RACSAM
Rev. R. Acad. Cien. Serie A. Mat.
VOL. **101** (1), 2007, pp. 81–86
Geometría y Topología / Geometry and Topology

On the family of cyclic trigonal Riemann surfaces of genus 4 with several trigonal morphisms

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Abstract. A closed Riemann surface which is a 3-sheeted regular covering of the Riemann sphere is called cyclic trigonal, and such a covering is called a cyclic trigonal morphism. Accola showed that if the genus is greater or equal than 5 the trigonal morphism is unique. Costa-Izquierdo-Ying found a family of cyclic trigonal Riemann surfaces of genus 4 with two trigonal morphisms. In this work we show that this family is the Riemann sphere without three points. We also prove that the Hurwitz space of pairs (X, f), with X a surface of the above family and f a trigonal morphism, is the Riemann sphere with four punctures. Finally, we give the equations of the curves in the family.

Sobre la familia de superficies de Riemann de género 4 cíclicas trigonales con varios morfismos trigonales

Resumen. Una superficie de Riemann que es una cubierta regular de 3 hojas de la esfera se llama *cíclica trigonal*, y la cubierta un *morfismo trigonal*. Accola probó que el morfismo trigonal es único si el género de la superficie es mayor o igual que 5. Costa-Izquierdo-Ying encontraron una familia de superficies de Riemann de género 4 cíclicas trigonales con varios morfismos trigonales. En este trabajo demostramos que dicha familia es, en efecto, la esfera de Riemann con tres punzamientos. Además demostramos que el espacio de Hurwitz de pares (X, f), con X una surperficie en la familia anterior y f un morfismo trigonal, es la esfera de Riemann con cuatro punzamientos. Finalmente encontramos las ecuaciones de las curvas en la familia.

1 Introduction

A closed Riemann surface X which can be realized as a 3-sheeted covering of the Riemann sphere is said to be *trigonal*, and such a covering is called a *trigonal morphism*. If the trigonal morphism is a cyclic regular covering, then the Riemann surface is called *cyclic trigonal*. This is equivalent to X being a curve given by a polynomial equation of the form $y^3 + c(x) = 0$.

Trigonal Riemann surfaces have been recently studied (see [2] and [12]). By Lemma 2.1 in [1], if the surface X has genus $g \geq 5$, then the trigonal morphism is unique. The Severi-Castelnouvo inequality is used in order to prove such uniqueness, but this technique is not valid for small genera. Costa-Izquierdo-Ying proved in [7] that this bound is sharp: Using the characterization of trigonality by means of Fuchsian groups ([6]), the family ${}^{36}\mathcal{M}_4^3 = \{X_4(\lambda)\}$ of cyclic trigonal Riemann surfaces of genus four admitting several cyclic trigonal morphisms was obtained.

Presentado por Pedro Jiménez Guerra.
Recibido: 10 de noviembre de 2006. *Aceptado*: 13 de diciembre de 2006.
Palabras clave / Keywords: Trigonal Riemann surfaces, trigonal morphisms, automorphisms group.
Mathematics Subject Classifications: Primary 14H15; Secondary 30F10

Our main result establishes that the space ${}^{36}\mathcal{M}_4^3$ of cyclic trigonal surfaces of genus 4 admitting two trigonal morphisms is a Riemann sphere without three points. To prove this, we prove that there is a unique class of actions of $D_3 \times D_3$ on the Riemann surfaces $\{X_4(\lambda)\}$. See [4], [5] and [13].

A Hurwitz space is a space formed by pairs (X_g, f), where X_g is a Riemann surface of genus g and $f : X \to \widehat{\mathbb{C}}$ a meromorphic function. These spaces are widely studied in algebraic geometry and mathematical physics. See, for instance, [3], [9] and [15]. We consider the Hurwitz space $\mathcal{H}$ of pairs (X, f), where $X \in {}^{36}\mathcal{M}_4^3$ and $f : X \to \widehat{\mathbb{C}}$ is a cyclic trigonal morphism. We obtain that $\mathcal{H}$ is a two fold connected covering of ${}^{36}\mathcal{M}_4^3$ and then $\mathcal{H}$ is a Riemann sphere without four points. Finally we obtain the equations for the algebraic curves in the family.

