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Combustion in Hydraulically Resisted Flows*

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Abstract. The effects of hydraulic resistance on premixed gas combustion in tubes and inert porous beds are discussed on the basis of recent research. It is found that the hydraulic resistance causes a gradual precompression and preheating of the unburned gas adjacent to the advancing deflagration which may lead (after an extended induction period) to a localized thermal explosion triggering an abrupt transition from deglagrative to detonative combustion.

The hydraulic resistance has a profound effect also on the structure and velocity of well-settled detonation. At a sufficiently high level of resistance the normal near Chapman-Jouguet detonation is found to undergo a jumpwise hysteretic transition to a low-velocity detonation driven by the developing pressure diffusivity. The lattter mode may even become subsonic, the phenomenon occasionally observed in porous bed combustion.

Combustión en flujos con resistencias hidraúlicas

Resumen. Se discuten, basado en recientes investigaciones, los efectos de la resistencia hidraúlica en la combustión de gases premezclados. Se ha encontrado que la resistencia hidraúlica provoca una precompresión y un precalentamiento gradual de los gases no quemados adyacentes a la deflagración que se propaga, lo que puede conducir (después de un extenso periodo de inducción) a una explosión térmica localizada que desencadena una transición abrupta de la combustión deflagrativa a la detonativa.

La resistencia hidraúlica también tiene un fuerte efecto en la estructura y velocidad de la detonación generada. A un nivel suficientemente alto de resistencia la detonación normal próxima a la de Chapman-Jouguet sufre una caída transicional con histéresis, a una detonación de baja velocidad dominada por la difusividad de presión generada. Este modo puede llegar a ser subsónico, fenómeno ocasionalmente observado en la combustión en lechos porosos.

"The mark of a good theory is to predict correctly
beyond the range of its validity".
— B. GEBHARD

1 Introduction

When modeling a combustion system one may try to include everything that is likely to be of quantitative importance, or one may intentionally ignore certain aspects in order to elucidate the impact of those that are retained.

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The first approach is necessary if the goal is to obtain numbers for comparison with experimental measurements or for the design of practical devices. The second approach is of great value when the goal is to gain physical insight by making the problem tractable.

Identification of the elementary mechanisms by the appropriate distillation is quite an art and often requires overcoming certain psychological barriers of conventional perception and reasoning. For example, no one would normally object to simplifications treating activation energy as a large parameter, since this is nothing but a formal recognition of the reaction rate's strong temperature dependence, a fact well known experimentally. However, if say, for studying the galloping detonation, one decides to adopt the small-heat-release approximation, this will, most likely be met with a great deal of skepticism by an experimentally oriented audience as a mathematical extravagance contradicting the very essence of detonative combustion. Indeed, in real-life systems the heat release is never small. And yet, the small-heat-release approximation often proves to be remarkably useful in interpreting even rather subtle and complex effects, and far beyond its nominal range of validity. This success is not entirely surprising. Stemming from a rational asymptotics, the small-heat-release based model relates to the original fully nonlinear formulation as, descriptively speaking, a tadpole relates to the fully developed frog. For all their morphological distinctions both entities share the same genetic code with all the intriguing consequences this entails. In the same vein, deliberately inflated or compressed parameters may be useful in securing physical understanding through numerical simulations when working with realistic numbers in prohibited by practical restrictions on the accessible resolution.

Let me now say a few words on what is called premixed gas combustion.

Premixed gas combustion is the combustion of gaseous reactants (say hydrogen and oxygen) which are perfectly mixed prior to ignition. Although premixed combustion may occur homogeneously throughout the volume, this mode is not typical. The most distinctive feature of premixed combustion is the ability to assume the form of a self-sustained reaction wave propagating subsonically or supersonically at a well-defined speed. The subsonic combustion waves are generally called premixed gas flames or deflagrations, while the supersonic combustion waves are called detonations. Typical flame velocities fall in the range from a few centimeters per second to a few meters per second. So the flames are essentially subsonic waves. Unlike flames, typical detonation velocities vary from a thousand to three thousand meters per second. So detonation waves are normally supersonic.

Flames and detonations differ not only in their velocities but also in the driving mechanisms. Flames are sustained by the heat flux from the hot burned gas to the cold unreacted mixture. Detonations are sustained by adiabatic compression (and thereby preheating) of the reactive mixture passing through the shock. Normally, flames are initiated by a mild energy discharge – for example by a spark, and detonations are generally provoked by shock waves – for example by a localized explosion.

Mathematically speaking, flames and detonations may be perceived as stable attractors, each being linked to its own base of initial data. Roughly speaking this is indeed the case, although in non-ideal real-life conditions the picture may be more involved. In practical applications, for safety reasons, premixing is generally avoided. Yet, there are several important applications of premixed gas combustion, the principal being: spark-ignition engines, lean-burn gas turbines household burners, and of recently renewed interest, pulsed detonation propulsion.

Apart from its technological relevance, premixed combustion constitutes a truly fascinating dynamical system, and displays an amazingly rich variety of phenomena such as: non-uniqueness of possible propagation regimes, their birth (ignition) and destruction (extinction), chaotic self-motion and fractal-like growth, and various hysteretic transitions.

In the last 30 years, the theory of combustion waves has developed to rather a high level of precision and conceptual clarity. Many perplexing phenomena have been analyzed and described, and many controversial questions have been resolved. Yet, for all these advances there are quite a few first-order effects, rooted in the very fundamentals of the field, whose first-principle understanding is still far from adequate. Among the challenges one may mention for example, understanding of the multiplicity of detonation regimes and spontaneous transition from deflagrative to detonative combustion. These are precisely the topics of my talk today.

