0$ such that $W(\cdot) \geq \eta$ a.e. in Ω. Here $2^* = 2N\, N-2$.

In the subcritical case the system (1) has been studied by various authors (see [Ro], [Si], [FM], [NT] and others). The review, even partial, of their results is out of the scope of this paper.

Assuming u to be known, the Dirichlet boundary value problem

$$\begin{cases} -\Delta v + \gamma v = \delta u & \text{in } \Omega \\ v = 0 & \text{on } \partial\Omega \end{cases}$$

is uniquely solved by $v = 1k\, Bu$ where the operator $B = k\delta(-\Delta + \gamma)^{-1}$ is bounded from $L^p(\Omega)$ to $W^{2,p}(\Omega)$ for all $1 \leq p < \infty$. Also, by the Schauder theory, B maps the Hölder space $C^\alpha\left(\overline{\Omega}\right)$ into $C^{1+\alpha}\left(\overline{\Omega}\right)$.

Moreover, it is easily checked that B is positive and self-adjoint in the sense that

$$\int_\Omega uBu\mathrm{d}x = \frac{1}{k\delta}\int_\Omega |\nabla w|^2 + \gamma w^2 \mathrm{d}x$$

for $u \in L^2(\Omega)$ and $w = Bu$; and if $w = Bu$, $z = Bv$ then

$$\int_\Omega uBv dx = \frac{1}{k\delta}\int_\Omega \nabla w \nabla z + \gamma wz \mathrm{d}x = \int_\Omega vBu\mathrm{d}x.$$

Let us define the operator

$$T \equiv -\Delta + B : L^2(\Omega) \to L^2(\Omega), \text{ with } D(T) = W^{2,2}(\Omega) \cap H_0^1(\Omega).$$

It is easy to observe that T is symmetric on its domain $D(T)$ i.e.

$$\langle Tu_1, u_2 \rangle = \langle u_1, Tu_2 \rangle \text{ for all } u_1, u_2 \in D(T),$$

where $\langle \cdot, \cdot \rangle$ denotes the L^2–inner product.

If $0 < \lambda_1 < \lambda_2 \leq \lambda_3 \leq \ldots$ and $(\varphi_k)_k$ denote respectively the eigenvalues and the eigenfunctions of $-\Delta$ in Ω under zero Dirichlet boundary conditions, then one can verify easily that the φ_k's are also eigenfunctions of T corresponding to the modified eigenvalues

$$\widehat{\lambda_k} = \lambda_k + \frac{k\delta}{\gamma + \lambda_k}, \; k = 1, 2, \ldots .$$

A more detailed analysis shows that the spectrum $\sigma(T)$ of T consists precisely of these eigenvalues (see [FM, Corollary 1.2.]).

From the above, we obtain that (P) is equivalent to the integro-differential equation

$$(\mathrm{P}') \qquad \begin{cases} -\Delta u + Bu = \lambda u + W(x) u |u|^{2^*-2} & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}.$$

We associate to the problem (P') the functional

$$I_\lambda(u) = \frac{1}{2} \int_\Omega |\nabla u|^2 + uBu - \lambda u^2 \mathrm{d}x - \frac{1}{2^*} \int_\Omega W(x) |u|^{2^*} \mathrm{d}x, \ \forall u \in H_0^1(\Omega).$$

In a standard way we can prove that $I_\lambda \in C^1(H_0^1(\Omega), \mathbf{R})$ and the critical points of I_λ are solutions of (P').

Note that $p = 2^*$ is the limiting Sobolev exponent for the embedding $H_0^1(\Omega) \hookrightarrow L^p(\Omega)$. Since this embedding is not compact, the functional I_λ does not satisfy the Palais-Smale condition in the energy range $(-\infty, +\infty)$. Hence there are serious difficulties when trying to find critical points by standard variational methods.

Using the ideas of Pohozaev (see [P]), Figueiredo and Mitidieri obtained a similar identity for the system (P) (see [FM, Lemma 4.1 and Remark 2.7]). From this identity, if Ω is starshaped, we can obtain that (P) admits only the trivial solution $u \equiv v \equiv 0$ for $\lambda \leq 0$.