In general, given a prime number p, a closed Riemann surface X which is a p-sheeted covering of the Riemann sphere is said to be *p-gonal*, and such a covering is called a *p-gonal morphism*. If the p-gonal morphism is a regular covering, then the Riemann surface is called *cyclic p-gonal*.

Again by Lemma 2.1 in [1], if the surface X has genus $g \geq (p-1)^2 + 1$, then the p-gonal morphism is unique. Costa-Izquierdo-Ying [8] have obtained uniparametric families ${}^{4p^2}\mathcal{M}^p_{(p-1)^2}$ of cyclic p-gonal Riemann surfaces of genus $(p-1)^2$ admitting two cyclic p-gonal morphisms, proving that the bound is sharp. We can prove [8] that the space ${}^{4p^2}\mathcal{M}^p_{(p-1)^2}$ is a Riemann sphere without three points. Considering the Hurwitz spaces $\mathcal{H}_p$ of pairs (X, f), where $X \in {}^{4p^2}\mathcal{M}^p_{(p-1)^2}$ and $f : X \to \widehat{\mathbb{C}}$ is a cyclic p-gonal morphism. We obtained ([8]) that $\mathcal{H}_p$ is a Riemann sphere without four points. Finally we obtain the equations for the algebraic curves in the families ${}^{4p^2}\mathcal{M}^p_{(p-1)^2}$.

2 Trigonal Riemann surfaces and Fuchsian groups

Let X_g be a compact Riemann surface of genus $g \geq 2$. The surface X_g can be represented as a quotient $X_g = \mathcal{D}/\Gamma$ of the complex unit disc $\mathcal{D}$ under the action of a (cocompact) Fuchsian group Γ, that is, a discrete subgroup of the group $\mathcal{G} = \mathrm{Aut}(\mathcal{D})$ of conformal automorphisms of $\mathcal{D}$. The algebraic structure of a Fuchsian group and the geometric structure of its quotient orbifold are given by the signature of Γ:

$$s(\Gamma) = (g;\ m_1, ..., m_r).$$

A group Γ with the above signature has a *canonical presentation:*

$$\langle x_1, \ldots, x_r, a_1, b_1, \ldots, a_g, b_g | x_i^{m_i}, i = 1, \ldots, r, x_1 \ldots\ x_r a_1 b_1 a_1^{-1} b_1^{-1} \ldots\ a_g b_g a_g^{-1} b_g^{-1} \rangle$$

The orbit space $\mathcal{D}/\Gamma$ is a surface of genus g, having r cone points. The integers m_i are the *periods* of Γ, the orders of the cone points of $\mathcal{D}/\Gamma$. The generators $x_1, \ldots, x_r$, are called the *elliptic generators*. Any elliptic element in Γ is conjugated to a power of some of the elliptic generators.

The hyperbolic area of the orbifold $\mathcal{D}/\Gamma$ equals:

$$\mu(\Gamma) = 2\pi \left(2g - 2 + \sum_{i=1}^{r} \left(1 - \frac{1}{m_i} \right) \right).$$

Given a subgroup Γ' of index N in a Fuchsian group Γ, we have the Riemann-Hurwitz formula

$$\mu(\Gamma')/\mu(\Gamma) = N. \tag{1}$$

A Fuchsian group Γ without elliptic elements is called a *surface group* and it has signature $(h; -)$. Given a Riemann surface represented as the orbit space $X = \mathcal{D}/\Gamma$, with Γ a surface Fuchsian group, a finite group G is a group of automorphisms of X if and only if there exists a Fuchsian group Δ and an epimorphism $\theta : \Delta \to G$ with $\ker(\theta) = \Gamma$.

We have the following characterization of cyclic trigonal Riemann surfaces using Fuchsian groups:

Theorem 1 ([6]) *Let X_g be a Riemann surface, X_g admits a cyclic trigonal morphism f if and only if there is a Fuchsian group Δ with signature $(0;3,\overset{(g+2)}{\ldots},3)$ and an index three normal surface subgroup Γ of Δ, such that Γ uniformizes X_g.*

