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Functions locally dependent on finitely many coordinates

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Abstract. The notion of functions dependent locally on finitely many coordinates plays an important role in the theory of smoothness and renormings on Banach spaces, especially when higher smoothness (C^∞) is involved. In this note we survey most of the main results in this area, and indicate many old as well as new open problems.

Funciones localmente dependientes de un número finito de coordenadas

Resumen. La noción de funciones localmente dependientes de un número finito de coordenadas juega un papel importante en la teoría de diferenciabilidad y renormamiento en los espacios de Banach, especialmente cuando aparece diferenciabilidad de orden superior (C^∞). En esta nota describimos una buena parte de los resultados en esta dirección, y listamos muchos problemas abiertos, nuevos y antiguos.

The present survey focuses on properties of Banach spaces which admit nontrivial real valued functions, locally dependent on finitely many coordinates. Let us give a precise definition.

Definition 1 *Let X be a Banach space, E be an arbitrary set, $M \subset X^*$ and $g\colon X \to E$. We say that g* depends only on *$\{f_1, \dots, f_n\} \subset X^*$* on *$U \subset X$ if there exists a mapping $G\colon \mathbb{R}^n \to E$ such that $g(x) = G(f_1(x), \dots, f_n(x))$ for all $x \in U$. We say that g* locally depends on finitely many coordinates from *M (LFC-M for short) if for each $x \in X$ there are a neighbourhood U of x and a finite subset $F \subset M$ such that g depends only on F on U. We say that g* locally depends on finitely many coordinates *(LFC for short) if it is LFC-X^*.*

The canonical example of a non-trivial LFC function is the norm $\|\cdot\|_\infty$ on c_0, which is LFC-$\{e_i^*\}$ away from the origin. Indeed, take any $x = (x_i) \in c_0$, $x \neq 0$. Let $n \in \mathbb{N}$ be such that $|x_i| < \|x\|_\infty /2$ for $i > n$. Then $\|\cdot\|_\infty$ depends only on $\{e_1^*, \dots, e_n^*\}$ on $U(x, \|x\|_\infty /4)$.

The first use of LFC in the literature is Kuiper's construction (which appeared in [2]) of a C^∞-smooth equivalent norm on c_0. Let us present an alternative construction which works also for $c_0(\Gamma)$. Choose a C^∞-smooth even convex function $\phi : \mathbb{R} \to \mathbb{R}$, with the properties: $\phi(t) = 0$ for $t \in [-\frac{1}{2}, \frac{1}{2}]$, $\phi(1) = 1$. Put $\Phi((x_\gamma)_{\gamma\in\Gamma}) = \sum_{\gamma\in\Gamma} \phi(x_\gamma)$. It is straightforward to verify that Φ is LFC and consequently C^∞-smooth convex function on $c_0(\Gamma)$, which separates the origin from the unit sphere. To get an equivalent renorming from Φ, it suffice to apply the Minkowski functional to the set $\Phi^{-1}([0, 1])$.

The single most important application of LFC is probably the use of the above (or similar) C^∞-smooth and LFC function on $c_0(\Gamma)$ in the proof, originally due to Torunczyk [38], of the existence of C^k-smooth partitions of unity for reflexive spaces admitting a C^k-smooth bump. This was later generalized in [12] to

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WCG spaces, but its proof applies in fact even to WLD class [4]. Apart from this fundamental contribution, there are two main directions where the LFC notion proved to be very useful. The first one is in the essentially separable theory of polyhedral spaces. The second one is in the theory of $C(K)$, K scattered spaces (or equivalently Asplund $C(K)$ spaces). In both cases, LFC serves as an ideal tool for obtaining highly (C^∞) smooth norms or bumps on the spaces in question. In fact, it is not an exaggeration to say that LFC is the main general constructive tool in theory of higher smoothness on Banach spaces. There are notable exceptions, some of which we will discuss later ([28], [29], [30]).

The LFC notion was first explicitely defined and used in [32]. In this paper the authors use the method originally due to Kurzweil ([24]), in order to prove the following fundamental result.