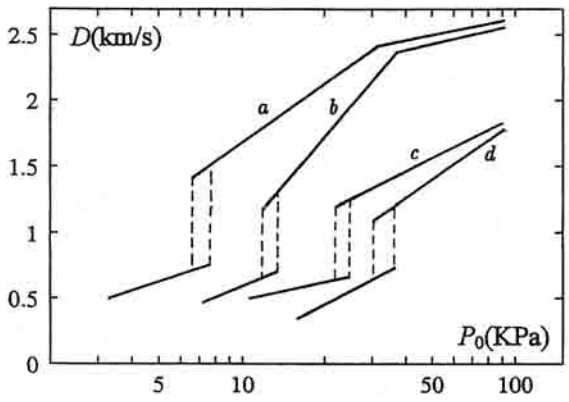

Figure 1. Detonation velocity vs. initial pressure in corrugated channels; $C_2H_2 + 1.5O_2$, (a, b); $C_3H_8 + 5O_2$, (c, d). Note the hysteretic character of the jumps ([15]).

2 Multiplicity of Detonation Regimes in Hydraulically Resisted Flows

Since the early works of Laffitte [8] and Shchelkin [13] it is known that the flow impediments may have a profound effect on gaseous detonation markedly reducing is propagation velocity compared to the associated Chapman-Jouguet value. Moreover, as was discovered later, the obstacle-affected detonation can exhibit fascinating jumpwise transitions from high-velocity sub-CJ quasi-detonation to a low-velocity detonation in a choking regime [15, 9, 4, 11].

On Figure 1 one can see the typical response of the detonation velocity to the initial pressure in channels filled with obstacles. Whereas both detonation modes are supersonic relative to the fresh mixture, the choking regime is subsonic relative to the burnt gas.

It transpires that this curious peculiarity of the obstacle-affected detonation may be successfully described within the framework of the quasi-one-dimensional ZND-Fanno model.

In ZND-Fanno formulation the presence of obstacles is accounted for through augmentation of the momentum equation by the appropriate drag-force term simulating momentum loss but leaving the equations of mass and energy conservation unaltered.

In the frame of reference attached to the advancing detonation, the set of equations describing its aero-thermo-chemical structure reads,

$$d[\rho(u-D)]/dx = 0, \quad \text{continuity} \tag{1}$$

$$d[\rho(u-D)u + P]/dx = f, \quad \text{momentum} \tag{2}$$

$$d[\rho(u-D)(c_vT + 1/2u^2) + Pu]/dx = Q\omega, \quad \text{energy} \tag{3}$$

$$d[\rho(u-D)C]/dx = -\omega, \quad \text{concentration} \tag{4}$$

$$P = (c_p - c_v)\rho T, \quad \text{state.} \tag{5}$$

Here D, u are the detonation and gas flow velocities in the laboratory frame of reference; T, C, P, ρ are the temperature, deficient reactance concentration, pressure and density. ω is the reaction rate defined by a one-step first-order Arrhenius kinetics, $\omega = A\rho C \exp(-E/RT)$. The drag force f is specified as $f = -2c_f\rho u|u|/d$ where d is the hydraulic diameter and c_f is the drag factor assumed to be constant. For porous beds $c_f = 0.58$. For well-shaken spherical packings $d = 0.44d_p$, where d_p is the particle diameter. To reduce the number of parameters involved the effects due to molecular transport and heat losses are discarded. Indeed in porous media, or very rough tubes, the momentum loss is a dominating influence, so the heat losses may often be ignored.

The detonation is assumed to propagate through an initially quiescent homogeneous premixture whose temperature, pressure, density, and deficient reactant concentration are regarded as prescribed. Hence the boundary conditions read,

$$T(+\infty) = T_u, \qquad C(+\infty) = C_u, \qquad \rho(+\infty) = \rho_u \ ,$$
$$P(+\infty) = P_u = (c_p - c_v)\rho_u T_u, \qquad u(+\infty) = 0 \ . \tag{6}$$

Far behind the combustion wave, due to the flow deceleration and the reactant's consumption

$$u(-\infty) = 0, \ C(-\infty) = 0 \ . \tag{7}$$

As may be easily shown, under these conditions,

$$T(-\infty) = T_u + QC_u/c_v = T_b \ ,$$
$$P(-\infty) = (c_p - c_v)\rho_u T_b = P_b \ . \tag{8}$$

Thus, the final temperature (T_b) and pressure (P_b) of the burned gas appear to be identical to those reached in the constant volume adiabatic explosion. Due to the conservation of the mass and enthalpy, the original set of equations is reduced to a single first-order o.d.e. for the density ρ as a function of the concentration C,

$$d\rho/dC = F(\rho, C) \ , \ 0 < C < C_u \ . \tag{9}$$

This equation should be solved subject to the following boundary conditions: For the supersonic propagation ($D > a_u$), $\rho(C_u) = \rho_s(D)$, the density behind the shock, and $\rho(0) = \rho_u$, the density in the stagnant combustion products, which because of the friction coincides with the density of the burned gas. For the subsonic propagation ($D < a_u$), $\rho(C_u) = \rho_u$, $\rho(0) = \rho_u$. One therefore ends up with an overdetermined boundary-value problem which should yield both the solution $\rho = \rho(C)$, and the propagation velocity D. The problem is solved numerically by a conventional shooting technique (Figure 2).