Denote

$$S_B = \inf_{u \in H_0^1(\Omega) \setminus \{0\}} \frac{\|u\|_B^2}{\|u\|_{2^*}^2}$$

where $\|u\|_B^2 =_\Omega |\nabla u|^2 + uBu\mathrm{d}x$, $\forall u \in H_0^1(\Omega)$. From the positivity of B we have that

$$S_B \geq S = \inf_{u \in H_0^1(\Omega) \setminus \{0\}} \frac{\|u\|^2}{\|u\|_{2^*}^2},$$

where S corresponds to the best constant for the Sobolev continuous embedding $H_0^1(\Omega) \hookrightarrow L^{2^*}(\Omega)$. Then $S_B > 0$ because it is well known that $S > 0$.

Under the above conditions and notations, the result proved in this paper is the following:

Theorem 1.1. *For* $\lambda > 0$ *denote* $\widehat{\lambda_+} = \min\left\{\widehat{\lambda_j} : \lambda < \widehat{\lambda_j}\right\}$ *and suppose that the multiplicity of* $\widehat{\lambda_+}$ *is* m. *Then, if*

$$\widehat{\lambda_+} - \lambda < \left(\frac{\eta}{\|W\|_\infty}\right)^{\frac{2}{2^*}} S_B \left[meas\left(\Omega\right)\right]^{-2/N},$$

the problem (P) *admits at least* m *pairs of nontrivial solutions*

$$\left\{\left(u_k\left(\lambda\right), v_k\left(\lambda\right)\right); \left(-u_k\left(\lambda\right), -v_k\left(\lambda\right)\right)\right\},\ k = 1, 2, \ldots, m.$$

Moreover

$$\|u_k\left(\lambda\right)\| \to 0 \text{ and } \|v_k\left(\lambda\right)\| \to 0, \text{ as } \lambda \nearrow \widehat{\lambda_+},$$

for every

$$k \in \{1, 2, \ldots, m\}.$$

The proof of the above theorem uses standard ideas and the techniques are essentially the same as those used in [CFS] and [CFP]. The main tool used is the following slightly modified result of Bartolo, Benci and Fortunato (see [BBF, Theorem 2.4]) contained in [CFS, Theorem 2.5]:

Theorem 2.2. *Let* H *be a real Hilbert space with norm* $\|\cdot\|_H$ *and suppose* $I \in C^1(H, \mathbf{R})$ *is a functional on* H *satisfying the following conditions:*
I1) I *is even,* $I(0) = 0$;
I2) *There exists a constant* $\beta > 0$ *such that the Palais-Smale condition (PS) holds in* $(0, \beta)$;
I3) *There exist two closed subspaces* $V, W \subset H$ *and positive constants* ρ, ξ, β' *with* $\xi < \beta' < \beta$ *such that*
i) $I(u) \leq \beta'$ *for any* $u \in W$;
ii) $I(u) \geq \xi$ *for any* $u \in V$, $\|u\|_H = \rho$;
iii) $codimV < \infty$ *and* $\dim W \geq codimV$.
Then there exists at least $\dim W - \operatorname{codim} V$ *pairs of critical points of* I *with critical values belonging to the interval* $[\xi, \beta']$.

2 Proof of Theorem 1

Step1.

First we show that although the Palais-Smale condition does not hold globally for I_λ it is satisfied locally in $\left(-\infty, 1\, N\, S_B^{N/2} \|W\|_\infty^{\frac{N-2}{2}}\right)$ in the following sense:

If $c < 1\, N\, S_B^{N/2} \|W\|_\infty^{\frac{N-2}{2}}$ *and* $(u_m)_{m\geq 1}$ *is a sequence in* $H_0^1(\Omega)$ *such that*

$$\begin{cases} I_\lambda(u_m) \to c \\ \mathrm{d}I_\lambda(u_m) \to 0 \textit{strongly in } H^{-1}(\Omega) \end{cases}, \textit{ as } m \to \infty,$$

then $(u_m)_{m\geq 1}$ *contains a subsequence converging strongly in* $H_0^1(\Omega)$.