Theorem 1 ([32]) *Let X be an infinitely dimensional Banach space X admitting a LFC bump function. Then X is saturated by c_0.*

Another important structural property is the following.

Theorem 2 ([8]) *Let X be a Banach space, $M \subset X^*$ and X admits an arbitrary LFC-M bump function. Then $\overline{\operatorname{span}}\, M = X^*$. In particular, if X has a LFC bump function, then it is an Asplund space.*

The above theorems are generalizations of earlier results of Fonf [9], [10], where the same conclusions were obtained for polyhedral spaces. We refer to the Fonf, Lindenstrauss an Phelps' article in [22] for references and results in the vast subject of infinite dimensional convexity. Let us explain the situation in some detail. Polyhedral spaces were introduced by Klee [23].

Definition 2 *Let X be a Banach space. We say that X is* polyhedral *if the unit ball of every finite dimensional subspace of X is a polyhedron (i.e. an intersection of finitely many half-spaces).*

Polyhedrality is an isometric property of a Banach space, i.e. depends on the particular norm. In our note we are interested in isomorphic properties of Banach spaces, and so we will use the term polyhedral space for all spaces that are polyhedral under some equivalent renorming. Polyhedral spaces include all isometric preduals of ℓ_1 (by this we mean preduals to the space $(\ell_1, \|\cdot\|_1)$), in particular all $C(K)$, K countable spaces. Polyhedrality clearly passes to closed subspaces. The following question is well-known.

Proposition 1 *Is every separable polyhedral Banach space X isomorphic to a subspace of an isometric predual of ℓ_1?*

On the other hand, very little is known about quotients of polyhedral spaces. We have

Proposition 2 *Let X be a quotient of a separable polyhedral Banach space Y. Is then X polyhedral? Does X at least contain a copy of c_0?*

No counterexample is known. An interesting example, due to Alspach [1], in this context is the quotient space of $C(\omega^\omega)$, which is not isomorphic to a subspace of $C(K)$, K scattered, but which is itself an isometric predual of ℓ_1. We refer to Rosenthal's article in the Handbook [22] for more examples and results around $C(K)$, K scattered. The deep and extensive theory of $C(K)$ spaces and operators on them (at least when K is countable, and so $C(K)$ is polyhedral) has a direct bearing on the problems discussed in this survey. A classical theorem of Pelczynski implies that all quotients of $C(K)$, K scattered contain a copy of c_0. However even the following is unknown.

Proposition 3 *Let X be a quotient of $C(K)$, K countable. Is then X c_0 saturated?*

Besides proving the above theorems for polyhedral spaces, Fonf [10] characterized separable polyhedral spaces as follows. Condition (iii) comes from a later paper [13].

Theorem 3 *Let X be a separable Banach space X. TFAE.*
(i) X is isomorphically polyhedral,
(ii) X has a LFC renorming,
(iii) X has a C^∞-smooth and LFC renorming.

Composing a norm with a suitable smooth real function is a standard technique for obtaining bumps of the same smoothness (resp. preserves LFC) as a given norm. Thus the above theorems, whose proofs use different techniques, are formally generalizing Fonf's results valid exactly for spaces with LFC norms. We can now ask analogous questions about spaces admitting a LFC bump instead of polyhedral ones. We will not make a full list, but for example it seems reasonable to ask the following.

Proposition 4 *Let X be a quotient of a separable polyhedral Banach space Y. Does X admit a LFC bump?*

Coming back to the previous theorem, whose proof strongly uses convexity (via the notion of countable boundary) the following analogue is open.

Proposition 5 *Suppose that a separable Banach space X admits a (continuous) LFC bump. Does X admit a C^∞-smooth bump?*

The answer is yes under the additional assumption that X has a Schauder basis [17]. This is a very strong indication that the answer should be positive in general. We may ask even more.