The multiplicity of detonation regimes arises as a product of the interplay between two mechanisms controlling adiabatic compression: shock and drag-induced diffusion of pressure.

For quasi-detonation (low hydraulic resistance) the process is dominated by the first (shock) mechanism, while for the choking regime (strong hydraulic resistance) by the second (diffusion of pressure).

In obstacle-laden systems (e.g. porous beds) the pressure diffusivity $\mathcal{D}_{\rm bar}$, may easily be as high as 10^6 times the thermal diffusivity $\mathcal{D}_{\rm th}$, which may well result in supersonic propagation.

In the choking regime the shock plays a totally subordinate role. In contrast to the CJ or quasi-detonation, here the shock is not a driving agency but rather a by-product of supersonic propagation. If the reaction wave is forced to move supersonically it has no choice but to be accompanied by a shock.

If the level of hydraulic resistance is high enough, the pressure diffusivity $\mathcal{D}_{\rm bar}$ drops which may well result in a fast but subsonic and therefore shockless propagation. Propagation velocity D falls below the sonic velocity of the unburnt gas, a_u. Fast subsonic regimes are indeed occasionally observed in porous bed combustion. In 1987 Lyamin and Pinaev published a paper, "*Fast Subsonic Combustion in an Inert Porous Media with a Smooth Pressure Rise in the Wave*", a title which speaks for itself.

The subsonic mode emerges as a smooth continuation of the supersonic one and hence may well be referred to as subsonic detonation, two words that one would have thought could never be connected! The inclusion of the word detonation in the new definition is intended to emphasize a crucial dynamical ingredient unifying all regimes, namely adiabatic compression. The idea of extending the concept of detonation over the subsonic domain is not new.

In the 1987 survey by Mitrofanov, when discussing the non-classical combustion waves the author wrote,

> "*It would seem appropriate to extend the notion of detonation over a certain subsonic range of wave velocities ($D < a_u$) where there is a continuous in D passage to this domain with the preservation of pressure and density peaks with the front.*"

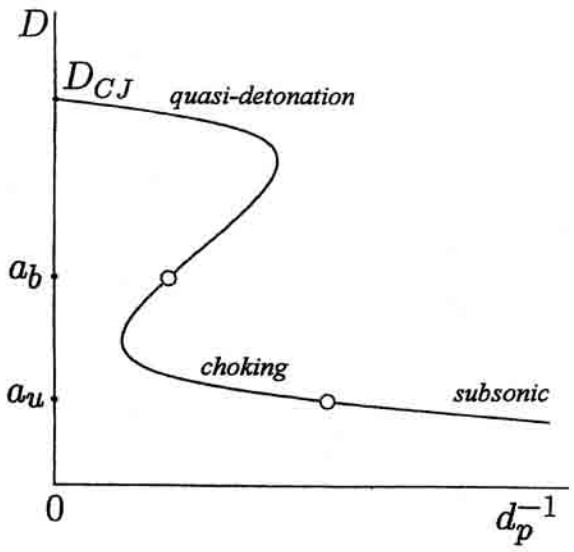

Figure 2. Detonation velocity D vs. reciprocal of the porous bed particle diameter d_p as implied by ZND-Fanno model; adiabatic limit. D_{CJ}, a_u, a_b correspond to the Chapman-Jouguet detonation and sound velocities in the unburnt and burnt gas, respectively. The appropriately scaled inverse diameter d_p^{-1} may serve as a measure of hydraulic resistance. Note that for bimolecular reactions the impact of the particle diameter d_p is similar to that of the initial pressure P_u ([2]).

It is interesting that this issue was raised by the experimentalists 10 years ahead of the theoretical substantiation of the phenomenon. Apart from being sustained by adiabatic compression, the subsonic detonation shares several other features with the supersonic one. Similar to the latter, the subsonic detonation is prone to galloping and spinning instabilities.

The instability onset is basically controlled by the product,

$$E(T_b - T_u)(1 - \gamma^{-1})/RT_b^2 \ ,$$

a structure similar to that arising in conventional detonation. This again reflects the common nature of both processes. Note, that the temperature appearing in the Arrhenius exponent is the temperature at the entrance to the reaction zone. At near-sonic propagation, $D \sim a_u$, the problem may be tackled analytically. For the Arrhenius kinetics, $W = A\rho C \exp(-(E/RT))$, and the Forchheimer (quadratic) drag law, $f = 2c_f \rho u|u|/d$, the propagation velocity D is given by the relation,

$$\frac{2c_f(1-\sigma)(1-\gamma^{-1})}{Aa_b^2 d} D^3 = \exp\left(-\frac{E}{RT_+}\right). \tag{10}$$

Here $\sigma = T_u/T_b$ and $T_+ = T_u + (1 - \gamma^{-1})(T_b - T_u)$. T_+ is the temperature at the 'entrance' to the reaction zone (Figure 3). With the appropriate choice of parameters the low-velocity propagation may be either supersonic or subsonic.