Let $c \in \left(-\infty, 1\, N\, S_B^{N/2} \|W\|_\infty^{\frac{N-2}{2}}\right)$ and let $(u_m)_{m\geq 1} \subset H_0^1(\Omega)$ be a sequence such that

$$\begin{aligned} I_\lambda(u_m) &\to c, \text{ as } m \to \infty, \text{ and} \\ \mathrm{d}I_\lambda(u_m) &\to 0, \text{ as } m \to \infty, \text{ in } H^{-1}(\Omega). \end{aligned}$$

It is easy to observe that there exists $M > 0$ a positive constant such that, for every $m \in \mathbf{N}^*, |I_\lambda(u_m)| \leq M$.

If we choose $\theta \in (12^*, 12)$ and $m \in \mathbf{N}^*$ sufficiently large, we obtain

$$M + \theta\|u_m\| \geq I_\lambda(u_m) - \theta\ \mathrm{d}\ I_\lambda(u_m)\, u_m \geq \frac{1}{2}\int_\Omega |\nabla u_m|^2 + u_m B u_m - \lambda u_m^2 \mathrm{d}x -$$

$$-\frac{1}{2^*}\int_\Omega W(x)\,|u_m|^{2^*}\,\mathrm{d}x - \theta\int_\Omega |\nabla u_m|^2 + u_m B u_m - \lambda u_m^2 \mathrm{d}x + \theta\int_\Omega W(x)\,|u_m|^{2^*}\,\mathrm{d}x \geq$$

$$\geq \left(\frac{1}{2} - \theta\right)\int_\Omega |\nabla u_m|^2 + u_m B u_m - \lambda u_m^2 \mathrm{d}x + \left(\theta - \frac{1}{2^*}\right)\int_\Omega W(x)\,|u_m|^{2^*}\,\mathrm{d}x \geq$$

$$\geq \left(\frac{1}{2} - \theta\right)\|u_m\|^2 - C_1\lambda\,\|u_m\|_{2^*}^2 + \eta\left(\theta - \frac{1}{2^*}\right)\|u_m\|_{2^*}^{2^*} \geq$$

$$\geq \left(\frac{1}{2} - \theta\right)\|u_m\|^2 + \inf_{\rho\geq 0}\left[\eta\left(\theta - \frac{1}{2^*}\right)\rho^{2^*} - C_1\lambda\rho^2\right],$$

where $C_1 > 0$ is a positive constant.

Then $(u_m)_{m\geq 1}$ is bounded in $H_0^1(\Omega)$. Hence we may extract a subsequence $(u_m)_{m\geq 1}$ (relabeled) such that

$$\begin{aligned} &u_m \rightharpoonup u \text{ weakly in } H_0^1(\Omega) \\ &u_m \to u \text{ strongly in } L^p(\Omega), \text{ for any } p \in [1, 2^*) \\ &u_m \to u \text{ a.e. in } \Omega \end{aligned}$$

Now, we prove that u is a solution of (P'). Let $\varphi \in C_0^\infty(\Omega)$. Then

$$|\mathrm{d}I_\lambda(u)\varphi| \leq \|\mathrm{d}I_\lambda(u_m)\|_{H^{-1}}\|\varphi\| + |(\mathrm{d}I_\lambda(u) - \mathrm{d}I_\lambda(u_m))\varphi| \to 0, \text{ as } m \to \infty.$$

Hence u weakly solves (P').

Let $v_m = u_m - u$. Clearly

$$v_m \rightharpoonup 0 \text{ weakly in } H_0^1(\Omega) \tag{2}$$
$$v_m \to 0 \text{ strongly in } L^p(\Omega), \text{ for any } p \in [1, 2^*) \tag{3}$$
$$v_m \to 0 \text{ a.e. in } \Omega$$

From (2) and (3) observe that

$$\begin{aligned} o(1) &= \mathrm{d}I_\lambda(u_m)v_m = \int_\Omega \nabla u_m \nabla v_m + v_m B u_m - \lambda u_m v_m \mathrm{d}x - \\ &- \int_\Omega W(x) v_m u_m |u_m|^{2^*-2} \mathrm{d}x \\ &= \int_\Omega |\nabla v_m|^2 + v_m B v_m \mathrm{d}x - \int_\Omega W(x) v_m u_m |u_m|^{2^*-2} \mathrm{d}x + o(1) \\ &= \|v_m\|_B^2 - \int_\Omega W(x) v_m u_m |u_m|^{2^*-2} \mathrm{d}x + o(1). \end{aligned}$$