Theorem 1 yields an algorithm to find cyclic trigonal Riemann surfaces:
Let $G = \mathrm{Aut}(X_4)$ and let $X_4 = \mathcal{D}/\Gamma$ be a Riemann surface of genus 4 uniformized by the surface Fuchsian group Γ. The surface X_4 admits a cyclic trigonal morphism f if and only if there is a maximal Fuchsian group Δ with signature $(0; m_1, \ldots, m_r)$, an order three automorphism $\varphi : X_4 \to X_4$, such that $\langle\varphi\rangle \leq G$ and an epimorphism $\theta : \Delta \to G$ with $\ker(\theta) = \Gamma$ in such a way that $\theta^{-1}(\langle\varphi\rangle)$ is a Fuchsian group with signature $(0; 3, 3, 3, 3, 3, 3)$. Furthermore the trigonal morphism f is unique if and only if $\langle\varphi\rangle$ is **normal** in G (see [10]). We use Singerman's method [16] to obtain a presentation of $\theta^{-1}\langle\varphi\rangle$.

Since we assume that there are at least two trigonal automorphims, by [7], $D_3 \times D_3 \leq G$. Consider Fuchsian groups Δ with signature $(0; 2, 2, 2, 3)$ and the group $D_3 \times D_3 = \langle a, b, s, t / a^3 = b^3 = s^2 = t^2 = [a, b] = [s, b] = [t, a] = (sa)^2 = (tb)^2 = (st)^2 = 1\rangle$. Consider the epimorphism $\theta : \Delta \to D_3 \times D_3$ defined by $\theta(x_1) = s$, $\theta(x_2) = tb$, $\theta(x_3) = sta$ and $\theta(x_4) = a^2b$. The action of $\theta(x_4) = a^2b$ on the $(\langle ab\rangle)$-cosets has six fixed cosets. Then, by the Riemann-Hurwitz formula $s(\theta^{-1}(\langle ab\rangle)) = (0; 3, 3, 3, 3, 3, 3)$. In the same way $s(\theta^{-1}(\langle a^2b\rangle)) = (0; 3, 3, 3, 3, 3, 3)$. Thus the Riemann surfaces uniformized by $\mathrm{Ker}(\theta)$ are cyclic trigonal Riemann surfaces that admit two different trigonal morphisms $f_1 : \mathcal{D}/\mathrm{Ker}(\theta) \to \hat{\mathbb{C}}$ and $f_2 : \mathcal{D}/\mathrm{Ker}(\theta) \to \hat{\mathbb{C}}$ induced by the subgroups $\langle ab\rangle$ and $\langle a^2b\rangle$ of $D_3 \times D_3$. The dimension of the family of surfaces $\mathcal{D}/\mathrm{Ker}(\theta)$ is given by the dimension of the space of groups Δ with $s(\Delta) = (0; 2, 2, 2, 3)$. This dimension is $3(0) - 3 + 4 = 1$. We have obtained:

Theorem 2 ([7]) *There is a uniparametric family ${}^{36}\mathcal{M}_4^3$ of Riemann surfaces $X_4(\lambda)$ of genus 4 admitting several cyclic trigonal morphisms. The surfaces $X_4(\lambda)$ have $G = Aut(X_4(\lambda)) = D_3 \times D_3$ and the quotient Riemann surfaces $X_4(\lambda)/G$ are uniformized by the Fuchsian groups Δ with signature $s(\Delta) = (0; 2, 2, 2, 3)$.*

3 Actions of finite groups on Riemann surfaces

Our aim is to show that the space ${}^{36}\mathcal{M}_4^3$ is connected and hence a Riemann surface. To do that we will prove, by means of Fuchsian groups, that there is exactly one class of actions of $D_3 \times D_3$ on the surfaces $X_4(\lambda)$.

Each *(effective and orientable) action* of $G = D_3 \times D_3$ on a surface $X = X_4(\lambda)$ is determined by an epimorphism $\theta : \Delta \to G$ from the Fuchsian group Δ with signature $s(\Delta) = (0; 2, 2, 2, 3)$ such that $\mathrm{Ker}(\theta) = \Gamma$, where $X_4(\lambda) = \mathcal{D}/\Gamma$ and Γ is a surface Fuchsian group.

Remark 1 *The condition Γ being a surface Fuchsian group imposes: $o(\theta(x_1)) = o(\theta(x_2)) = o(\theta(x_3)) = 2$, $o(\theta(x_4)) = 3$, and $\theta(x_1)\theta(x_2)\theta(x_3) = \theta(x_4)^{-1}$.*

Two actions ϵ, ϵ' of G on X are *(weakly) topologically equivalent* if there is an $w \in \mathrm{Aut}(G)$ and an $h \in \mathrm{Hom}^+(X)$ such that $\epsilon'(g) = h\epsilon w(g)h^{-1}$.

