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A converse to Amir-Lindenstrauss theorem in complex Banach spaces

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Abstract. We show that a complex Banach space is weakly Lindelöf determined if and only if the dual unit ball of any equivalent norm is weak* Valdivia compactum. We deduce that a complex Banach space X is weakly Lindelöf determined if and only if any nonseparable Banach space isomorphic to a complemented subspace of X admits a projectional resolution of the identity. These results complete the previous ones on real spaces.

Un recíproco del Teorema de Amir-Lindenstrauss en espacios de Banach complejos

Resumen. Probamos que un espacio de Banach complejo es débilmente Lindelöf determinado si y solamente si la bola cerrada unidad dual de cualquier norma equivalente es, en la topología débil*, un compacto de Valdivia. Deducimos que un espacio de Banach complejo X es débilmente Lindelöf determinado si y solamente si cualquier espacio de Banach no separable isomorfo a un subespacio complementado de X admite una resolución proyectiva de la identidad. Estos resultados complementan los obtenidos para espacios de Banach reales.

1. Introduction

Projectional resolutions of the identity (shortly, PRI, for a definition see Section 2.) provide a powerful tool to study nonseparable Banach spaces. First PRIs were constructed by Lindenstrauss [12, 13]. The importance of this notion became obvious after the paper by Amir and Lindenstrauss [1] where a PRI is constructed in every weakly compactly generated space. This result was extended by Vašák [19] to weakly countably determined spaces and later by Valdivia [16] to spaces with dual unit ball being Corson compactum (in the weak* topology).

Recall that a compact space K is *Corson* if it is homeomorphic to a subset of

$$\Sigma(\Gamma) = \{x \in \mathbb{R}^\Gamma : \{\gamma \in \Gamma : x(\gamma) \neq 0\} \text{ is countable}\}$$

for a set Γ. It follows from [14, Proposition 4.1] that the dual unit ball of a Banach space X is Corson if and only if the space X is *weakly Lindelöf determined*, i.e. if there is a set $M \subset X$ with $\operatorname{span} M$ dense in X such that $\{x \in M : \xi(x) \neq 0\}$ is countable for each $\xi \in X^*$.

Further extensions hold for spaces related to Valdivia compacta. A compact space K is called *Valdivia* if there is a homeomorphic embedding $h : K \to \mathbb{R}^\Gamma$ with $h^{-1}(\Sigma(\Gamma))$ dense in K. Any set of the form

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$h^{-1}(\Sigma(\Gamma))$ for a homeomorphic embedding $h : K \to \mathbb{R}^\Gamma$ is called Σ-*subset* of K. Hence K is Valdivia if and only if it admits a dense Σ-subset.

Valdivia [17] constructed PRI in the space of real-valued continuous functions on a Valdivia compact space. Later, in [18], he extended this result to those Banach spaces X for which there is a linear subspace $S \subset X^*$ with $S \cap B_{X^*}$ being a dense Σ-subset of (B_{X^*}, w^*).

All these results were originally proved for real spaces (this was sometimes explicitly stated and sometimes implicitly supposed). The author is convinced that the same proofs with some obvious minor changes would work for complex spaces as well. However he did not check it. Instead of this we show in Section 2**.** that all the complex spaces from the mentioned classes do have PRI.

The situation is less easy when we consider converse theorems.

Fabian, Godefroy and Zizler noticed in [4, Lemma 2] that a Banach space X with density $\aleph_1$ has a PRI (if and) only if there is a linear subspace $S \subset X^*$ with $S \cap B_{X^*}$ being a dense Σ-subset of (B_{X^*}, w^*), which is a converse to the above quoted result of Valdivia. They proved it for real spaces but in the complex case the same proof can be used (see Proposition 2 below).

The author [7] showed that (B_{X^*}, w^*) is Corson provided $(B_{(X,|\cdot|)^*}, w^*)$ is Valdivia for each equivalent norm $|\cdot|$ on X. This result was proved for real spaces. The complex case requires some additional work and it belongs to the main results of the present paper (see Theorem 3).

