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Distinguished Preduals of Spaces of Holomorphic Functions

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ABSTRACT. For U open in a locally convex space E it is shown in [13] that there is a complete locally convex space $G(U)$ such that $G(U)'_i = (\mathcal{H}(U), \tau_\delta)$. We will show that when U is balanced there is an $\mathcal{S}$-absolute decomposition for $G(U)$ in terms of the preduals of the spaces of homogeneous polynomials. For U balanced open in a Fréchet space we investigate neccessary and sufficient conditions for $(\mathcal{H}(U), \tau_\delta)$ to be equal to $G(U)'_b$.

1. INTRODUCTION

Let U be an open subset of a locally convex space E over $\mathbb{C}$ and let $\mathcal{H}(U)$ be the space of holomorphic functions from U into $\mathbb{C}$. We will denote by τ_o the compact-open topology on $\mathcal{H}(U)$. A semi-norm p on $\mathcal{H}(U)$ is said to be *ported* by the compact subset K of U if for each open $V, K\subset V\subseteq U$, there is $C_V>0$ such that

$$p(f)\leq C_V\|f\|_V$$

for all $f\in\mathcal{H}(U)$. The τ_ω*-topology* on $\mathcal{H}(U)$ is the topology generated by all semi-norms ported by compact subsets of U.

If K is a compact subset of E we denote by $\mathcal{H}(K)$ the space of holomorphic germs on K. The τ_o and τ_ω topologies are defined by

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$$(\mathscr{H}(K), \tau_o) = \varinjlim_{K \subset U} (\mathscr{H}(U), \tau_o)$$

and

$$(\mathscr{H}(K), \tau_\omega) = \varinjlim_{K \subset U} (\mathscr{H}(U), \tau_\omega)$$

Let U be an open subset of a locally convex space E. We say that a seminorm p on $\mathscr{H}(U)$ is *τ_δ-continuous*, if for each countable increasing open cover $\{U_n\}_n$ of U there is an integer n_o and $C>0$ such that

$$p(f) \leq C\|f\|_{U_{n_o}}$$

for every f in $\mathscr{H}(U)$. The *τ_δ-topology* on $\mathscr{H}(U)$ is the topology generated by all τ_δ continuous semi-norms.

In [10], Mazet shows that there is a locally convex space $G(U)$ and a holomorphic map δ_U from U into $G(U)$ with the following universal property: Given any complete locally convex space F and $f \in \mathscr{H}(U, F)$ there is a unique $T_f \in \mathscr{L}(G(U), F)$ such that $f = T_f \circ \delta_U$. In particular if we take $F = \mathbb{C}$, we see that $G(U)$ is a predual of $\mathscr{H}(U)$. Mujica and Nachbin [13] give a new proof of this theorem and show that $G(U)$ is also a topological predual and the inductive dual of $G(U), G(U)'_i$, is equal to $(\mathscr{H}(U), \tau_\delta)$. In §2 we show that the spaces $(P(^nE), \tau_\omega)$ and $(\mathscr{H}(K), \tau_\omega)$ also have topological preduals which we denote by $Q(^nE)$ and $G(K)$ respectively. We show that the spaces $\{Q(^nE)\}_n$ are an $\mathscr{S}$-absolute decomposition for $G(U)$ when U is balanced and therefore many of the topological properties of $G(U)$ can be obtained from the topological properties of $Q(^nE)$.

In the final section we assume U is a balanced open subset of a Fréchet space. We show that we can construct $(\mathscr{H}(U), \tau_o)'_b$ from the $G(K)$'s, and use this result to show that $G(U)'_b = (\mathscr{H}(U), \tau_\delta)$ if and only $(\mathscr{H}(U), \tau_\delta)$ is the bidual of $(\mathscr{H}(U), \tau_o)$.

If E is a locally convex space and n a positive integer, $\hat{\bigotimes}_{n,\pi} E$ will denote the n-fold tensor product of E with itself completed with respect to and endowed with the π or projective topology. We denote by $\hat{\bigotimes}_{s,n,\pi} E$ the completion of the subspace generated by the symmetric tensors.