Hence

$$\|v_m\|_B^2 = \int_\Omega W(x) v_m u_m |u_m|^{2^*-2} \mathrm{d}x + o(1) \leq \|W\|_\infty \int_\Omega |v_m|^{2^*} \mathrm{d}x + o(1). \tag{4}$$

Since

$$\mathrm{d}I_\lambda(u_m)u_m = o(1),$$

we have that

$$\int_{\Omega} W(x)\,|u_m|^{2^*}\,\mathrm{d}x = \int_{\Omega} |\nabla u_m|^2 + u_m B u_m - \lambda u_m^2 \mathrm{d}x + o\,(1)\,.$$

Using this last equality we obtain

$$\begin{aligned} I_\lambda\,(u_m) &= \frac{1}{2}\left(\|u_m\|_B^2 - \lambda\,\|u_m\|_2^2\right) - \frac{1}{2^*}\int_{\Omega} W(x)\,|u_m|^{2^*}\,\mathrm{d}x \geq \\ &\geq \frac{\eta}{N}\,\|u\|_{2^*}^{2^*} + \frac{1}{N}\,\|v_m\|_B^2 + o\,(1) \geq \frac{1}{N}\,\|v_m\|_B^2 + o\,(1)\,. \end{aligned}$$

Then

$$\|v_m\|_B^2 \leq N\,I_\lambda\,(u_m) + o\,(1) < S_B^{N/2}\,\|W\|_\infty^{\frac{N-2}{2}}\,,\ \text{for } m \text{ sufficiently large.} \tag{5}$$

From (4) we have

$$\|v_m\|_B^2 \leq \|W\|_\infty\,S_B^{-\frac{2^*}{2}}\,\|v_m\|_B^{2^*} + o\,(1) \Longleftrightarrow$$

$$\|v_m\|_B^2\left(S_B^{\frac{2^*}{2}} - \|W\|_\infty\,\|v_m\|_B^{2^*-2}\right) \leq o(1).$$

Since, from (5),

$$S_B^{\frac{2^*}{2}} > \|W\|_\infty\,\|v_m\|_B^{2^*-2} \ \text{for } m \text{ large enough,}$$

we obtain that

$$v_m \to 0,\ \text{strongly in } H_0^1\,(\Omega)\ ,\ \text{as } m \to \infty,$$

and this ends the proof of the fact that I_λ satisfies the Palais-Smale condition on $\left(-\infty,\,1\,N\,S_B^{N/2}\,\|W\|_\infty^{\frac{N-2}{2}}\right)$.

Step 2.

Set

$$H_1 = \overline{\widehat{\lambda_j} \geq \widehat{\lambda_+} \oplus M\left(\widehat{\lambda_j}\right)} \text{ and } H_2 = \widehat{\lambda_j} \leq \widehat{\lambda_+} \oplus M\left(\widehat{\lambda_j}\right),$$

where $M\left(\widehat{\lambda_j}\right)$ denotes the eigenspace of T corresponding to the eigenvalue $\widehat{\lambda_j}$. Denote $\beta_\lambda = H_2 \sup I_\lambda$ and observe that, if $u = \sum\limits_{\widehat{\lambda_i} \leq \widehat{\lambda_+}} a_i \varphi_i \in H_2$, we have

$$\begin{aligned} I_\lambda(u) &= \frac{1}{2}\|u\|_B^2 - \lambda\|u\|_2^2 - \frac{1}{2^*}\int\limits_\Omega W(x)|u|^{2^*}\,\mathrm{d}x \leq \frac{1}{2}\left(\widehat{\lambda_+} - \lambda\right) \\ &\int\limits_\Omega u^2 \mathrm{d}x - \frac{\eta}{2^*}\|u\|_{2^*}^{2^*} \leq \frac{1}{2}\left(\widehat{\lambda_+} - \lambda\right)(\text{ meas }(\Omega))^{2/N}\|u\|_{2^*}^2 - \frac{\eta}{2^*}\|u\|_{2^*}^{2^*} \\ &\leq \rho \geq 0 \sup\left[\frac{1}{2}\left(\widehat{\lambda_+} - \lambda\right)(\text{ meas }(\Omega))^{2/N}\rho^2 - \frac{\eta}{2^*}\rho^{2^*}\right] \\ &= \frac{1}{N}\eta^{\frac{2-N}{2}}\left(\widehat{\lambda_+} - \lambda\right)^{N/2}(\text{ meas }(\Omega)). \end{aligned}$$