For example, at $A = 10^{10} s^{-1}$, $E/R = 10,000^\circ K$, $a_u = 350\,\mathrm{m/s}, \sigma = T_u/T_b = 0.15$, $\gamma = 1.3, \varphi = 0.4$,

$$D = 432\,\mathrm{m/s} \text{ at } d_p = 0.5\,\mathrm{cm} \qquad \text{(supersonic)}$$
$$D = 253\,\mathrm{m/s} \text{ at } d_p = 0.1\,\mathrm{cm} \qquad \text{(subsonic)}$$

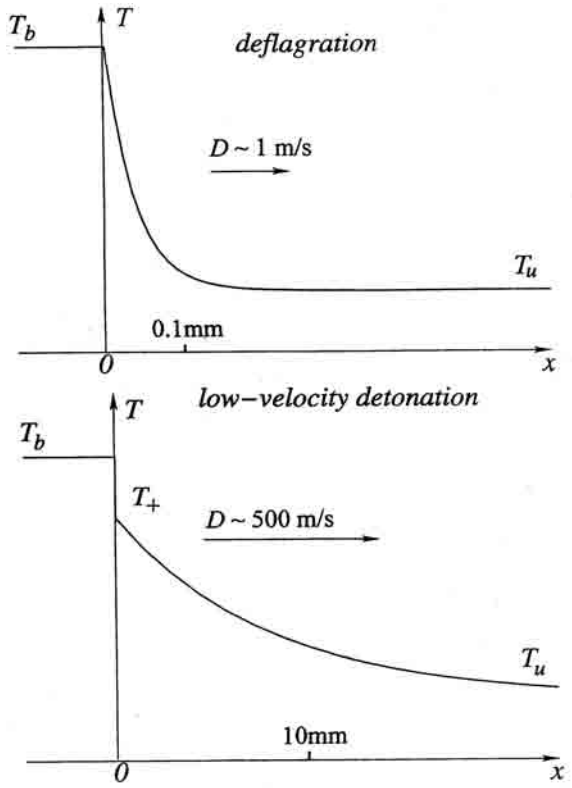

Figure 3. Temperature profiles in deflagration and low-velocity detonation waves. Note a strong disparity between the preheat zone widths and propagation velocities.

In the low-velocity detonation the inertial effects ahead of the reaction zone are weak compared to the effects due to hydraulic resistance. As a result one ends up with the following equation for the pressure $P(x)$,

$$-D\frac{d}{dx}\left(\frac{P}{a^2}\right)=\frac{d}{dx}\left(\frac{d}{\gamma c_f|u|}\frac{dP}{dx}\right) \tag{11}$$

Comparing its right and left sides, the combination $a^2 d/\gamma c_f|u|$ may naturally be viewed as an effective pressure diffusivity,

$$\mathcal{D}_{\rm bar}=\frac{a^2 d}{\gamma c_f|u|} \tag{12}$$

At the entrance to the reaction zone the pressure diffusivity is given by the relation,

$$\mathcal{D}_{\rm bar}(T_+)=a^2(T_+)d/\gamma c_f u(T_+) \tag{13}$$

where $u(T_+)=D(1-a^2(T_+)/a_b^2)$. In terms of $\mathcal{D}_{\rm bar}(T_+)$ the expression for the propagation velocity D may be recast as

$$D^2=\frac{A\mathcal{D}_{\rm bar}(T_+)}{1-\gamma^{-1}(\sigma+(1-\sigma)(1-\gamma^{-1}))}\exp\left(-\frac{E}{RT_+}\right) \tag{14}$$

which is quite similar structurally to the equation for deflagrative combustion driven by the thermal diffusivity. Recall that for the deflagrative combustion there is a classical Zel'dovich-Frank-Kamenetsky relation ([16]),

$$D^2=\frac{2A\mathcal{D}_{\rm th}}{(1-\sigma)^2}\left(\frac{RT_b}{E}\right)^2\exp\left(-\frac{E}{RT_b}\right) \tag{15}$$

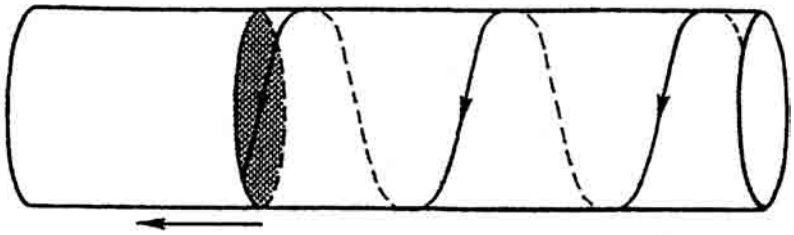

Figure 4. Profiles of the reduced pressure $\Pi = (P-P_u)/(P_b-P_u)$ at several equidistant instants of time adjacent to the point of transition from deflagrative to detonative, yet subsonic, propagation. Here P_u and P_b are the pressures in the unburned and burned gas, respectively; ξ is the scaled coordinate, in units of $\mathcal{D}_{\rm bar}/D^0_{\rm det}$; $D^0_{\rm det}$ is the velocity of steady detonation, and $\mathcal{D}_{\rm bar}$ is the pressure diffusivity ([1])