We refer the reader to [6] for further reading on infinite dimensional holomorphy and to [9] for further reading on locally convex spaces.

2. PREDUALS OF HOMOGENEOUS POLYNOMIALS AND SPACES OF GERMS

Just as the space of holomorphic functions on each open subset of a locally convex space E has a predual, the space of n-homogeneous polynomials on E, for each integer n, and the space of holomorphic germs on each compact subset K of E will also have a predual. In fact, for n-homogeneous polynomials, by taking $Q(^nE)$ to be the space of all linear forms on $P(^nE)$ which when restricted to each locally bounded set is τ_o-continuous, the proof of Theorem 2.1 of [13] is easily adapted to show the following:

Proposition 1. *Let E be a locally convex space, then for each positive integer n, there is a complete locally convex space $Q(^nE)$ and an n-homogeneous polynomial $\delta_n \in P(^nE, Q(^nE))$ with the property that given any complete locally convex space F and any $P \in P(^nE, F)$ there is a unique $L_P \in \mathscr{L}(Q(^nE), F)$ such that $P = L_P \circ \delta_n$.*

This result has previously been proved by Mujica, [12], for Banach spaces and Ryan in [14] with $Q(^nE)$ replaced by $\hat{\bigotimes}_{s,n,\pi} E$. By the uniqueness of L_P it will follow that $Q(^nE)$ is topologically isomorphic to $\hat{\bigotimes}_{s,n,\pi} E$.

Let us define $G(K)$ to be the space of linear maps from $\mathscr{H}(K)$ to $\mathbb{C}$ which are τ_o-continuous on each set of holomorphic germs which are defined and uniformly bounded on some neighbourhood of K. Applying Theorem 1.1 of [13] to the inductive limit $(\mathscr{H}(K), \tau_\omega) = \varinjlim_{K \subset V} \mathscr{H}^\infty(V, \| \|_V)$ we get:

Proposition 2. *Let K be a compact subset of a locally convex space E, then $G(K)'_i = (\mathscr{H}(K), \tau_\omega)$.*

This result had been previously proved by Mujica [11] for E a Fréchet space. In [11], Mujica points out that in this special case $G(K) = (\mathscr{H}(K), \tau_o)'_b$.

The concept of $\mathscr{S}$-absolute decomposition ([5]) allows us to obtain topological properties of $(\mathscr{H}(U), \tau)$ from the corresponding properties of $(P(^nE), \tau)$, $\tau = \tau_o$, τ_ω or τ_δ and U balanced. To show that $\{Q(^nE)\}_n$ is an $\mathscr{S}$-absolute decomposition for $G(U)$ we require the following lemma. (See Proposition 3.15 of [6] for a related result.) We let $\mathscr{S} = \{(\alpha_n) \in \mathbb{C}^{\mathbb{N}}:$ $\limsup_{n\to\infty} |\alpha_n|^{\frac{1}{n}} \leq 1\}$.

Lemma 3. *Let U a balanced open subset of a locally convex space E, $(\alpha_n)_n \in \mathscr{S}$ and $\{f_\beta\}_\beta$ be a family of functions in $\mathscr{H}(U)$ uniformly bounded on some neighbourhood of a compact balanced set K. Then there is an $M > 0$ such that*

$$\sum_{n=0}^{\infty} |\alpha_n| \left\| \frac{\hat{d}^n f_\beta(0)}{n!} \right\|_V \leq M$$

for every β and some neighbourhood V of K.

For U open in any locally convex space E it can be shown that, for each n, the map $\tilde{\phi} : Q(^nE) \to G(U)$ defined by $\tilde{\phi}(f) = \phi\left(\frac{\hat{d}^n f(0)}{n!}\right)$ for $\phi \in Q(^nE)$, $f \in \mathscr{H}(U)$ identifies $Q(^nE)$ with a closed subspace of $G(U)$.

