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Scattering problems in a domain with small holes

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Abstract. In this paper, we consider a family of scattering problems in perforated unbounded domains Ω_ε. We assume that the perforation is contained in a bounded region and that the holes have a 'critical' size. We study the asymptotic behaviour of the outgoing solutions of the steady-state scattering problem and we prove that an extra term appears in the limit equation. Finally, we obtain convergence results for scattering frequencies and solutions.

Problemas de difracción en un dominio con pequeños agujeros.

Resumen. En este artículo consideramos una familia de problemas de difracción en un dominio Ω_ε no limitado y perforado. Suponemos que las perforaciones están contenidas en una región limitada y que los agujeros tengan una talla crítica. Estudiamos el comportamiento asintótico de las soluciones que emergen del problema estacionario de difracción y probamos que en la ecuación límite, aparece un término nuevo. Finalmente, obtenemos algunos resultados de convergencia para las frecuencias y las soluciones de difracción.

1. Introduction

In this paper we study the limit behaviour of a family of scattering problems (P_ε), defined in perforated domains Ω_ε, where $\varepsilon \to 0_+$. The domain $\Omega_\varepsilon = \mathbb{R}^3 \setminus T_\varepsilon$, where $T_\varepsilon = \cup_k T_\varepsilon^k$ is the union of the sets T_ε^k (the so-called 'holes'), that are assumed to be contained in a bounded region B independent of ε. We consider homogeneous Dirichlet boundary conditions on the boundary of the holes ∂T_ε and the Sommerfeld radiation condition at infinity (see formula (3)). Our aim is to prove that, under specific assumptions on the asymptotic behaviour of T_ε, the scattering frequencies and the corresponding scattering solutions (see Section 4) converge to the ones related to a limit problem on the whole of $\mathbb{R}^3$, where an extra-term appears in the differential operator, and the Sommerfeld radiation condition at infinity is preserved.

The behaviour of Dirichlet boundary-value problems for the Laplace operator on *bounded* perforated domains $B_\varepsilon = B \cap \Omega_\varepsilon$ is well-known and it is deeply analyzed in the paper [2]. The appearance of the extra-term for this situation is proved by Cioranescu and Murat under the assumption that the holes T_ε have the so called 'critical size', while the cases where the holes are ”too small” or ”too large” exhibit a different behaviour.

In the present paper we are concerned with sets T_ε^k that are homothetic to a given set T, have diameter proportional to a small parameter r_ε, and are evenly distributed along a periodic network of period ε. Different phenomena occur in dependence of the limit of $r_\varepsilon/\varepsilon^3$, as $\varepsilon \to 0$. More precisely, if $r_\varepsilon \sim \varepsilon^3$,

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we say that T_ε have the 'critical size', and we show that the phenomenon of the appearance of the extra-term extends to the case of scattering problems. The same problem for the cases where $r_\varepsilon/\varepsilon^3 \to 0$ and $r_\varepsilon/\varepsilon^3 \to \infty$ was studied in [8]. A general reference about scattering theory is [6]. Other results connected with perforated domains can be found in [1] and [12]. The plan of the paper is the following: in Section 2 we introduce problem (P_ε), and recall the main properties concerning existence and uniqueness of solutions; in Section 3 we deal with the limit problem (P_0) and the study of the convergence of the solutions of the above problems (P_ε); finally, in Section 4 we introduce the concept of scattering frequency and scattering solution, and study the corresponding limit behaviour.

2. Statement of the problem

Let $Y = (0,1)^3$ and let $T \subset\subset Y$ be a closed set with Lipschitz boundary. Given two real parameters $\varepsilon, r_\varepsilon$, with $0 < r_\varepsilon < \varepsilon$, we denote the r_ε-homothetic contraction of T by $r_\varepsilon T$, and for each integer vector $k \in \mathbb{Z}^3$ we denote the ε-translations by $T_\varepsilon^k = r_\varepsilon T + \varepsilon k$ (see figure 1). Given a bounded open set $B \subset \mathbb{R}^3$, we consider the set of integer vectors $I_\varepsilon(B) = \{k \in \mathbb{Z}^3 : T_\varepsilon^k \subset B\}$ and we denote by Ω_ε the perforated domain $\Omega_\varepsilon = \mathbb{R}^3 \setminus T_\varepsilon$, where $T_\varepsilon = \cup_{k \in I_\varepsilon(B)} T_\varepsilon^k$.