Here, $\mathcal{D}_{\rm th}$ is the thermal diffusivity; $T_b = T_u + QC_u/c_p$, adiabatic temperature of combustion products; $\sigma = T_u/T_b$. On Figure 3 one can see the typical profiles of temperature in deflagrative and low-velocity detonative combustion. For the sonic propagation $D = a_u$, and for example, at $a_u = 350\,\mathrm{m/s}$, $\sigma = 0.15$, $\varphi = 0.4$, $\gamma = 1.3$, $d_p = 0.5\,\mathrm{cm}$,

$$\mathcal{D}_{\rm bar}(T_+) = 1.8 \cdot 10^4\,\mathrm{cm}^2/s\,.$$

The developing pressure diffusivity thus may be as high at 10^4 times the typical thermal diffusivity $\mathcal{D}_{\rm th}$ at the same pressure and temperature, which explains its ability to support supersonic propagation effectively without input from the upstream shock. The presence or absence of the shock in the upstream flow does not affect the picture in the reactive layer. Both subsonic and low-velocity supersonic modes are sustained by the same mechanism: *the drag-induced diffusion of pressure.*

When the high-activation-energy limit $E/RT_b \gg 1$ is combined with the small-heat-release approximation, so that the product $\beta = E(T_b - T_u)/RT_b^2$ (Zel'dovich number) remains finite, the dynamics of the subsonic detonation may be described by a free-interface problem for a single diffusion equation. In the appropriately chosen units it reads,

$$\Pi_\tau = \nabla^2\Pi + \Omega(\Pi)\delta_F\ ,\ V_F = -\Omega(\Pi)\ . \tag{16}$$

Here $\gamma = c_p/c_v$, $\Pi = (P - P_u)/(P_b - P_u)$ is the reduced pressure that vanishes far ahead of the front, $F = 0$, and approaches unity far behind. δ_F is the surface δ - function, $V_F = F_\tau/|\nabla F|$ is the normal velocity of the interface, and Ω is the Arrhenius-type source intensity,

$$\Omega(\Pi) = \exp[\alpha(\Pi - 1)/(\Sigma + (1 - \Sigma)\Pi)] \tag{17}$$

where $\alpha = 1/2\beta(1 - \gamma^{-1})/(1 - \gamma^{-1}(1 - \sigma))^2$, and $\Sigma = \gamma\sigma(\gamma - 1 + \sigma) < 1\ ,\ \sigma = T_u/T_b < 1$.
At $\alpha > \alpha_{\rm cr} = 4$ the steady planar detonation becomes unstable, and one ends up with the oscillatory propagation ([3]). At higher α the oscillations undergo period-doubling and become chaotic ([5]). In the $3D$-geometry the oscillatory subsonic detonation may assume a spinning mode (Figure 4). However, the most remarkable aspect of subsonic detonation is its ability to evolve spontaneously from deflagrative combustion. Interesting in itself, this outcome brings one closer to understanding the deflagration-to-detonation transition in the supersonic domain, still one of the major challenges in the field.

3 Deflagration-to-Detonation Transition

The transition from deflagrative to detonative combustion remains one of the major puzzles of combustion theory.

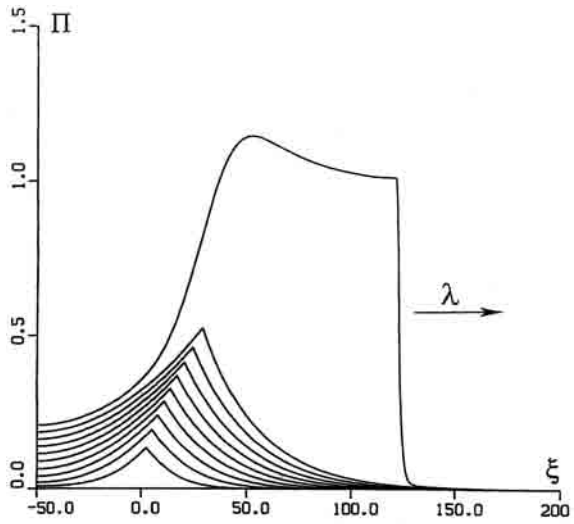

Figure 5. Trajectory of point of maximum temperature in spinning detonation. Shaded cross-section corresponds to instantaneous position of wave front. Horizontal arrow indicates direction of wave propagation.

It has long been observed that the transition is reluctant to occur in unconfined obstacle-free systems. Yet, the transition may be significantly facilitated in the presence of walls and flow impediments.

Apart form inducing hydrodynamic disturbances and thereby affecting the deflagration speed, the obstacles also exert resistance to the gas flow causing reduction of its momentum. As discussed earlier, the hydraulic resistance is the principal agency controlling the detonation velocity deficit and multiplicity of detonation regimes. It transpires that the hydraulic resistance (friction) is also of relevance to the transition problem. The friction appears to be capable of triggering the transition even if the multidimensional effects, such as the flame acceleration due to folding, are completely suppressed and the system is regarded as effectively one-dimensional with the confinement being accounted for through the velocity-dependent drag-force term added to the momentum equation (ZND – Fanno model). To demonstrate the phenomenon in its most 'stripped down' form, it is helpful to begin with the limit of strong hydraulic resistance where the emerging detonation falls into the subsonic range. In this situation one may neglect the inertial effects and take Darcy's law as the momentum equation.