Proposition 4. *Let U be a balanced open subset of a locally convex space E; then $\{Q(^nE)\}_n$ is an $\mathscr{S}$-absolute decomposition for G(U).*

Proof. Let $B = \{f_\beta\}$ be a family of locally bounded function in $\mathscr{H}(U)$. Recall that the topology on $G(U)$ is the topology of uniform convergence on locally bounded subsets of $\mathscr{H}(U)$. As

$$(1, 2^2, \ldots, n^2, \ldots) \in \mathscr{S}$$

it follows by Lemma 3 that for each x we can choose a neighbourhood V_x of Γ_x, the balanced hull of $\{x\}$, such that

$$\sup_\beta \sum_{n=0}^{\infty} n^2 \left\| \frac{\hat{d}^n f_\beta(0)}{n!} \right\|_{V_x} = M_x < \infty .$$

Therefore for every m and every β we have,

$$\left\| m^2 \sum_{n=m}^{\infty} \frac{\hat{d}^n f_\beta(0)}{n!} \right\|_{V_x} \leq \sum_{n=m}^{\infty} n^2 \left\| \frac{\hat{d}^n f_\beta(0)}{n!} \right\|_{V_x} \leq M_x.$$

Thus the set

$$\tilde{B} = \left\{ m^2 \sum_{n=m}^{\infty} \frac{\hat{d}^n f_\beta(0)}{n!} \right\}_{m,\beta}$$

is locally bounded.

For $\phi \in G(U)$, let $\phi_n = \phi|_{P(^nE)}$. For each semi-norm α let $U_\alpha = \{x \in E: \alpha(x) \leq 1\}$ and $B_\alpha^n = \{P \in P(^nE): \|P\|_{U_\alpha} \leq 1\}$. Then B_α^n is a locally bounded set of holomorphic functions on $\mathscr{H}(U)$ and $\phi_n | B_\alpha^n = \phi | B_\alpha^n$ is τ_o-continuous and $\phi_n \in Q(^nE)$. Since ϕ is τ_o-continuous on $\tilde{B}$ and the Taylor series expansion of f_β about 0 converges to f_β in the τ_o-topology we have that,

$$\left\| \phi - \sum_{k=0}^{m-1} \phi_k \right\|_B = \sup_\beta \left| \left(\phi - \sum_{k=0}^{m-1} \phi_k \right)(f_\beta) \right|$$

$$= \sup_\beta \left| \phi \left(\sum_{n=m}^{\infty} \frac{\hat{d}^n f_\beta(0)}{n!} \right) \right| \leq \frac{1}{m^2} \|\phi\|_{\tilde{B}} \to 0$$

as $m \to \infty$. Thus $\phi = \sum_{n=0}^{\infty} \phi_n$ in $G(U)$.

As

$$(\alpha_1, 2^2\alpha_2, \ldots, n^2\alpha_n, \ldots) \in \mathscr{S}$$

for $(\alpha_n)_n \in \mathscr{S}$, it follows by Lemma 3 that for every $x \in U$, we can find $N_x > 0$ such that

$$\sup_\beta \sum_{n=0}^{\infty} n^2 \alpha_n \left\| \frac{\hat{d}^n f_\beta(0)}{n!} \right\|_{W_x} \leq N_x$$

for some neighbourhood W_x of Γ_x. In particular the set

$$B' = \left\{ n^2 \alpha_n \frac{\hat{d}^n f_\beta(0)}{n!} \right\}_{n,\beta}$$

is locally bounded. Let $\phi_\eta \to 0$ in $G(U)$. As every locally bounded subset of $P(^nE)$ is locally bounded in $\mathscr{H}(U)$ and

$$\|(\phi_\eta)_n\|_{\left\{\frac{\hat{d}^n f_\beta(0)}{n!}\right\}_\beta} = \sup_\beta \left|\phi_\eta\left(\frac{\hat{d}^n f_\beta(0)}{n!}\right)\right| \le \frac{1}{n^2\alpha_n}\|\phi_\eta\|_{B'},$$