Thus

$$\beta_\lambda \leq \frac{1}{N}\eta^{\frac{2-N}{2}}\left(\widehat{\lambda_+} - \lambda\right)^{N/2}(\text{meas}(\Omega)).$$

If $u = \sum\limits_{\widehat{\lambda_i} \geq \widehat{\lambda_+}} a_i \varphi_i \in H_1$, a simple computation shows that

$$I_\lambda(u) \geq \left(1 - \frac{\lambda}{\widehat{\lambda_+}}\right)\|u\|_B^2 - C_2\|u\|_B^{2^*},$$

where $C_2 > 0$ is a positive constant. Clearly, there exist constants ρ_λ, $\xi_\lambda \in (0, \beta_\lambda)$ such that

$$I_\lambda(u) \geq \xi_\lambda, \text{ for any } u \in H_1,\ \|u\|_B = \rho_\lambda.$$

Step 3.

Now, it is easy to observe that the hypothesis of Theorem 2 are satisfied for $H = H_0^1(\Omega)$, $f = I_\lambda, \beta = 1\, N\, S_B^{N/2}\|W\|_\infty^{\frac{N-2}{2}}$, $V = H_1$, $W = H_2$, $\xi = \xi_\lambda$, $\rho = \rho_\lambda$, $\beta' = \beta_\lambda$ and so, for

$$\widehat{\lambda_+} - \lambda < \left(\frac{\eta}{\|W\|_\infty}\right)^{\frac{2}{2^*}} S_B[\text{ meas }(\Omega)]^{-2/N},$$

the problem (P') admits at least

$$m = \dim\left(H_1 \cap H_2\right) - \operatorname{codim}\left(H_1 + H_2\right) = \dim M\left(\widehat{\lambda_+}\right)$$

pairs of nontrivial solutions

$$\left\{u_k(\lambda), -u_k(\lambda)\right\}, \; k = 1, 2, ..., m.$$

Since

$$I_\lambda\left(u_k(\lambda)\right) \in [\delta, \beta'] \text{ and } \beta' \le \frac{1}{N}\eta^{\frac{2-N}{2}}\left(\widehat{\lambda_+} - \lambda\right)^{N/2}\left(\text{meas}\,(\Omega)\right) \to 0, \text{ as } \lambda \nearrow \widehat{\lambda_+},$$

we obtain that

$$I_\lambda\left(u_k(\lambda)\right) \to 0, \text{ as } \lambda \nearrow \widehat{\lambda_+}, \; \forall k \in \{1, 2, ..., m\}.$$

From this and from $\mathrm{d}I_\lambda\left(u_k(\lambda)\right) = 0$, we obtain that

$$u_k(\lambda) \to 0, \text{ strongly in } H_0^1(\Omega), \text{ as } \lambda \nearrow \widehat{\lambda_+}. \tag{6}$$

since I_λ satisfies the (PS) condition in the interval

$$\left(-\infty, 1\, N\, S_B^{N/2}\, \|W\|_\infty^{\frac{N-2}{2}}\right).$$

Now, from the equivalence between (P') and (P), it is easy to observe that if $\widehat{\lambda_+} - \lambda < \left(\eta\|W\|_\infty\right)^{\frac{2}{2^*}} S_B\left[\text{ meas }(\Omega)\right]^{-2/N}$, then (P) admits at least m pairs of nontrivial solutions $\left\{\left(u_k(\lambda), v_k(\lambda)\right); \left(-u_k(\lambda), -v_k(\lambda)\right)\right\}$, $k = 1, 2, \ldots, m$, where $v_k(\lambda) = 1\, kB\left(u_k(\lambda)\right)$. Moreover, from (6) and the continuity of B, we also obtain that

$$v_k(\lambda) \to 0, \text{ strongly in } H_0^1(\Omega), \text{ as } \lambda \nearrow \widehat{\lambda_+}.$$

and this ends the proof.

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