As an additional simplification is is helpful to adopt the small-heat-lease (SHR) approximation where the variations of temperature, pressure, density and flow-velocity are regarded as small and, hence, the nonlinear effects may be ignored everywhere but in this reaction-rate term, generally highly sensitive even to minor temperature changes. In the SHR formulation the problem becomes much more tractable mathematically without, however, compromising the crucial features of the original fully nonlinear system. In suitably chosen units the resulting set of governing equations reads

$$\gamma\Theta_\tau - (\gamma-1)\Pi_\tau = \gamma\varepsilon\Theta_{\xi\xi} + \Omega(\Phi,\Theta) \tag{18}$$

$$\Phi_\tau = \varepsilon Le^{-1}\Phi_{\xi\xi} - \Omega(\Phi,\Theta) \tag{19}$$

$$\Pi_\tau - \Theta_\tau = \Pi_{\xi\xi} \tag{20}$$

Equations (18) and (19) represent the partially linearized conservation equations for energy and the deficient reactant. Equation (20) is a linearized continuity equation, incorporating the equation of state and momentum (Darcy's law). $\Theta = (T-T_u)/(T_b-T_u)$ is the scaled temperature, $\Pi = (P-P_u)/(P_b-P_u)$ the scaled pressure, $\Phi = C/C_u$ the scaled concentration, $\Omega(\Phi,\Theta) \sim C\exp(-E/RT)$ the scaled reaction-rate.

ξ, τ are the appropriately scaled spatio-temporal coordinates; $\gamma = c_p/c_\nu$, $Le =$ Lewis number.

$$\varepsilon = \mathcal{D}_{\rm th}/\mathcal{D}_{\rm bar}$$

is the thermal diffusivity/pressure diffusivity ratio.

In the adopted formulation the pressure diffusivity is defined as

$$\mathcal{D}_{\rm bar} = K a_u^2/\gamma\nu$$

where, $K =$ porous bed permeability, $\nu =$ kinematic viscosity, $a_u =$ sonic velocity in the unburnt gas.

Because of the a_u^2 - factor, the pressure diffusivity $\mathcal{D}_{\rm bar}$ may be enormous compared with the thermal diffusivity $\mathcal{D}_{\rm th}$. For many realistic porous systems $\varepsilon = \mathcal{D}_{\rm th}/\mathcal{D}_{\rm bar}$ varies within the range 10^{-4}–10^{-7} which makes it a natural small parameter of the problem. At small ε one may single out two distinct modes of combustion,

(i) the fast wave sustained by the diffusive transfer of pressure (subsonic detonation), and

(ii) the slow wave sustained by the diffusive transfer of heat (deflagration).

In the subsonic detonation regime for the leading order asymptotics the original model (18)–(20) simplifies to

$$\gamma\Theta_\tau - (\gamma-1)\Pi_\tau = \Omega(\Phi,\Theta)\,, \tag{21}$$

$$\Phi_\tau = -\Omega(\Phi,\Theta)\,, \tag{22}$$

$$\Pi_\tau - \Theta_\tau = \Pi_{\xi\xi}\,. \tag{23}$$

This shortened system admits to the traveling wave solution

$$\Theta = \Theta(\xi - \lambda_\tau), \Pi = \Pi(\xi - \lambda\tau), \Phi = \Phi(\xi - \lambda\tau)$$

propagating at velocity $\lambda \sim 1$. In the deflagration regime $\Pi \sim \sqrt{\varepsilon}, \xi \sim \sqrt{\varepsilon}$ and for the leading order asymptotics the original model, yields

$$\gamma\Theta_\tau = \gamma\varepsilon\Theta_{\xi\xi} + \Omega(\Phi,\Theta)\,, \tag{24}$$

$$\Phi_\tau = \varepsilon Le^{-1}\Phi_{\xi\xi} - \Omega(\Phi,\Theta)\,, \tag{25}$$

$$\Pi_{\xi\xi} = 0\,. \tag{26}$$

The system is obviously nothing but a conventional constant-density model for the free-space deflagration.

The associated traveling-wave solution spreads at a velocity proportional to the square root of the thermal diffusivity, $\lambda \sim \sqrt{\varepsilon}$. The higher-order approximation for subsonic detonation, i.e. incorporation of the thermal diffusivity effects, does not produce any significant change in the overall dynamical picture. There still exists a steady traveling wave solution with $\lambda \sim 1$. For the deflagration the picture is different. Here, the higher order approximation, i.e., accounting for the porous bed resistance, leads to the local elevation of pressure. The pressure, however, does not stabilize at some low level but rather keeps growing as $\Pi \sim \sqrt{\varepsilon\tau}$. (Figure 5). The gradual pile-up of the pressure results in the formation of an extended preheat zone ahead of the advancing and slightly accelerating flame.

The fresh premixture adjacent to the flame undergoes a low-gradient precompression and preheating. This slowly proceeding development ultimately ends up as an adiabatic explosion which abruptly converts the burning process from deflagration to subsonic detonation (Figures 5 and 6). Subsequently it was realized that the frictional mechanism 'works' also at low hydraulic resistance where the detonation spreads at a supersonic speed (Figures 7 and 8). Here the pertinent model, understandably, involves both chemical and hydrodynamic nonlinearities (inertial effects). In both subsonic and supersonic cases the transition is triggered by a localized thermal explosion, which in turn is conditioned by the spatial gradient of induction time