$(\phi_\eta)_n \to 0$ in $Q(^nE)$ for every n. This shows that $\{Q(^nE)\}_n$ is a Schauder decomposition for $G(U)$. For $\sum_{n=0}^\infty \phi_n \in G(U)$ and $(\alpha_n)_n \in \mathscr{S}$,

$$\begin{aligned}\left\|\sum_{n=k}^\infty \alpha_n\phi_n\right\|_B &\le \sum_{n=k}^\infty |\alpha_n| \|\phi_n\|_B \\ &= \sum_{n=k}^\infty \sup_\beta \left|\phi\left(\alpha_n \frac{\hat{d}^n f_\beta(0)}{n!}\right)\right| \\ &= \sum_{n=k}^\infty \frac{1}{n^2} \sup_\beta \left|\phi\left(n^2\alpha_n \frac{\hat{d}^n f_\beta(0)}{n!}\right)\right| \\ &\le \|\phi\|_{B'} \sum_{n=k}^\infty \frac{1}{n^2}.\end{aligned}$$

This shows that $\{Q(^nE)\}_n$ is an $\mathscr{S}$-decomposition for $G(U)$, and taking $k=0$, we see that the decomposition is $\mathscr{S}$-absolute. □

In a way similar to that in which each $Q(^nE)$ can be identified with a closed subspace of $G(U)$ it can be shown that each $Q(^nE)$ can be identified with a closed subspace of $G(K)$ for K a compact subset of E and E any locally convex space. By a modification of Proposition 4 we have:

Proposition 5. *Let K be a balanced compact subset of a locally convex space E, then $\{Q(^nE)\}_n$ is an $\mathscr{S}$-absolute decomposition for $G(K)$.*

Corollary 6. *If E is a Fréchet space then $Q(^nE) = (P(^nE), \tau_o)'_b$.*

Proof. Both $\{Q(^nE)\}_n$ and $\{(P(^nE), \tau_o)'_b\}_n$ are $\mathscr{S}$-absolute decompositions for $G(K) = (\mathscr{H}(K), \tau_o)'_b$. □

3. DISTINGUISHED PREDUALS OF SPACES OF HOLOMORPHIC FUNCTIONS

In this section we investigate conditions for $G(U)'_b$ to be equal to $(\mathscr{H}(U), \tau_\delta)$. We begin by relating $(\mathscr{H}(U), \tau_o)'_b$ to the $G(K)$'s when U is a balanced open subset of a Fréchet space. We denote by $R_{K,U}$ the map from $\mathscr{H}(U)$ to $\mathscr{H}(K)$ which assigns to each f in $\mathscr{H}(U)$ its restriction to K.

Proposition 7. *Let U be a balanced open subset of a Fréchet space E, then* $(\mathscr{H}(U), \tau_o)'_b = \varinjlim_{\substack{K\subset U \\ K \text{ balanced}}} G(K)$.

Proof. Since $(\mathscr{H}(U), \tau_o)$ and $(\mathscr{H}(K), \tau_o)$ are semi-Montel, we have that $(\mathscr{H}(U), \tau_o)'_b = (\mathscr{H}(U), \tau_o)'_\tau$ and $(\mathscr{H}(K), \tau_o)'_\tau = (\mathscr{H}(K), \tau_o)'_b = G(K)$, where τ denotes the Mackey topology. If K is a balanced compact subset of U then the Taylor series of f about 0 converges to f in $(\mathscr{H}(K), \tau_o)$. Since $\mathscr{H}(U)$ contains all polynomials, $R_{K,U}((\mathscr{H}(U), \tau_o))$ is dense in $(\mathscr{H}(K), \tau_o)$. Therefore the inductive limit

$$(\mathscr{H}(U), \tau_o) = \varprojlim_{\substack{K\subset U \\ K \text{ balanced}}} (\mathscr{H}(K), \tau_o)$$

is reduced. Therefore by IV.4.4 of [15] we have that

$$(\mathscr{H}(U), \tau_o)'_b = \varinjlim_{\substack{K\subset U \\ K \text{ balanced}}} (\mathscr{H}(K), \tau_o)'_b = \varinjlim_{\substack{K\subset U \\ K \text{ balanced}}} G(K).$$

□

Lemma 8. *Let U be a balanced open subset of a Fréchet space E, then* $G(U)'_b = ((\mathscr{H}(U), \tau_o)'_b)'_b$.