In terms of groups: two epimorphisms $\theta_1, \theta_2 : \Delta \to G$ define two topologically equivalent actions of G on X if there exist automorphisms $\phi : \Delta \to \Delta$, $w : G \to G$ such that $\theta_2 = w \cdot \theta_1 \cdot \phi^{-1}$. With other words, let $\mathcal{B}$ be the subgroup of $\mathrm{Aut}(\Delta)$ induced by orientation preserving homeomorphisms. Then two epimorphisms $\theta_1, \theta_2 : \Delta \to G$ define the same class of G-actions if and only if they lie in the same $\mathcal{B} \times \mathrm{Aut}(G)$-class. See [4, 11, 14]. We are interested in finding elements of $\mathcal{B} \times \mathrm{Aut}(G)$ that make our epimorphisms $\theta_1, \theta_2 : \Delta \to G$ equivalent. We can produce the automorphism $\phi \in \mathcal{B}$ ad hoc. In our case the only elements $\mathcal{B}$ we need are compositions of $x_j \to x_{j+1}$ and $x_{j+1} \to x_{j+1}^{-1} x_j x_{j+1}$, where we write down only the action on the generators moved by the automorphism.

Lemma 1 *There is an epimorphism $\theta : \Delta \to G$ satisfying the Remark 1 if and only if $\theta(x_4) = a^{\varepsilon} b^{\delta}$, where $\varepsilon, \delta \in \{-1, +1\}$.*

PROOF. The elements of order three in G are $a^{\varepsilon}b^{\delta}$, $a^{\pm 1}$ and $b^{\pm 1}$. If $\theta(x_4) = a^{\pm 1}$ or $\theta(x_4) = b^{\pm 1}$ then the action of $\theta(x_4)$ on the $\langle a \rangle$- and $\langle b \rangle$-cosets leaves twelve fixed cosets which is geometrically imposible. ■

Using Lemma 1 and Remark 1 we obtain all the epimorphisms $\theta : \Delta \to G$. We list them in 6 types:

1.	$\theta(x_1) = sa^i$,	$\theta(x_2) = tb^j$,	$\theta(x_3) = sta^h b^k$
2.	$\theta(x_1) = tb^j$,	$\theta(x_2) = sa^i$,	$\theta(x_3) = sta^h b^k$
3.	$\theta(x_1) = tb^j$,	$\theta(x_2) = sta^i b^k$,	$\theta(x_3) = sa^h$
4.	$\theta(x_1) = sa^i$,	$\theta(x_2) = sta^h b^j$,	$\theta(x_3) = tb^k$
5.	$\theta(x_1) = sta^i b^j$,	$\theta(x_2) = tb^k$,	$\theta(x_3) = sa^h$
6.	$\theta(x_1) = sta^i b^j$,	$\theta(x_2) = sa^h$,	$\theta(x_3) = tb^k$

where $0 \le i \le 2$, $0 \le j \le 2$, $i \ne h \mod 3$ and $j \ne k \mod 3$.

Theorem 3 *There is a unique class of actions of the finite group $G = D_3 \times D_3$ on the surfaces $X = X_4(\lambda)$.*

PROOF. First of all, $1_d \times w \in \mathcal{B} \times \mathrm{Aut}(G)$, where the automorphism $w : G \to G$ is defined by $w(s) = t$, $w(t) = s$, $w(a) = b$ and $w(b) = a$ commutes epimorphisms of type 1 with epimorphisms of type 2; epimorphisms of type 3 with epimorphisms of type 4, and finally epimorphisms of type 5 with epimorphisms of type 6.