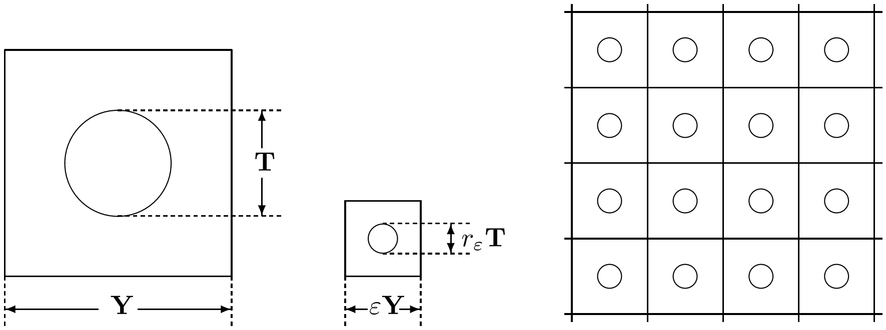

Figure 1. The reference period $\mathbf{Y}$ and the periodical reproduction of T_ε^k.

Note that Ω_ε is an unbounded perforated domain where the 'holes' T_ε^k are distributed in the bounded region B along a periodic lattice of side-length ε, and have diameter proportional to r_ε (see figure 2).

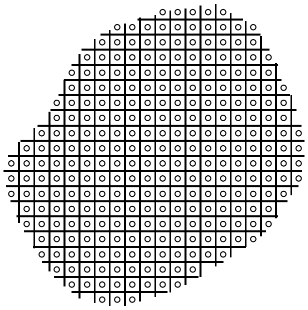

Figure 2. The perforated domain Ω_ε.

In the sequel we use the standard notation for Lebesgue and Sobolev spaces. We shall deal with functions that, a priori, may take values in the complex plane $\mathbb{C}$, but, for the sake of simplicity, we shall not indicate $\mathbb{C}^3$ in the notation for the function spaces. Hence, for instance, $L^2(A)$ stands for $L^2(A;\mathbb{C})$, and so on. The scalar product in $L^2(A)$ is denoted by

$$(u,v)=\int_A u\bar{v}dx.$$

Finally, we denote by $B_R=B(0,R)=\{x\in\mathbb{R}^3 : |x|<R\}$ the open ball of $\mathbb{R}^3$, centered at 0, with radius $R>0$.

We now consider the reduced wave equation in Ω_ε with the outgoing radiation condition (see [10]). The dependence in time is supposed of the type exp($-i\omega t$). The problem $(P_\varepsilon(\omega))$ for a fixed $\varepsilon>0$ reads: find u^ε such that

$$-\Delta u^\varepsilon-\omega^2u^\varepsilon=f \quad \text{in} \quad \mathcal{D}'(\Omega^\varepsilon) \tag{1}$$

$$u^\varepsilon=0 \quad \text{on} \quad \partial\mathbf{T}_\varepsilon \tag{2}$$

$$u^\varepsilon(x)=\frac{1}{4\pi}\int_{|y|=R}\left(-u^\varepsilon\frac{\partial}{\partial|y|}\left(\frac{\mathrm{e}^{\,\mathrm{i}\,\omega|x-y|}}{|x-y|}\right)+\frac{\partial u^\varepsilon(y)}{\partial|y|}\,\frac{\mathrm{e}^{\,\mathrm{i}\,\omega|x-y|}}{|x-y|}\right)\mathrm{d}\,S_y \quad \text{with} \quad |x|>R \tag{3}$$

where $f\in L^2(\mathbb{R}^3)$ is such that $\operatorname{support} f\subset B(0,R)$ and $\omega\in\mathbb{C}$ satisfies $-\pi/2<arg\sqrt{\omega^2}\leq\pi/2.$ The properties of the solutions u^ε to the problem $(P_\varepsilon(\omega))$ for a given value of $\omega\in\mathbb{C}$ will be addressed at the end of this Section. Moreover the asymptotic behavior of u^ε as $\varepsilon\to0$ will be presented at the end of Section 3. The problem considered in this paper is related in some sense to perturbation or homogenization problems studied in [5], [7] and [9].

Remark 1 Since

$$-\Delta u^\varepsilon-\omega^2u^\varepsilon=0 \tag{4}$$

for $|x|>R$, then by the interior regularity of the solution of the elliptic equations, $u^\varepsilon\in H^2(B_{R+1}\setminus\overline{B_R})$. Hence, by the trace theorem, we have that $u^\varepsilon|_{\partial B_R}\in H^{3/2}(\partial B_R)$ and $(\partial u^\varepsilon/\partial n)|_{\partial B_R}\in H^{1/2}(\partial B_R)$, where $\vec{n}=(n_1,n_2,n_3)$ is the unitary outer normal to ∂B_R. Therefore the integral expression (3) in $P_\varepsilon(\omega)$ makes sense and shows the behaviour of u^ε at infinity. ■

Remark 2 If $\omega\in\mathbb{R}$, the requested decay in (3) does not guarantee that $u^\varepsilon\in L^2(\mathbb{R}^3)$. Moreover, in this case the expression (3) is equivalent to the outgoing Sommerfeld radiation condition

$$\left|\frac{\partial u^\varepsilon}{\partial|x|}-\mathrm{i}\,\omega u^\varepsilon\right|=\mathrm{O}(R^{-2}) \quad \text{and} \quad |u^\varepsilon|=\mathrm{O}(R^{-1})$$

as $R\to+\infty$. For this reason we call the expression (3) the outgoing radiation condition in integral form; this condition is valid both for ω real and ω complex (see [10]). ■