Proposition 6 *Suppose that a separable Banach space X admits a (continuous) LFC bump. Is then X polyhedral?*

A tentative counterexample to this question, described below in some detail, is in [17].

Condition (iii) above has motivated subsequent research by several authors. The next theorem comes from [3].

Theorem 4 *Let X be a separable polyhedral Banach space X. Then X has a real analytic renorming. Moreover, every equivalent norm on X can be uniformly on bounded sets approximated by real analytic (resp. C^∞-smooth and LFC) norms.*

It was known since the work of Kurzweil [23] and Bonic and Frampton [2], that on separable Banach spaces the existence of a C^k (resp. C^∞) smooth bump implies the uniform approximability of all continuous functions by means of C^k (resp. C^∞) smooth functions. The above theorem is however considerably more difficult, and relies on different techniques, since the approximating functions remain convex.

Leung [25] has constructed an important example of an Orlicz sequence space h_M, which is Asplund, c_0 saturated, but nonpolyhedral. We will discuss his example below; at this point let us mention that it follows from the work of Maleev and Troyanski ([28], [29], [30]), that Leung's space h_M has an equivalent C^∞-smooth renorming. Their important characterization of the best smoothness for h_M in terms of the behaviour of M itself, is (in one direction) based of direct calculation. Of course, none of its renormings can be LFC, and in [18] it is shown that h_M does not have a real analytic renorming either.

Let us now pass to the discussion of the main result of Leung's work. The condition (iv) was stated in [25] without proof. For a proof see [17].

theorem[[26]] The following statements are equivalent for every non-degenerate Orlicz function M:

1. There exists a constant $K > 0$ such that $\lim_{t\to 0+} \frac{M(Kt)}{M(t)} = \infty$.

2. The Orlicz sequence space h_M is isomorphic to a subspace of $C(\omega^\omega)$.

3. The Orlicz sequence space h_M is isomorphic to a subspace of $C(K)$ for some scattered compact K.

4. There is a sequence $\{\eta_k\}$ of real numbers decreasing to 1 such that the norm on h_M defined by

$$|||x||| = \sup_k \eta_k \|P_k \widehat{x}\|$$

is LFC-$\{e_i^*\}$.

theorem

All spaces satisfying (ii) are subspaces of isometric preduals of ℓ_1 and thus polyhedral. Leung conjectured that conversely all polyhedral Orlicz sequence spaces satisfy his description. Besides the suggestive condition (iv) the following is true.

Negating the condition in (i) we obtain the following formula

$$(\forall K > 0)(\exists \{t_n\}_{n=1}^{\infty}, t_n \searrow 0) \lim_{n\to\infty} \frac{M(Kt_n)}{M(t_n)} < \infty.$$

Reversing the order of the quantifiers we obtain the following stronger (less general) condition

$$(\exists \{t_n\}_{n=1}^{\infty}, t_n \searrow 0)(\forall K > 0) \lim_{n\to\infty} \frac{M(Kt_n)}{M(t_n)} < \infty.$$

Leung proved [25] that Orlicz sequence spaces satisfying the last condition are not polyhedral (although they may be c_0 saturated). His example mentioned above of a nonpolyhedral c_0 saturated Asplund space is in fact an Orlicz space satisfying this condition.

Leung's theorem comes close to a characterization of polyhedrality for Orlicz sequence spaces. The gap lies in the exchange of quantifiers. In [17] we construct an Orlicz sequence space with C^∞-smooth and LFC bump lies strictly in between the above conditions. This space is either a non-polyhedral space admitting a LFC bump (we are inclined to believe this alternative), or Leung's polyhedral conjecture is false. The construction in [17] is summarized in the next two theorems.

Theorem 5 *Let M be a non-degenerate Orlicz function for which there exist sequences $F_k \subset (0,1]$ and $G_k \supset (\overline{F_k} \setminus \{0\})$, G_k open, such that*

1. $\lim_{k\to\infty}(\sup G_k) = 0$,
2. *there is a sequence $K_k > 1$ such that*

$$\lim_{\substack{t\to 0+\\ t\notin F_k}} \frac{M(K_k t)}{M(t)} = \infty,$$

3. *there is a $K > 1$ and a sequence $C_k \to \infty$ such that $M(Kt) \geq C_k M(t)$ for all $t \in G_k$.*

Then there exists a C^∞-smooth LFC lattice bump function on the Orlicz sequence space h_M.