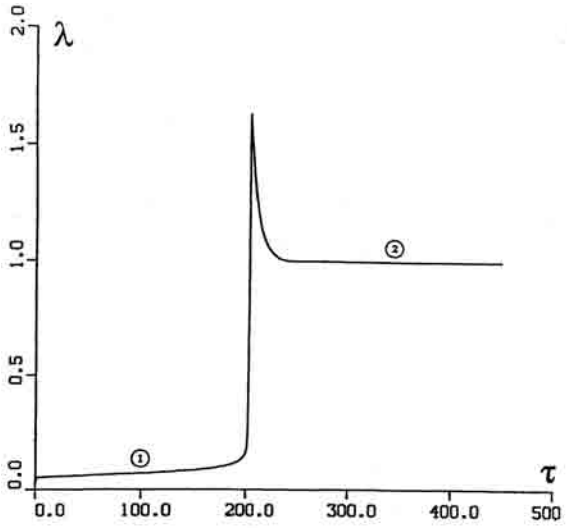

Figure 6. Time-record of the reaction wave speed. Here λ and τ are the scaled velocity and time in units of D^0_{det} and $\mathcal{D}_{\rm bar}/(D^0_{\rm det})^2$ respectively, (1) (2) mark deflagrative and detonative propagation, respectively, ([1]).

in the friction-induced extended preheat-zone formed ahead of the advancing deflagration. The DDT problem therefore transpires to be physically related to the shockless initiation of detonation in non-uniformly preconditioned gas, a topic that has been energetically explored in recent years.

Unlike the latter systems, however, in the DDT case the non-homogeneous environment, which provides the required spatial gradient of induction time, is not prescribed but arises as a product of the flame-confinement interaction. Mathematically speaking, the friction simply destroys deflagration as an equilibrium traveling-wave solution.

Yet, at low friction and appropriate initial conditions the deflagrative mode is feasible but only as a transient slowly evolving wave with a long but limited life-span.

Understandably, various assumptions underlying the ZND–Fanno model make its implications more of a qualitative guide than a quantitative prediction. Besides, there are some salient features of the transition which are not covered by the model. One of them is the predetonational acceleration of the advancing flame, which is virtually absent in the one-dimensional description ignoring the continuous growth of the flame area. It is therefore desirable to invoke the spatial picture of the transition where the hydraulic resistance is determined directly by the boundary conditions rather than through the effective drag-force.

Yet, since real-life DDT is hardly ever laminar, its direct numerical simulation not unexpectedly meets with formidable difficulties. In the survey of Shepherd and Lee ([14]), when discussing the state of the art in the numerical modeling of DDT, the authors wrote,

> "*No method has been developed yet that is capable of accurately predicting a complete flame acceleration and transition to detonation event ... This problem is due in large measure to the practical limitations on spatial and temporal resolutions in multidimensional computations*".

The theoretical findings based on the ZND–Fanno model suggest that the complexities due to turbulence are likely to be largely irrelevant to the transition which is presumably triggered by the flow deceleration in the boundary layer, irrespective of whether the bulk flow is turbulent or not.

Hence, in order to reproduce the spatial picture of the transition it may be instructive to begin with a 2D thermally-insulated channel, narrow enough to ensure the laminar character of the developing flow with all

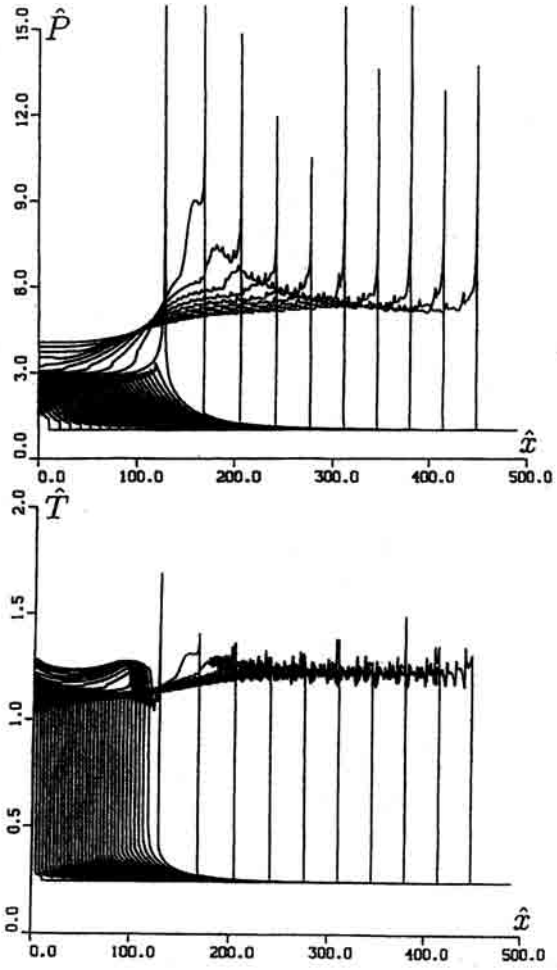

Figure 7. Friction affected dynamics. Profiles of scaled pressure $(\hat{P})$ and temperature $(\hat{T})$ at several consecutive equidistant instants of time adjacent to the point of DDT ([2]).

the technical advantages this entails. Figure 9 shows the first results obtained along these lines. It transpires that even extremely narrow (10 flame-width wide) adiabatic capillaries are perfectly capable of capturing the transition.

The two-dimensional simulations reproduce the formation of the tulip-like flame and its predetonational acceleration, well-know experimentally but clearly unobtainable within the one-dimensional approach employed in the ZND – Fanno formulation.

As expected, the detonation first develops in the boundary layer where the effect of hydraulic resistance is stronger, and thereupon spreads over the channel interior.