Now, all the epimorphisms within the same type are conjugated to each other by a conjugation in some of the following elements of $D_3 \times D_3$: $sta^{2(i'-i)}b^{2(j'-j)}$, $a^{2(i'-i)}b^{2(j'-j)}$, $sa^{2(i'-i)}b^{2(j'-j)}$ or $ta^{2(i'-i)}b^{2(j'-j)}$. So any epimorphism is equivalent to one of the following epimorphisms:

$\theta_0(x_1) = s$,	$\theta_0(x_2) = t$,	$\theta_0(x_3) = stab$,
$\theta_1(x_1) = t$,	$\theta_1(x_2) = stab$,	$\theta_1(x_3) = sa^2$
$\theta_2(x_1) = stab$,	$\theta_2(x_2) = sa^2$,	$\theta_2(x_3) = tb^2$

It is enough to show that there are elements of $\mathcal{B}$ commuting θ_0 with θ_1, and θ_0 with θ_2.

Consider $\phi_{1,2} : \Delta \to \Delta$ defined by $\phi_{1,2}(x_1) = x_2$, $\phi_{1,2}(x_2) = x_2^{-1}x_1x_2$, $\phi_{1,2}(x_3) = x_3$, $\phi_{1,2}(x_4) = x_4$, and $\phi_{2,3} : \Delta \to \Delta$ defined by $\phi_{2,3}(x_1) = x_1$, $\phi_{2,3}(x_2) = x_3$, $\phi_{2,3}(x_3) = x_3^{-1}x_2x_3$, $\phi_{2,3}(x_4) = x_4$.

Firstly, $\phi_{2,3} \cdot \phi_{1,2}$ takes the epimorphism $\theta_0(x_1) = s$, $\theta_0(x_2) = t$, $\theta_0(x_3) = stab$ to the epimorphism $\theta_1(x_1) = t$, $\theta_1(x_2) = stab$, $\theta_1(x_3) = sa^2$. Secondly, $\phi_{1,2} \cdot \phi_{2,3}$ takes the epimorphism θ_0 to the epimorphism $\theta_2(x_1) = stab$, $\theta_2(x_2) = sa^2$, $\theta_2(x_3) = tb^2$. ■

As a consequence of the previous theorem we obtain:

Theorem 4 *The space ${}^{36}\mathcal{M}_4^3$ is a Riemann surface. Furthermore it is the Riemann sphere with three punctures.*

PROOF. By Theorem 3, ${}^{36}\mathcal{M}_4^3$ is a connected space of complex dimension 1. The space ${}^{36}\mathcal{M}_4^3$ can be identified with the moduli space of orbifolds with three cone points of order 2 and one of order 3. Each cone point of order 2 corresponds to a conjugacy class of involutions in $D_3 \times D_3$: $[s], [t]$ and $[st]$. Using a Möbius transformation we can assume that the three order two cone points are 0, 1 and ∞. Thus ${}^{36}\mathcal{M}_4^3$ is parametrized by the position λ of the order three cone point Hence ${}^{36}\mathcal{M}_4^3$ is the Riemann sphere with three punctures. ■

We consider the pairs (X, f), where $X \in {}^{36}\mathcal{M}_4^3$ and $f : X \to \widehat{\mathbb{C}}$ is a cyclic trigonal morphism. Two pairs (X_1, f_1) and (X_2, f_2) are equivalent if there is an isomorphism $h : X_1 \to X_2$ such that $f_1 = f_2 \circ h$. The space of classes of pairs (X, f) given by the above equivalence relation and with the topology induced by the topology of ${}^{36}\mathcal{M}_4^3$ is a Hurwitz space $\mathcal{H}$.

Theorem 5 *The space $\mathcal{H}$ is a two-fold connected covering of ${}^{36}\mathcal{M}_4^3$ and then $\mathcal{H}$ is a Riemann sphere without four points.*

PROOF. By Theorem 2 each surface of ${}^{36}\mathcal{M}_4^3$ admits two cyclic trigonal morphisms, then $\mathcal{H}$ is a two-fold covering of ${}^{36}\mathcal{M}_4^3$.