Theorem 6 *There is a non-degenerate Orlicz function M such that $\liminf_{t\to 0+} \frac{M(Kt)}{M(t)} < \infty$ for any $K > 1$, yet the function M satisfies the conditions of the previous theorem. In particular, the corresponding Orlicz sequence space h_M admits a C^∞-smooth LFC lattice bump.*

Let us now pass to applications of LFC to $C(K)$, K scattered, spaces. The first result in this direction was a striking theorem due to Talagrand [37], according to which all spaces $C(\alpha)$, where α is an arbitrary ordinal segment with its natural order topology, admits a C^1-smooth renorming. The result is important for several reasons. First, if $\alpha \geq \omega_1$, the spaces $C(\alpha)$ do not admit a dual rotund renorming, and so these spaces are important examples in renorming theory. All previous smooth renormings were based on dual rotund renormings [4]. Second, the idea of the proof became a powerful tool for creating a whole theory through an impressive work on C^∞-smooth and LFC norms, bumps and partitions of unity due to Haydon [19], [20], [21].

At the core of this work lies the notion of (linear) Talagrand operator (abstracted from Talagrand's result by Haydon).

Definition 3 *Given a locally compact space L and a set M, we say that a bounded linear operator $T : C_0(L) \to c_0(L \times M)$ is a (linear) Talagrand operator if for every nonzero $f \in C_0(L)$, there exist $t \in L$ and $m \in M$ with $|f(t)| = \|f\|_\infty$ and $(Tf)(t,m) \neq 0$.*

Let us explain the main idea behind the applications. Given a set Γ, put $U(\Gamma) \subset \ell_\infty(\Gamma) \oplus c_0(\Gamma)$ to be a set of all pairs (f,g), such that $\|f\|_\infty, \|x\|_\infty < \||f| + \frac{1}{2}|x|\|_\infty$. Haydon proved the next [20]

Theorem 7 *The space $\ell_\infty(\Gamma) \oplus c_0(\Gamma)$ admits an equivalent lattice norm $\|\cdot\|$, whose restriction to $U(\Gamma)$ is C^∞-smooth and LFC.*

A simple argument shows that a $C_0(L)$ space admitting a Talagrand operator can be linearly isomorphically mapped into a subset of $U(L \times M)$, which in particular means that $C_0(L)$ has a C^∞-smooth and LFC renorming. For compact trees, Haydon proved [21] the stunning converse, namely as soon as the $C(T)$ has even just a Fréchet smooth renorming, then it has a Talagrand operator.

In general however, the converse to this statement is false. Indeed, Ciesielski and Pol [6] constructed a scattered compact space K, with the Cantor derivative $K^{(3)} = \emptyset$, such that $C(K)$ admits no continuous injections into any $c_0(\Gamma)$. Yet, by Godefroy, Pelant, Whitfield and the second author [11] all $C(K)$ spaces with $K^{(\omega_0)} = \emptyset$ admit a C^∞-smooth and LFC renorming. [11] has been subsequently generalized in [14] to all compacts with $K^{(\omega_1)} = \emptyset$. Further generalizations require imposing additional structural conditions on K, due to the fact that there exist examples [19], Kunen (published in [31]) of compact sets K, card$K^{(\omega_1)} = 1$, which admit no Gateaux differentiable, nor LFC renorming. One such condition is the existence of dual LUR renorming for $C(K)$. This condition has been characterized by Raja in [33] in terms of the compact space K, namely K has to be a descriptive compact. Then of course $C(K)$ has a C^1-smooth renorming, but using the structure of the dual ball it is proved in [16] that $C(K)$ admits even C^∞-smooth LFC renorming. Smith's work [34] characterizing those finite products of ordinal segments $K = [0,\alpha_1] \times \ldots [0,\alpha_k]$ for which $C(K)$ admits a linear Talagrand operator leads to other examples of $C(K)$ with C^∞-smooth and LFC norms without a Talagrand operator. The following question was asked already by Haydon, and the mentioned results add greatly to its credibility.