The previous findings pertain to the adiabatic limit where the system is regarded as thermally insulated. This is a major idealization. Realistic confined systems are invariably affected by heat losses. The hydraulic resistance and heat losses exert opposite effects on the transition. The resistance raises the local temperature (through adiabatic compression) and thereby promotes autoignition. The heat loss tends to reverse this trend by reducing the temperature. In smooth channels, due to Reynolds analogy, both mechanisms are of comparable influence. Therefore one cannot be certain about the final outcome of the competition between resistance and heat losses. Experimentally however, an often successful transition is an undeniable fact. As observed in our recent studies [7], with the channel walls maintained at the ambient temperature, and

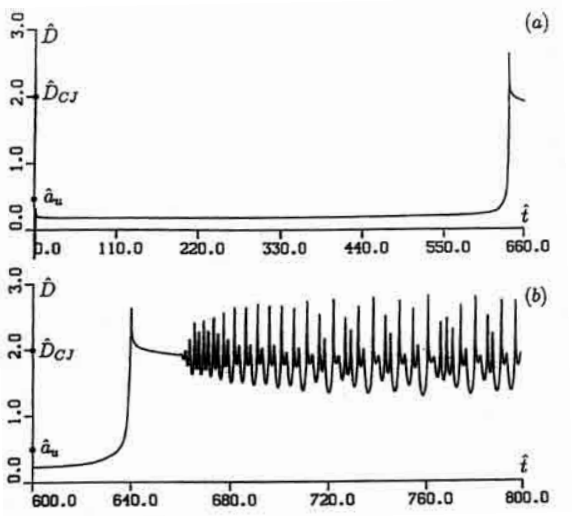

Figure 8. Time-record of the reaction wave speed $\hat{D}$ (in units of a_b) from the ignition $(\hat{t} = 0)$ to the point of thermal explosion (a), and formation of galloping detonation (b). $\hat{a}_u$, $\hat{D}_{CJ}$ correspond to the sonic velocity in the fresh mixture, and the Chapman-Jouguet detonation, respectively ([2]).

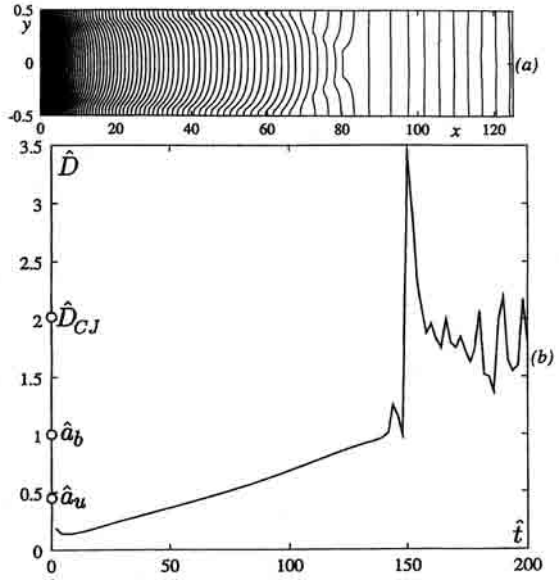

Figure 9. Numerical simulation of the transition in a narrow channel with thermally insulating and non-slip boundary conditions. (a) Reaction zone configurations at several consecutive equidistant instants of time, and (b) the corresponding velocity-time record. Note the disparity between the transversal and longitudinal scales: 25-fold compression ([6]).

the reaction kinetics assumed monomolecular, the transition does not occur, at least within the parameter range explored. However, for the bimolecular kinetics (other conditions as in the monomolecular case) the transition proved readily feasible (Figures 10 and 11). Higher molecularity implies a higher sensitivity of the explosive mixture to the pressure change which in these problems is quite significant. In contrast to the

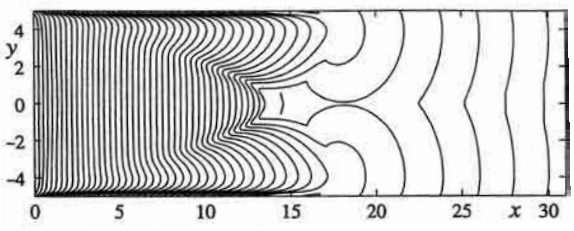

Figure 10. The reaction zone configuration at several consecutive equidistant instants of time near the transition point ([7]).

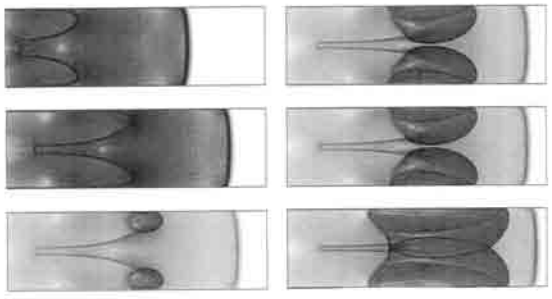

Figure 11. A sequence of the Schlieren-like tonal images of the pressure gradient norm. Here one readily identifies the evolving flame, leading shock, emergence of detonation and retonation waves, as well as transverse shocks induced by the shock-wall interaction ([7]).

adiabatic case, the incipient detonation emerges at a certain distance from the channel's wall. At the wall itself (due to the temperature drop) the reaction is suppressed.

The above numerical solution pertains to a parameter range where certain features typical of the transition in wide tubes are suppressed (e.g. turbulence, remote initiation). This certainly doesn't mean that the found transition mode, and that occurring in wide tubes, are different animals. What one encounters here, I believe, is precisely the relation between the frog and its tadpole. Moreover, as Ghengis Khan used to say, "*The world is too small a place to afford two rulers*", or in our language, "*Confined combustion is too simple a system to afford two mechanisms of the transition*!"

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