**RAC**SAM
*Rev. R. Acad. Cien. Serie A. Mat.*
VOL. **98** (1), 2004, pp. 229–237
Ciencias de la Computación / Computational Sciences

# A terminal area topology-independent GB-based conflict detection system for A-SMGCS

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**Abstract.** A module for conflict detection in A-SMGCS is presented. It supervises the operations that the ground controller has to perform. It doesn't depend on the topology of the terminal area. The system guarantees the safety of the proposed situation, that is, the impossibility that a conflict arises among aircrafts (and also road vehicles) obeying the signaling. We suppose that the terminal area has stop bars (or semaphores) controlling all intersections and accesses between runways, taxiways, exits, high speed exits, platform, apron's aircraft stands, air side vehicles service roads... plus stop bars controlling the intermediate taxiway's holding points and runway-holding positions. It is surprising that the use of semaphores for road vehicles traffic control is not frequent (although, they can be fully integrated in the conflict detection system of a A-SMGCS). The model proposed uses digraphs and the implementation is based on the algebraic theory of Gröbner bases.

### Un sistema de detección de conflictos para A-SMGCS independiente de la topología del área terminal basado en GB

**Resumen.** Se describe un módulo de A-SGMCS encargado de la detección de conflictos. Él supervisa las operaciones que el controlador de rodadura debe realizar. No depende de la topología del área terminal. El sistema garantiza la seguridad de la situación propuesta, esto es, la imposibilidad de que surja un conflicto entre aeronaves (y también vehículos de carretera) que respeten la señalización. Suponemos que el área terminal está dotada de barras de parada (o incluso semáforos) que controlan todas las intersecciones y accesos entre pistas, calles de rodaje, calles de salida, plataforma, puestos de estacionamiento de aeronaves, viales exclusivos para vehículos de carretera,... así como barras de parada que controlan los puestos de espera en rodaje y los puestos de espera en acceso a pista. Es sorprendente que no sea habitual el uso de semáforos en aeropuertos, como elementos de control del tráfico de vehículos de carretera (éstos no obstante, se pueden integrar con plena solvencia en el sistema de detección de conflictos A-SMGCS). El modelo utiliza digrafos y la implementación está basada en la teoría algebraica de las bases de Gröbner.

## 1. Introduction

We believe there is a surprising contrast between the great development and generalized use of computer-based *ATM* (*Air Traffic Management*) systems and the relatively scarce use of *A-SMGCS* (*Advanced Surface Movement Guidance and Control Systems*).

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Presentado por Manuel López Pellicer.
*Recibido*: February 29, 2004. *Aceptado*: October 13, 2004.
*Palabras clave / Keywords*: Air Traffic Control, Gröbner Bases, Computer Algebra, Artificial Intelligence, Problem Solving.
*Mathematics Subject Classifications*: 332701 (UNESCO Sci. & Tech. Class.), 13P10, 68T20.

Surface movement control is usually supervised manually by the ground controller. Moreover, controlling surface movements through the massive use of stop bars (and semaphores for road vehicles in air side vehicles service roads) and automated detection of the position of aircrafts and road vehicles (through the use of induction loops, ground radars, information sent from on-board GPSs...) is only used in some huge airports such as Frankfurt [8].

This isn't a secondary problem. As is well known, the worst accident of the history of aviation took place in an airport (Los Rodeos). In fact, the development of A-SGMCSs is a highly topical question. See, for instance, [9] or the projects of *Eurocontrol*: *TaxiCap* and *A-SMGCS APR Project 1* [5].

This article will be focused on the development of the software corresponding to the module of the A-SMGCS in charge of supervising the ground controller's operation (conflict detection module).

## 2. Focusing the Problem

There are some non-trivial questions that are not treated in this article:

- coordination between the ATM and the A-SMGCS
- hardware security (processors reliability and equipment redundancy)
- communications interfaces:
  computer ↔ stop bars and semaphores
  computer ↔ induction loops, ground radar, on-board GPSs
- software verification

and are crucial in a safety-critical task like this.

This article focuses on the development of the software corresponding to the module of the A-SGMCS in charge of supervising the operations that the ground controller has to perform (conflict detection module) in a terminal area with any topology.