Proposition 7 *Suppose that $C(K)$ space admits a C^1-smooth renorming. Does it admit also a C^∞-smooth (resp. LFC) renorming?*

Proposition 8 *Suppose that $C(K)$ space admits a LFC renorming. Does it admit also a C^∞-smooth and LFC renorming?*

The use of the Talagrand operator inspires the following weaker version of the above question.

Proposition 9 *Let X be a Banach space. Suppose there exists $\{f_\gamma\}_{\gamma\in\Gamma} \subset S_{X^*}$ such that for every $0 \neq x \in X$ there exists a finite set $A \subset \Gamma$ such that $\|x\| = max_{\gamma\in A}|f_\gamma(x)| > sup_{\gamma\in\Gamma\setminus A}|f_\gamma(x)|$. (In particular, $\|\cdot\|$ is clearly LFC). Does there exist a C^∞-smooth and LFC renorming of X?*

Proposition 10 *Find a general approach to C^∞-smooth and LFC renormings for the $C(K)$, K scattered spaces, which includes both the results obtained via Talagrand operator and the dual LUR case.*

We believe that the last problem is resolvable through careful analysis of the existing results. In fact, Smith [35] has recently made progress in this direction for the related case of Gateaux smooth renormings of trees, and his results also make use of LFC techniques.

Let us now turn to the question of bumps and partitions of unity. In general, the following is a well-known problem in smoothness.

Proposition 11 *Let X be a Banach space admitting a C^k-smooth (resp. LFC etc.) bump. Does then X admit C^k-smooth (resp. LFC) partitions of unity?*

It is known that partitions of unity are equivalent to uniform approximations of all continuous functions by functions of the given smoothness class ([4]). The above problem has a positive solution for WCG spaces ([4]). The case when $X = C(K)$ has received some attention in [11], [5], [19], [20], [21], [15]. It was finally resolved in the positive in [16].

Theorem 8 *Let K be a scattered compact, TFAE.*

1. *$C(K)$ has a C^k-smooth (resp. LFC) bump.*
2. *$C(K)$ has a C^k-smooth (resp. LFC) partitions of unity (in particular every continuous function can be uniformly approximated by C^k-smooth functions).*
3. *$C(K)$ has a C^k-smooth (resp. LFC) nonlinear Talagrand operator.*

Nonlinear Talagrand operator is defined analogously to the linear one, except that the injection into $c_0(L \times M)$ is not necessarily linear. Instead, we require that the coordinate functions be of the prescribed smoothness, whenever they are nonzero. However, one of the main unresolved problems left in this area is the following.

Proposition 12 *Does every $C(K)$, K scattered, admit a LFC (resp. C^∞-smooth and LFC) bump?*

The example of Haydon in [19] has no LFC renorming, but as it was proved in [21], every $C(T)$ space where T is a tree compact has an C^∞-smooth and LFC bump. A tentative counterexample (under the continuum hypothesis) would be the $C(K)$, for K Kunen compact space ([31]). It is not clear at this point, whether the additional axioms of set theory might play a role in this investigation. It is clear that at least the LFC version of the problem (i.e. not requiring any smoothness of the bump) is an essentially combinatorial question. And there is an important precedent where axioms proved to play a role. Indeed, Todorčević [37] has shown under a strong version of Martin's axiom (Martin's maximum axiom), that every nonseparable Banach space contains an uncountable biorthogognal system. On the other hand, Kunen's space $C(K)$ [31], under continuum hypothesis, yields a nonseparable counterexample to the last statement. Needless to say, continuum hypothesis and Martin's axiom are mutually exclusive.

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