Proof. It follows by Grothendieck's Completeness Theorem, Theorem 3.11.1 of [9], that $G(U)$ is the completion of $(\mathscr{H}(U), \tau_o)'_b$. Therefore, $((\mathscr{H}(U), \tau_o)'_b)' = G(U)' = \mathscr{H}(U)$. To show that these two topologies agree it is sufficient to prove that each bounded subset of $G(U)$ is contained in the closure, in $G(U)$, of a bounded subset of $(\mathscr{H}(U),\tau_o)'_b$. Let B be a bounded subset of $G(U)$. Define $\tilde{B}$ by

$$\tilde{B} := \left\{ \sum_{n=0}^{m} \phi_n : \sum_{n=0}^{\infty} \phi_n \in B,\, m \in \mathbb{N} \right\},$$

where $\phi_n = \phi|_{P(^nE)}$. By the proof of Proposition 3.13 of [6], $\tilde{B}$ is bounded and as $\phi_n \in Q(^nE) = (P(^nE), \tau_o)'_b$, $\tilde{B} \subset (\mathscr{H}(U),\tau_o)'_b$. Since $\sum_{n=0}^{m} \phi_n \to \phi$ on the locally bounded (and therefore τ_o-bounded) subsets of $\mathscr{H}(U)$, B is contained in the closure of $\tilde{B}$. This completes the proof. □

The space $G(U)$ is not in general a Fréchet space when U is an open subset of a Fréchet space E. However, our next result shows that $G(U)$ behaves very like a Fréchet space vis-a-vis necessary and sufficient conditions needed in order to show it is distinguished. We will denote by τ_d the topology on $\mathscr{H}(U)$ defined by $(\mathscr{H}(U), \tau_d) = G(U)'_b$.

Theorem 9. *Let U be a balanced open subset of a Fréchet space E, then the following are equivalent:*

(a) $G(U)'_b = G(U)'_i = (\mathscr{H}(U), \tau_\delta)$,
(b) $G(U)$ is distinguished ($G(U)'_b$ is barrelled),
(c) $G(U)'_b$ is infrabarrelled,
(d) $G(U)'_b$ is bornological.

Proof. Since $(\mathscr{H}(U), \tau_\delta)$ is barrelled and bornological we see that (a) will imply (b), (c) and (d).

It follows from Lemma 8 and the fact that $G(U)'_i = (\mathscr{H}(U), \tau_\delta)$, that τ_d is a topology on $\mathscr{H}(U)$ which satisfies $\tau_o \le \tau_d \le \tau_\delta$. It now follows from Corollary 3 of [1] that (b) implies (a). Since $G(U)$ is barrelled, see Theorem 4.4 of [13], it follows by Theorem 3.6.1. of [9], that $G(U)'_b$ is quasi-complete and so (b) is equivalent to (c). Finally, we note that (d) always implies (c). □

Conditions (a) and (b) of the above Theorem are also true in the case where U is a balanced open subset of a DF space where we replace Corollary 3 of [1] by Corollary 5 of [1].

From Lemma 8 and Theorem 9 we see that if U a balanced open subset of a Fréchet space E, then $G(U)'_b = (\mathscr{H}(U), \tau_\delta)'_b$ if and only if the bidual of $(\mathscr{H}(U), \tau_o)$ is equal to $(\mathscr{H}(U), \tau_\delta)$. The question of when $((\mathscr{H}(U), \tau_o))'_b)'_b$ is equal to $(\mathscr{H}(U), \tau_\delta)$, was investigated by Dineen and Isidro in [8]. There they proved the following Proposition.