## 3. Related Work of this Research Team

We have worked in various research lines related to traffic control and traffic simulation:

- We have applied Commutative Algebra Techniques to knowledge extraction and verification of Knowledge-Based Systems (Expert Systems), the underlying logic being either Boolean or modal multivalued. The implementations are developed in Computer Algebra Systems (CASs) like Maple, CoCoA,.... Although the underlying mathematical theory (Gröbner bases) has a high complexity [4], the implementations are surprisingly brief. Moreover, this approach can be adapted to decision making in a railway interlocking system [11] (this work was inspired by D. Bayer's approach to 3-coloring graphs [2, 1], but in the railway interlocking system we consider directed graphs). The resulting code is far shorter than other well known topology-independent approaches [3, 6, 7]. In fact, [11] is the starting point of the present article, although in this case there can be multiple edges between two nodes. This article is an extended version of [14].
- We have also treated decision making in a railway interlocking system using a Boolean matrix-based approach [10]. In this case the implementation can be developed in a CASs as well as in a Numeric System such as MatLab. The conflict detection module of an A-SMGCS has been studied from this point of view in [13].
- The collaboration with *AERTEC Ingeniería y Desarrollos* began with the development of a simulation package for AENA-Málaga (AENA is the Spanish National Airport Authority). This package simulates, passenger by passenger, the movements of departing passengers within the terminal building [12].

## 4. About the Conflict Detection Module

**Definition 1** *We shall denote by "sections" the regions of the runways, taxiways, exits, high speed exits, platform, apron's aircraft stands, air side vehicles service roads... delimited by stop bars, semaphores and NO-ENTRY signals.*

**Example 1** *Let us consider the simple terminal area in Figure 1 (it has no air side vehicles service roads).*

*The runway ($AB$) and the taxiway $A'B'$ are mainly used in the directions $B \to A$ and $A' \to B'$, respectively. There are two orthogonal exits ($ML$ and $GF$) connecting the runway and the taxiway and two high speed exits ($HI$ for the usual direction of use and $JK$ for the unusual). Exploitation is therefore performed counterclockwise in quadrilateral $MLFG$.*

*Meanwhile, the access to the platform from the taxiway takes place through $NN'$ and the exit from the taxiway to the platform through $OO'$.*

*The platform is considered to be connected to isolated branches (apron's aircraft stands in front of the gates).*

*We have also considered some more special sections: the sections of the airport's airspace corresponding to approach and take off from the runway(s) ($DB$ and $AE$ in Figure 1). These sections can be considered to be as short as desired: for instance, section $DB$ can be the imaginary section where an aircraft that has already been authorized to land in runway $BA$ is allocated.*

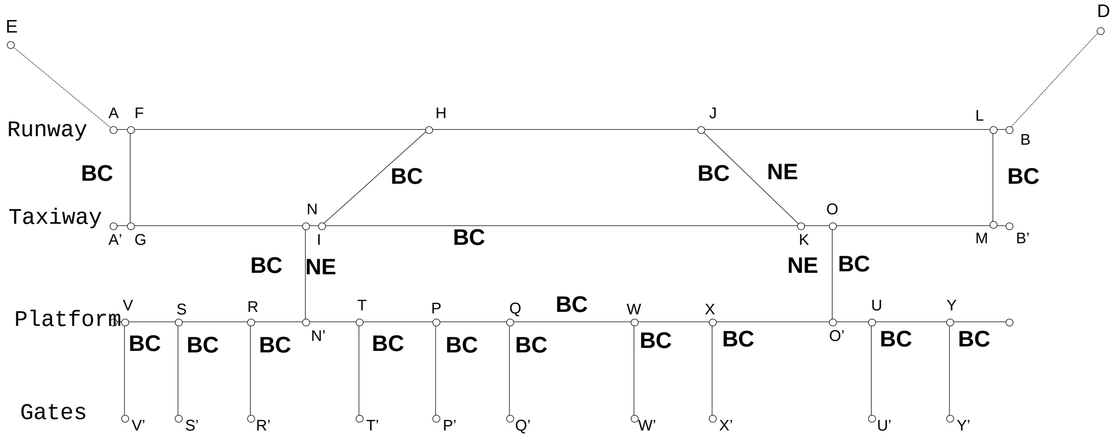

Figure 1. Sketch of a simple terminal area

*$BC$ signals correspond to control bars and $NE$ to "NO-ENTRY" signals.*

*For example, there is a control bar in the exit $HI$ that controls the movements $H \to I$ and $I \to H$ and a "NO-ENTRY" signal in exit $JK$ that forbids entering the runway from the taxiway from $K$.*