A further result of the author [10] says that a Banach space X is weakly Lindelöf determined if (and only if) any nonseparable space isomorphic to a complemented subspace of X admits a PRI. The result was proved for real spaces but the complex case can be done copying the proof. It is formulated in Theorem 4 below.

Let us now fix some terminology and notation. By a Banach space we mean a real or complex Banach space unless one possibility is explicitly chosen. If X is a complex Banach space, we denote by X_R the space X considered as a real space. We will need the following standard proposition relating the weak topologies of X and X_R and the weak* topologies of X^* and X_R^*.

Proposition 1 *Let X be a complex Banach space.*

- *The identity X onto X_R is a real-linear, isometric and weak-to-weak homeomorphic map.*
- *The mapping $\phi : X^* \to X_R^*$ defined by $\phi(\xi) = \operatorname{Re} \xi$, $\xi \in X^*$, is a real-linear, isometric and weak*-to-weak* homeomorphic map.*

The 'isometric part' of the first point is obvious. The 'isometric part' of the second assertion is a standard part of the proof of complex Hahn-Banach theorem (see e.g. [5, p. 28-29]). The 'weak' and 'weak*' parts are then easy to check.

2. Projectional resolutions in complex Banach spaces

Let us start with giving the definition. Let X be a Banach space (real or complex) with $\operatorname{dens} X = \kappa > \aleph_0$. A *projectional resolution of the identity* (or, shortly *PRI*) on X is an indexed family $(P_\alpha)_{\alpha \in [\omega, \kappa]}$ of linear operators on X satisfying the following conditions:

(i) $P_\omega = 0$, $P_\kappa = \mathrm{Id}_X$;

(ii) $P_\alpha P_\beta = P_\beta P_\alpha = P_\alpha$ whenever $\omega \le \alpha \le \beta \le \kappa$;

(iii) $\|P_\alpha\| = 1$ for $\alpha \in (\omega, \kappa]$;

(iv) $\operatorname{dens} P_\alpha X \le \operatorname{card} \alpha$;

(v) $P_\lambda X = \overline{\bigcup_{\alpha < \lambda} P_\alpha X}$ whenever $\lambda \in (\omega, \kappa]$ is a limit ordinal.

The original constructions of PRIs [1, 19, 16, 17, 18] used various methods. A unifying approach was proposed in [15] and is described in detail in [3, Section 6.1]. It uses the notion of *projectional generator*. Following [3, Definition 6.1.6] we define a projectional generator on a (real or complex) Banach space X to be a pair (W, Φ) satisfying the following conditions:

- $\overline{W}$ is a linear subspace of X^*;
- W is a 1-norming subset of X^* (i.e., $\|x\| = \sup\{|\xi(x)| : \xi \in W \cap B_{X^*}\}$ for each $x \in X$);
- Φ is a mapping defined on W whose values are countable subsets of X;
- if $B \subset W$ is such that $\overline{B}$ is linear, then $\Phi(B)^\perp \cap \overline{B}^{w^*} = \{0\}$ (note that $\Phi(B)$ is the standard abbreviation for $\bigcup\{\Phi(b) : b \in B\}$).

The relationship of projectional generator and PRI is described by the following theorem.

Theorem 1 *A nonseparable Banach space with a projectional generator admits a PRI.*

This theorem is proved in [3, Proposition 6.1.7] for real spaces. However, the same proof works in complex case. One should make only some obvious changes – at few places add absolute value and use numbers $p + iq$, $p, q \in \mathbb{Q}$, instead of rational numbers.