Proposition 10. *(Dineen-Isidro) Let U be a balanced open subset of a locally convex space E, then $((\mathscr{H}(U), \tau_o)'_b)'_b = (\mathscr{H}(U), \tau_\delta)$ if and only if $(\mathscr{H}(U), \tau_\delta)$ has a basis of absolutely convex τ_o-closed neighbourhoods of* 0.

Thus, in the case where U is a balanced open subset of a Fréchet space E, we see that the sufficient condition given in [13] Theorem 1.1 for $G(U)'_b$ to be equal to $(\mathscr{H}(U), \tau_\delta)$ is in fact also necessary.

As a complemented subspace of a distinguished space is distinguished, and $Q(^nE)$ is complemented in $G(U)$ for every integer n, we see that a necessary condition for τ_d to equal τ_δ is that each $Q(^nE)$ is distinguished. This gives us a means of obtaining Fréchet spaces E such that $G(U)'_b \neq G(U)'_i$ for any balanced open subset U of E. We begin with the observation that $Q(^1E) = E$, and therefore we have that $G(U)'_b \neq (\mathscr{H}(U), \tau_\delta)$ for any balanced open subset of a non-distinguished Fréchet space.

Taskinen, [16], constructs a Fréchet-Montel space F_o, such that $Q(^2F_o) = F_o \hat{\otimes}_{s,\pi} F_o$ is not distinguished. Thus we see that $G(U)'_b \neq (\mathscr{H}(U), \tau_\delta)$ for any balanced open subset U of F_o. This also means that there are Fréchet-Montel spaces such that $G(U)$ is not distinguished.

To apply Theorem 9 and obtain examples of open subsets of Fréchet spaces where $G(U)'_b = (\mathscr{H}(U), \tau_\delta)$ is very difficult. This is because it is hard to show that an arbitrary locally convex space is distinguished. A necessary condition for τ_d to be equal to τ_δ is that each $Q(^nE)$ is distinguished. When this holds we have the following Proposition showing that τ_d is finer than τ_ω.

Proposition 11. *Let E be a Fréchet space such that $Q(^nE)$ is distinguished for every integer n, then, for every balanced open set U in E, τ_d is finer than τ_ω on $\mathscr{H}(U)$.*

Proof. For every balanced compact subset K of E, $G(K)$ is a Fréchet space, and it follows by Proposition 2 of [3], that $G(K)$ is distinguished. Therefore $G(K)'_i = G(K)'_b$, for every balanced compact set K of E. Hence

$$(\mathscr{H}(U), \tau_\omega) = \lim_{\substack{\leftarrow \\ K \subset U}} (\mathscr{H}(K), \tau_\omega) = \lim_{\substack{\leftarrow \\ K \subset U}} G(K)'_i = \lim_{\substack{\leftarrow \\ K \subset U}} G(K)'_b.$$

By the definition of projective limit, $\lim\limits_{\substack{\leftarrow \\ K \subset U}} G(K)'_b$. is weaker than

$$(\lim_{\substack{\rightarrow \\ K \subset U}} G(K))'_b = ((\mathscr{H}(U), \tau_o)'_b)'_b = G(U)'_b.$$

□

If E is a Banach space, $Q(^nE)$ will also be a Banach space for every integer n. In particular, each $Q(^nE)$ is distinguished and therefore by Proposition 11 we have that $\tau_\omega \leq \tau_d \leq \tau_\delta$ on $\mathscr{H}(U)$ for every balanced open subset U of E. Therefore if we know that $\tau_\omega = \tau_\delta$ on $\mathscr{H}(U)$, we can conclude that $G(U)'_b = (\mathscr{H}(U), \tau_\delta)$. For examples of Banach spaces and products of Banach spaces with nuclear spaces with this property we refer to [4,2,7].

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