*One section would consist in the part of the taxiway from $A'$ to the control bar between $I$ and $H$, together with the space from $G$ to the control bar in $GF$, and from $I$ to the control bar in $IH$ and from $N$ to the control bar in $NN'$.*

*The terminal area would be divided in the following sections:*

$$EAFHJLBD, A'GNI, KOMB', VSRN'TPQ, WXO'UY, V', S', R', T', P', Q', W', X', U', Y' \ .$$

*Note that the sections correspond to the nodes of the associated graph. The connectivity among sections can be found in Figure 2. Observe that multiple arcs exist and only the possibility to access has been represented.*

*But, in fact, what we have to consider is a digraph instead of a graph, because "NO-ENTRY" signaling is directional. For instance, such a signal in the high speed exit $JK$ forbids entering but not exiting the runway (i.e., it forbids movement $KOMB' \rightarrow EAFHJLBD$).*

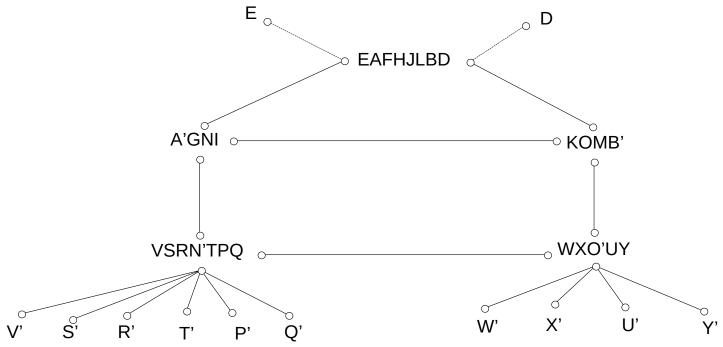

Figure 2. Graph associated to the scheme of Figure 1

*Just a few control bars have been included in this simple example. To take into practice a denser exploitation following this philosophy of suppressing movements secured exclusively in a visual way, it would be a good idea to install more control bars. For example one could be installed at the runway's threshold, and some more could be installed along the taxiway (intermediate taxiway's holding points).*

# 5. Input Data

## 5.1. Fixed Input

Part of the input of the problem is always fixed (except if the terminal area is reformed):

- topology of the terminal area (runways, taxiways, exits, high speed exits, platform, apron's aircraft stands, air side vehicles service roads... and their connections)
- position of control bars (that control the passage of aircrafts and road vehicles) and semaphores (that control road vehicles traffic in air side vehicles service roads).

## 5.2. Variable Input

However, another part of the input is time-dependent and must be updated:

- "color" of the stop bars and semaphores
- presence of aircrafts and road vehicles in the different sections (including the sections of the airport's airspace corresponding to approach and take off from the runway(s))

# 6. Accessibility

**Definition 2** *Two sections are called "consecutive" if and only if a stop bar or semaphore or "NO-ENTRY" signal separates them.*

**Example 2** *In the terminal area of figures 1 and 2, $AG'NI$ and $KOMB'$ are consecutive sections. Meanwhile, sections $EAFHJLBD$ and $WXO'UY$ are not consecutive.*

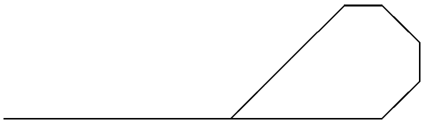

Figure 3. Reversing loop

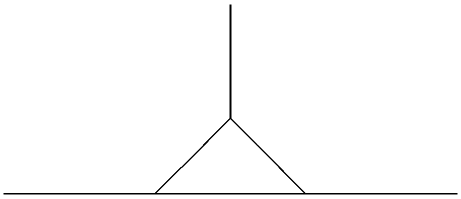

Figure 4. Reversing triangle

### 6.1. Accessibility to a Next Section

A digraph (directed graph) $\mathcal{G}$ is considered. The vertices of the graph are the sections of the line and the directed edges represent movements to a next section that are allowed (at least through one path) according to the current color of the stop bars and semaphores.

Graph $\mathcal{G}$ is reflexive, as any aircraft or road vehicle can stay where it is.