Now we are going to define the classes of Banach spaces associated to Valdivia compacta. Let X be a (real or complex) Banach space. We say that $S \subset X^*$ is a Σ-*subspace* of X^* if there is $M \subset X$ with $\operatorname{span} M$ dense in X such that

$$S = \{\xi \in X^* : \{x \in M : \xi(x) \neq 0\} \text{ is countable}\}.$$

If X^* admits a 1-norming Σ-subspace, the space X is called 1-*Plichko*. *Plichko spaces* are those spaces X for which X^* admits a norming Σ-subspace. Recall that $S \subset X^*$ is *norming* if

$$\|x\|_S = \sup\{|\xi(x)| : \xi \in S \cap B_{X^*}\}, \qquad x \in X,$$

defines an equivalent norm on X. If $\|x\|_S = \|x\|$ for all $x \in X$, the subspace S is called 1-*norming*.

Note that a Banach space is 1-Plichko if and only if there is a linear subspace $S \subset X^*$ with $S \cap B_{X^*}$ being a dense Σ-subset of (B_{X^*}, w^*) (see [6, Theorem 2.3] or [8, Theorem 2.7] for the case of real spaces and [11, Theorem 3.2] for complex spaces).

Using Theorem 1 we get the following.

Theorem 2 *Let X be a (real or complex) nonseparable* 1*-Plichko Banach space. Then X admits a PRI.*

PROOF. Let $M \subset X$ be such that $\operatorname{span} M$ is dense in X and the Σ-subspace S defined by M is 1-norming. For any $\xi \in S$ set

$$\Phi(\xi) = \{x \in M : \xi(x) \neq 0\}.$$

Then the pair (S, Φ) is a projectional generator on X. Indeed, the first three conditions are trivially satisfied. Let us show that the fourth one is fulfilled as well.

Take $B \subset S$ and $\xi \in \overline{B}^{w^*} \cap \Phi(B)^\perp$. If $\xi \neq 0$, there is some $x \in M$ with $\xi(x) \neq 0$. As $\xi \in \overline{B}^{w^*}$, there is some $\eta \in B$ with $\eta(x) \neq 0$. But then $x \in \Phi(\eta) \subset \Phi(B)$ and hence $\xi(x) = 0$ (as $\xi \in \Phi(B)^\perp$). This contradiction finishes the proof that (S, Φ) is a projectional generator. Hence X admits a PRI by Theorem 1. ■

Remark. The previous theorem shows that the results on existence of a PRI in real spaces mentioned in the introduction hold for complex spaces as well. Note, however, that this theorem together with the presented proof cannot be viewed as a substitute of the previous results. Indeed, to prove, say, that complex weakly compactly generated (WCG) spaces admit a PRI, one should argue as follows. Let X be a complex WCG space. Then X_R is WCG and hence, by the Amir-Lindenstrauss theorem, X_R is also weakly Lindelöf determined (WLD). Thus X is WLD as well. Finally, X is 1-Plichko and we can apply the previous theorem.

For spaces of density $\aleph_1$ we have the following.

Proposition 2 *Let X be a Banach space with $\operatorname{dens} X = \aleph_1$. Then X is 1-Plichko if and only if X admits a PRI.*

PROOF. The 'only if' part follows from Theorem 2. We show the 'if' part repeating the argument of [4, Lemma 2]. Let $(P_\alpha)_{\alpha\in[\omega,\omega_1]}$ be a PRI on X. Then $(P_{\alpha+1} - P_\alpha)X$ is separable for each $\alpha \in [\omega, \omega_1)$. Let M_α be a countable dense subset. Set $M = \bigcup_{\alpha\in[\omega,\omega_1)} M_\alpha$. Then it is easy to check that $\operatorname{span} M$ is dense in X. Let S be the Σ-subset of X^* defined by M.

We claim that S contains $\bigcup_{\alpha\in[\omega,\omega_1)} P^*_\alpha X^*$. Indeed, let $\xi \in X^*$ and $\alpha \in [\omega, \omega_1)$ be such that $P^*_\alpha \xi = \xi$. Suppose that $\beta \geq \alpha$ and $x \in M_\beta$. Then

$$\xi(x) = P^*_\alpha \xi((P_{\beta+1} - P_\beta)x) = \xi(P_\alpha(P_{\beta+1} - P_\beta)x) = \xi(P_\alpha x - P_\alpha x) = 0.$$

Hence

$$\{x \in M : \xi(x) \neq 0\} \subset \bigcup_{\beta\in[\omega,\alpha)} M_\beta$$

which is a countable set.