We only need to prove that the covering space is connected. We need to show that the monodromy of the covering $\mathcal{H} \to {}^{36}\mathcal{M}_4^3$ is not trivial. Each $(X(\lambda), f) \in \mathcal{H}$ is given by a point $\lambda \in \widehat{\mathbb{C}} - \{0, 1, \infty\}$ and a trigonal cyclic morphism $f : X \to \widehat{\mathbb{C}}$. The trigonal cyclic morphism is given by the projections $f_{ab} : X \to X/\langle ab \rangle$ or $f_{a^2b} : X \to X/\langle a^2b \rangle$. There is an action of $\pi_1({}^{36}\mathcal{M}_4^3) = \pi_1(\widehat{\mathbb{C}} - \{0, 1, \infty\})$ on the set of representations $R = \{r : \pi_1(\widehat{\mathbb{C}} - \{0, 1, \infty, \lambda\}) \to D_3 \times D_3\}$. The group $\pi_1({}^{36}\mathcal{M}_4^3)$ is generated by three meridians and each one acts on R as the action induced by a braid in $\widehat{\mathbb{C}} - \{0, 1, \infty, \lambda\}$. The braid $\Phi_{34}^{-1}\Phi_{23}^2\Phi_{34}$ (given by the action of one of the meridians of ${}^{36}\mathcal{M}_4^3$) sends $r_1 : \pi_1(\widehat{\mathbb{C}} - \{0, 1, \infty, \lambda\}, *) \to D_3 \times D_3$ defined by: $x_1 \to s$, $x_2 \to t$, $x_3 \to stab$, $x_4 \to a^2b^2$ to $r_2 : \pi_1(\widehat{\mathbb{C}} - \{0, 1, \infty, \lambda\}) \to D_3 \times D_3$ defined by: $x_1 \to s$, $x_2 \to tb$, $x_3 \to sta^2b^2$, $x_4 \to ab^2$

The representations r_1 and r_2 are conjugated by sb^2, but such a conjugation sends $\langle ab \rangle$ to $\langle a^2b \rangle$. ■

4 Equations of the algebraic curves in ${}^{36}\mathcal{M}_4^3$

Let X be a curve of ${}^{36}\mathcal{M}_4^3$ with automorphisms group $\mathrm{Aut}(X) = D_3 \times D_3$.

The subgroup $\langle a, b \rangle$ of $D_3 \times D_3$ is a normal subgroup of $D_3 \times D_3$ and it isomorphic to $C_3 \times C_3$. The quotient group $D_3 \times D_3/\langle a, b \rangle$ is isomorphic to the Klein group $C_2 \times C_2$. We can factorize the covering $X \to X/D_3 \times D_3$ by two regular coverings: $X \to X/C_3 \times C_3$, $X/C_3 \times C_3 \xrightarrow{C_2 \times C_2} X/D_3 \times D_3$.

The quotient space $X/C_3 \times C_3$ is a 2-orbifold with four conic points of order 3 and genus 0. The orbifold $X/D_3 \times D_3 = (X/C_3 \times C_3)/C_2 \times C_2$ has three conic points of order 2, one conic point of order 3 and genus 0. Using a Möbius transformation we can consider that the action of $C_2 \times C_2$ on $X/C_3 \times C_3$ is the given by the transformations $\{z \to \pm z, z \to \pm \frac{1}{z}\}$. Since the set of the four conic points of order three is an orbit of the action of $C_2 \times C_2$ on $X/C_3 \times C_3$, then the conic points of $X/C_3 \times C_3$ are: $\{\pm\lambda, \pm\frac{1}{\lambda}\}$ for $\lambda \in \mathbb{C} - \{0, \pm 1, \pm i\}$.

To obtain X from $X/C_3 \times C_3$ we factorize $X \to X/C_3 \times C_3$ by the coverings: $X \xrightarrow{C_3} X/C_3$, $X/C_3 \times C_3 \xrightarrow{C_3} X/C_3 \times C_3$.

The cyclic three-fold covering $g : X/C_3 \to X/C_3 \times C_3$ branched on $\pm\lambda$ is: $g(z) = (\frac{-\lambda z + \lambda}{z+1})^3$. The orbifold X/C_3 has six conic points of order 6 that are the preimages by g of $\pm\frac{1}{\lambda}$. If ζ_1 is a primitive cubic root of $\lambda - 1$, ζ_2 is a primitive cubic root of $\frac{\lambda^2+1}{\lambda-1}$ and ξ is a primitive cubic root of the unity, then X has equation as algebraic complex curve:

$$y^3 = \prod_{i=1}^{3}(x - \zeta_1^i)\prod_{i=1}^{3}(x - \zeta_2^i)^2.$$

Acknowledgement. Antonio F. Costa has been partially supported by MTM2005-01637, Milagros Izquierdo has been partially supported by the Swedish Research Council (VR) and Daniel Ying has been partially supported by The Royal Swedish Academy of Sciences (KVA)

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