### 6.2. General Accessibility

Nevertheless, the problem is not so simple, as the possibility of an aircraft (or road vehicle) moving from the section it occupies to another one, then to another one and again to another one, etc... has to be taken into account. Think for instance about moving along the taxiway if all the stop bars along it allow going on. Therefore, the general movements of the aircrafts (or road vehicles) will be given by the transitive closure of graph $\mathcal{G}$.

Observe that considering directed graphs allows this approaches to deal with conflicting situations like reversing loops and reversing triangles without problems (see figures 3 and 4).

## 7. Safety of a Proposed Situation

Under this approach, an accident could only take place if two aircrafts or an aircraft and a road vehicle obeying the signaling could access the same section at the same time.

Let us denote by $Z(\alpha)$ the set of sections accessible by an aircraft or road vehicle located in section $\alpha$. Then, a proposed coloring of control bars and semaphores will be safe if and only

$$Z(\alpha) \cap Z(\beta) = \emptyset$$

for every pair of sections $\alpha$ and $\beta$ ($\alpha \neq \beta$) occupied by an aircraft or a road vehicle (*).

Therefore, if we start with a safe situation (for instance, an empty terminal area or a terminal area with all control bars and semaphores forbidding movements and at most one aircraft or road vehicle in each section) and we follow an inductive process: whenever any change is to take place, it has to be analyzed as suggested in (*) before being authorized, all situations will always be safe.

In fact we shall consider that more than one road vehicle can be in the same section at the same time; otherwise lots of semaphores should have to be installed along the air side vehicles service roads. In such cases the group of road vehicles in the section will be assigned a single number (**).

# 8. Digraphs and Polynomial Ideals

## 8.1. Representing the Digraph and the Position of Aircrafts and Road Vehicles with a Polynomial Ideal

Let us associate polynomial variables such as $x_1, x_2, ..., x_n$ to the sections in which the terminal area is divided. The graph $\mathcal{G}$ will be interpreted as a polynomial ideal $I \subseteq Q[x_1, x_2, ..., x_n]$ that is initialized as $\langle 0 \rangle$ . For instance, that it is possible to move from section $x_i$ to a (next) section $x_j$ (according to the position and "color" of the stop bars and semaphores and the position of the "NO-ENTRY" signals) will be represented by including polynomial

$$x_i \cdot (x_i - x_j)$$

in the ideal $I$ .

Nevertheless, a certain preprocessing of ideal $I$ is recommended. If it is possible to move both from $x_i$ to $x_j$ and from $x_j$ to $x_i$ then we shall include $x_i - x_j$ in ideal $I$ , instead of both $x_i \cdot (x_i - x_j)$ and $x_j \cdot (x_j - x_i)$.

The ideal $P$, corresponding to the position of the aircrafts and road vehicles is also initialized as $\langle 0 \rangle$. Aircrafts and road vehicles (or groups of road vehicles in the same section) will be denoted by (different) positive integers. For instance, if aircraft (or road vehicle) $\alpha$ is in section $x_i$ , then the polynomial

$$x_i - \alpha$$

will be included in the list of generators of the ideal, $P$.

Observe that no $x_i - \alpha$ and $x_i - \beta$ with $\alpha \neq \beta$ can appear together as generators of $P$, because a section can't be occupied by more than one aircraft or road vehicle at the same time (except the case (**)).

## 8.2. Deciding the Accessibility with the Polynomial Model

**Proposition 1** *If aircraft or road vehicle of number $\alpha$ is in section $x_i$ and it is possible to pass from section $x_i$ to a next section $x_j$, then $x_j - \alpha \in I + P$ (Accessibility to a next section).*

PROOF. If aircraft or road vehicle of number $\alpha$ is in section $x_i$ , then $x_i - \alpha \in I + \langle x_i - \alpha \rangle \subseteq I + P$ (***). Now:

i) if it is possible to pass from section $x_i$ to section $x_j$ , but it is not possible to pass from section $x_j$ to section $x_i$ , we have

$$x_i \cdot (x_i - x_j) \in I \subseteq I + P$$

and then it follows from (***) that

$$\alpha \cdot (\alpha - x_j) \in I + P$$

but $\alpha$ is a positive integer (multiplicative unit), and consequently

$$\alpha - x_j \in I + P \ .$$

ii) if it is possible to pass both from section $x_i$ to $x_j$ and from $x_j$ to $x_i$ , then we have

$$x_i - x_j \in I$$

and then it follows from (***) that

$$\alpha - x_j \in I + P \ .$$

■

Therefore, the value $\alpha$ "propagates" through the (directed) edges of $\mathcal{G}$. Moreover, this happens not only through the (directed) edges of $\mathcal{G}$ but through the directed edges of the transitive closure of $\mathcal{G}$ (what can be proven by finite induction).