Moreover, $\bigcup_{\alpha\in[\omega,\omega_1)} P^*_\alpha X^*$ is a 1-norming subspace of X^*. Let $x \in X$ and $\varepsilon > 0$. By the condition (v) of the definition of a PRI there is some $\alpha \in [\omega, \omega_1)$ and $y \in P_\alpha X$ such that $\|x - y\| < \varepsilon$. Choose some $\xi \in X^*$ with $\|\xi\| = 1$ and $|\xi(y)| = \|y\|$ and set $\eta = P^*_\alpha \xi$. Then $\|\eta\| \leq 1$ and we have

$$|\eta(x)| \geq |\eta(y)| - |\eta(y - x)| \geq \|y\| - \|x - y\| \geq \|x\| - 2\|x - y\| > \|x\| - 2\varepsilon.$$

This completes the proof. ■

Remarks on the above proof.

(1) It can be shown that $S = \bigcup_{\alpha\in[\omega,\omega_1)} P^*_\alpha X^*$. But we do not need it.

(2) If $x \in X$, then necessarily $x \in P_\alpha X$ for some $\alpha \in [\omega, \omega_1)$. This follows from the condition (v) of the definition of a PRI, the fact that closures in X are described by limits of sequences and ω_1 has uncountable cofinality. Thus one can take $y = x$. This simplifies a bit the argument. However, the present proof works for spaces of arbitrary density which is useful in proving the complex analogue of [10, Lemma 3] used in the proof of Theorem 4.

Note also that there is a Banach space with Valdivia dual unit ball which has no PRI (see [9] for the real case and [11, Example 3.9] for the complex case).

3. Main results

In this section we state and prove our main results. We formulate them only for complex spaces as they are already known in the real case.

The first one is the following theorem which was proved in [7, Theorem 1] for real spaces.

Theorem 3 *Let X be a complex Banach space. The following assertions are equivalent.*

1. *X is weakly Lindelöf determined.*
2. *$(X, |\cdot|)$ is 1-Plichko for each equivalent norm $|\cdot|$ on X.*

3. *$(B_{(X,|\cdot|)^*}, w^*)$ is Valdivia for each equivalent norm $|\cdot|$ on X.*

Moreover, if $\operatorname{dens} X = \aleph_1$*, then the previous assertions are equivalent with the following one.*

4. *$(X, |\cdot|)$ has a projectional resolution of the identity for each equivalent norm $|\cdot|$ on X.*

The following theorem was proved in [10, Theorem 1] for real spaces.

Theorem 4 *Let X be a complex Banach space. The following assertions are equivalent*

1. *X is weakly Lindelöf determined.*
2. *Each nonseparable complex Banach space isomorphic to a subspace of X admits a projectional resolution of the identity.*
3. *Each nonseparable complex Banach space isomorphic to a complemented subspace of X admits a projectional resolution of the identity.*

Proof of Theorem 3. The implications $1 \implies 2 \implies 3$ are trivial. The implication $2 \implies 4$ follows from Theorem 2. If $\operatorname{dens} X = \aleph_1$, the implication $4 \implies 2$ follows from Proposition 2.

$3 \implies 1$ Let X be a complex Banach space which is not weakly Lindelöf determined. Then (B_{X^*}, w^*) is not Corson. If (B_{X^*}, w^*) is not Valdivia, we are done. Hence suppose that (B_{X^*}, w^*) is Valdivia but not Corson. Let A be a dense Σ-subset of (B_{X^*}, w^*).