Reciprocally, as the polynomials that generate the ideal $I$ are given by the directed edges of $\mathcal{G}$, the value $\alpha$ can not "propagate" if there is no directed edge linking them in the transitive closure of $\mathcal{G}$.

So we have the the following:

**Proposition 2** *An aircraft or road vehicle of number $\alpha$ , situated in section $x_i$ , can reach section $x_j$ if and only if $x_j - \alpha \in I + P$ (General Accessibility).*

Using the well known radical membership criterion [4] and Gröbner bases (GB), the previous proposition can be expressed as follows (all the ideals treated here are radical).

**Corollary 1** *Let $t$ be a new variable, and let us consider the polynomial ring $Q[x_1, x_2, ..., x_n, t]$ . An aircraft or road vehicle of number $\alpha$ , situated in section $x_i$ , can reach section $x_j$ if and only if*

$$GB(\ \langle 1 - t \cdot (x_j - \alpha) \rangle + I + P\ ) = \{1\}$$

*(General Accessibility).*

## 8.3. Decision Making about the Safety in the Polynomial Model

**Theorem 1** *A proposed situation given by the ideal $I$ and a position of aircrafts and road vehicles given by ideal $P$ is safe if and only if*

$$I + P \ \neq \ \langle 1 \rangle$$

*i.e., if and only if*

$$GB(I + P) \neq \{1\}$$

*(Safety).*

PROOF. Proposition 2 can also be used to check the safety of a proposed situation. A proposed situation is not safe if and only if two different aircrafts or road vehicles, $\alpha, \beta$ ($\alpha \neq \beta$), respectively located in sections $x_i$ and $x_j$ ($i \neq j$), and a certain section $x_k$ exist such that the two aircrafts or road vehicles can reach section $x_k$. But this is equivalent to

$$x_k - \alpha \ , \ x_k - \beta \in I + P$$

what implies

$$\alpha - \beta \in I + P \ .$$

As $\alpha$ and $\beta$ are different positive integers, $\alpha - \beta$ is a multiplicative unit, and therefore the previous statement is equivalent to the degeneration of the ideal $I + P$ into the whole ring, i.e., to

$$I + P = \langle 1 \rangle \ .$$

Reciprocally, if $I + P = \langle 1 \rangle$, then $x_k - \alpha \ , \ x_k - \beta \in I + P$, that is, $x_k$ is accessible by two different vehicles or road vehicles ($\alpha$ and $\beta$), and therefore the situation is not safe. ■

**Remark 1** *Now it is clear why it is convenient to preprocess ideal* $I$*: the GB corresponding to* $\{x_i \cdot (x_i - x_j), x_j \cdot (x_j - x_i)\}$ *is something like* $\{x_i \cdot x_j - x_j^2, x_i^2 - x_j^2\}$ *, that, although leading to the same result as* $x_i - x_j$ *(in this particular application), is far more laborious to handle. This is specially important in case no integer value is "propagated" through those edges because, in such case, these polynomials are carried along the subsequent computations.* ■

## 9. Implementation

The code has been developed in the Computer Algebra System (CAS) *Maple*. It uses Maple's Gröbner bases implementation and is extremely brief (one page of code).

It handles in a matter of seconds proposed situations of similar size to that of figures 1 and 2.

The implementation relies on the correctness of the code of this application and of Maple's Gröbner bases package. Therefore this work can't be considered as a real application but as a prototype: in safety-critical packages like this there are many topics involved, apart from verification of software, such as verification of the information transmitted to the computer from the induction loops, ground radars, on-board GPSs,...; redundancy of equipments;... that have not been discussed.

## 10. Conclusions

In our opinion, a simple but exciting application of Gröbner bases to a non-trivial transportation engineering decision problem has been developed. Moreover, the corresponding code is surprisingly brief.

**Acknowledgement.** This work was partially supported by the research project PR3/04-12410 (Universidad Complutense de Madrid, Spain).

We would also like to thank Antonio Gómez-Guillamón (AERTEC Ingeniería y Desarrollos) for his support of this research work.

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