Let ϕ be the mapping introduced in Proposition 1. Then it follows from that proposition that $\phi(A)$ is a dense Σ-subset of $(B_{X_R^*}, w^*)$. Clearly $\phi(A)$ is closed to taking limits of weak* converging sequences (see e.g. [8, Lemma 1.6]). Therefore, if $\phi(A) \supset S_{X_R^*}$, then $\phi(A) = B_{X_R^*}$ by a corollary to Josefson-Niessenzweig theorem [2, Chapter XII, Exercise 2(i)]. By Bishop-Phelps theorem functionals attaining their norm are norm dense in $S_{X_R^*}$, hence there is some $\eta_0 \in S_{X_R^*} \setminus A$ and $x \in S_X$ such that $\eta_0(x) = 1$.

Set $\xi_0 = \phi^{-1}(\eta_0)$. Then $\xi_0 \in S_{X^*} \setminus A$ and $\operatorname{Re} \xi_0(x) = 1$. As $|\xi_0(x)| \le 1$, necessarily $\xi_0(x) = 1$.

Let $L = B_{X^*} \cap \{\xi \in X^* : \xi(x) = 1\}$. Then L is a weak* G_δ weak* compact subset of B_{X^*}, hence $L \cap A$ is dense in L (see e.g. [8, Lemma 1.11]). It follows that L is Valdivia. As $\xi_0 \in L \setminus A$, L is not Corson. Moreover, it is clear that L is convex. By [7, Proposition 3] there is a convex weak* compact $K \subset L$ which is not Valdivia.

Set
$$B = \operatorname{conv}\left(\frac{1}{2} B_{X^*} \cup \overline{\operatorname{conv} \bigcup \{\alpha K : |\alpha| = 1\}}^{w^*}\right).$$

It is clear that B is a convex weak* compact convex set satisfying $\alpha B = B$ for each $\alpha \in \mathbb{C}$, $|\alpha| = 1$ and $\frac{1}{2} B_{X^*} \subset B \subset B_{X^*}$. Hence B is a dual unit ball of an equivalent norm on X. It suffices to show that B is not Valdivia.

We claim that
$$B \cap \{\xi \in X^* : \xi(x) = 1\} = K. \tag{1}$$

The inclusion $\supset$ is obvious. Let us show the inverse one. Let ξ belong to the set on the left-hand side. Then there is $b \in X^*$, $\|b\| \le \frac{1}{2}$, $h \in \overline{\operatorname{conv} \bigcup \{\alpha K : \|\alpha\| = 1\}}^{w^*}$ and $t \in [0, 1]$ with $\xi = tb + (1 - t)h$. We have
$$1 = \xi(x) = |\xi(x)| \le t|b(x)| + (1 - t)|h(x)| \le \frac{t}{2} + 1 - t = 1 - \frac{t}{2}.$$

It follows that $t = 0$ and hence $\xi \in \overline{\operatorname{conv} \bigcup \{\alpha K : |\alpha| = 1\}}^{w^*}$. Therefore there is a net
$$\xi_\tau = \sum_{j=1}^{n_\tau} t_\tau^j \alpha_\tau^j k_\tau^j,$$

where $|\alpha_\tau^j| = 1$, $k_\tau^j \in K$, $t_\tau^j \ge 0$ and $\sum_{j=1}^{n_\tau} t_\tau^j = 1$, weak* converging to ξ. In particular, $\xi_\tau(x) \to 1$.

For each τ set

$$k_\tau = \sum_{j=1}^{n_\tau} t_\tau^j k_\tau^j.$$

As K is convex, $k_\tau \in K$ for each τ. Up to passing to a subnet we may suppose that k_τ converges to a point $k \in K$. We will show that $k = \xi$.

Choose $\varepsilon > 0$ arbitrary. For each τ denote

$$A_\tau = \{j \in \{1, \dots, n_\tau\} : \operatorname{Re} \alpha_\tau^j > 1 - \varepsilon\}$$

and $B_\tau = \{1, \dots, n_\tau\} \setminus A_\tau$. Then

$$\begin{aligned}\operatorname{Re}(1 - \xi_\tau(x)) &= \operatorname{Re}\left(1 - \sum_{j=1}^{n_\tau} t_\tau^j \alpha_\tau^j k_\tau^j(x)\right) = \operatorname{Re}\left(\sum_{j=1}^{n_\tau} t_\tau^j (1 - \alpha_\tau^j)\right) \\ &= \sum_{j \in B_\tau} t_\tau^j (1 - \operatorname{Re} \alpha_\tau^j) + \sum_{j \in A_\tau} t_\tau^j (1 - \operatorname{Re} \alpha_\tau^j) \geq \varepsilon \sum_{j \in B_\tau} t_\tau^j.\end{aligned}$$

Hence

$$\lim_\tau \sum_{j \in B_\tau} t_\tau^j = 0.$$

Choose τ_0 such that

$$\sum_{j \in B_\tau} t_\tau^j < \varepsilon \text{ whenever } \tau \geq \tau_0.$$

Suppose now $\tau \geq \tau_0$. We have

$$\begin{aligned}\|\xi_\tau - k_\tau\| &= \|\sum_{j=1}^{n_\tau} t_\tau^j (1 - \alpha_\tau^j) k_\tau^j\| \leq \sum_{j \in B_\tau} 2 t_\tau^j + \sum_{j \in A_\tau} t_\tau^j |1 - \alpha_\tau^j| \\ &< 2\varepsilon + \sum_{j \in A_\tau} t_\tau^j \sqrt{(1 - \operatorname{Re} \alpha_\tau^j)^2 + (\operatorname{Im} \alpha_\tau^j)^2} \\ &= 2\varepsilon + \sum_{j \in A_\tau} t_\tau^j \sqrt{1 - 2 \operatorname{Re} \alpha_\tau^j + (\operatorname{Re} \alpha_\tau^j)^2 + (\operatorname{Im} \alpha_\tau^j)^2} = 2\varepsilon + \sum_{j \in A_\tau} t_\tau^j \sqrt{2(1 - \operatorname{Re} \alpha_\tau^j)} \\ &\leq 2\varepsilon + \sum_{j \in A_\tau} t_\tau^j \sqrt{2\varepsilon} \leq 2\varepsilon + \sqrt{2\varepsilon}.\end{aligned}$$

As $\varepsilon > 0$ is arbitrary, we have

$$\lim_\tau \|\xi_\tau - k_\tau\| = 0,$$

hence

$$\xi_\tau - k_\tau \xrightarrow{w*} 0,$$

therefore $\xi = k$ and thus $\xi \in K$. This completes the proof of the equality (1).

It follows that K is a weak* G_δ subset of B. Hence, if B were a Valdivia compactum, K would be Valdivia as well by [8, Lemma 1.11]. Therefore B is not Valdivia and the proof is completed. ■

Theorem 4 is now an easy consequence of Theorem 3 and of the proof of [10, Theorem 1]. Indeed, 1 $\Longrightarrow$ 2 follows from Theorem 2 (together with the fact that weakly Lindelöf determined spaces are stable to taking subspaces – see e.g. [8, Example 4.39]) and 2 $\Longrightarrow$ 3 is trivial. For 3 $\Longrightarrow$ 1 we can repeat the proof of [10, Theorem 1]. Indeed, Lemmata 1–4 of [10] are clearly true also for complex spaces. The proof continues by transfinite induction on the density of the space X. As separable spaces are weakly Lindelöf determined, it holds for $\aleph_0$. Suppose that κ is an uncountable cardinal such that $3 \implies 1$ holds for all spaces with density strictly less than κ. Let $\operatorname{dens} X = \kappa$ and X satisfy (3). Then, copying the proof given in [10], we show that $(X, |\cdot|)$ is 1-Plichko for each equivalent norm $|\cdot|$ on X. Hence X is weakly Lindelöf determined by Theorem 3.

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