Remark 3 If, in the integral expression in (3), we take $\mathrm{e}^{-\,\mathrm{i}\,\omega|x-y|}$ in place of $\mathrm{e}^{\,\mathrm{i}\,\omega|x-y|}$ we get the *incoming radiation condition*. ■

Remark 4 In this paper we work in the space $\mathbb{R}^3$, but we may present the problem in $\mathbb{R}^N$ with $N>3$. In this case the radiation condition at infinity would be

$$u^\varepsilon(x)=\int_{|y|=R}\left(u^\varepsilon\frac{\partial G_\omega(x,y)}{\partial|y|}-\frac{\partial u^\varepsilon(y)}{\partial|y|}\,G_\omega(x,y)\right)\mathrm{d}\,S_y$$

where $G_\omega(x,y)$ is the Green's function given by

$$G_\omega(x,y) = \frac{1}{4}\left(\frac{\omega}{2\pi|x-y|}\right)^{(N-2)/N} H^{(1)}_{(N-2)/2}(\omega|x-y|)$$

and $H^{(1)}$ is the Hankel function of the first kind. ■

It is known (see for instance [10], [13]) that, if the imaginary part of ω is greater than or equal to zero, the problem $(P_\varepsilon(\omega))$ has a unique solution. More precisely, we have the following

Proposition 1 *For every $\varepsilon > 0$, the problem (P_ε) has one and only one solution $u^\varepsilon \in H^1(B_R \setminus \mathbf{T}^\varepsilon)$, ($R > 0$ such that $B \subset B_R$) for any complex ω except from a discrete set of complex numbers with* $\operatorname{Im}\omega < 0$. ■

For the proof see in particular [10], Chapter 15, Theorem 2.3.

Remark 5 Starting from Proposition 1, it is proved that the solution of problem (P_ε) depends analytically on ω (see [10], [11]). Then we can emphasize the dependence on ω and on f, writing $u^\varepsilon(f,\omega)$ as the solution of problem $P_\varepsilon(f,\omega)$. Moreover, in the reference [10] it is proved that $u^\varepsilon(f,\omega)$ is meromorphic in ω, and its poles have imaginary part less then zero. ■

3. A priori estimates and convergence of solutions with $r_\varepsilon \approx \varepsilon^3$

Let us denote by $\tilde{u}^\varepsilon$ the extension by zero, to the whole of $\mathbf{R}^3$, that is:

$$\tilde{u}^\varepsilon = \begin{cases} u^\varepsilon & \text{in} \quad \Omega^\varepsilon \\ 0 & \text{in} \quad \mathbf{T}_\varepsilon. \end{cases}$$

Then we have that, for all ε, $\tilde{u}^\varepsilon \in H^1_{\text{loc}}(\mathbb{R}^3)$.

The limit analysis depends on the behaviour, as $\varepsilon \to 0$, of $r_\varepsilon / \varepsilon^3$, and three different situations occur:

(i) when $r_\varepsilon \ll \varepsilon^3$, then $\tilde{u^\varepsilon} \to u$, where u solves the equation

$$-\Delta u - \omega^2 u = f \qquad \text{in} \qquad \mathcal{D}'(\mathbb{R}^3),$$

with the Sommerfeld radiation condition at infinity (3);

(ii) when $r_\varepsilon \approx \varepsilon^3$, an extra term appears in the limit problem;

(iii) when $r_\varepsilon \gg \varepsilon^3$, then $\tilde{u}^\varepsilon \to 0$ in $L^2(B)$.

Afterwards we consider the more interesting case (ii), where an extra term appears in the limit equation. Some information about the case (i) and (iii) may be found in [8].

Lemma 1 *For $r_\varepsilon \approx \varepsilon^3$ and for ω real and positive, the extension $\tilde{u}^\varepsilon$ of the solution u^ε of the problem (P_ε) satisfies the estimate*

$$\|\tilde{u}^\varepsilon\|_{H^1(B_{R+5})} < M \tag{5}$$

where M is a constant independent of ε.

PROOF. By contradiction, we suppose that

$$\|\tilde{u}^\varepsilon\|_{H^1(B_{R+5})} = A_\varepsilon \to +\infty \qquad\qquad \varepsilon \to 0,$$

and normalize

$$w^\varepsilon = \frac{\tilde{u}^\varepsilon}{A_\varepsilon}; \qquad\qquad \|w^\varepsilon\|_{H^1(B_{R+5})} = 1 \qquad\qquad \forall\varepsilon. \tag{6}$$

Then we have a subsequence of w^ε (still denoted with w^ε) such that

$$w^\varepsilon \rightharpoonup w_0 \qquad\qquad \text{weakly in } H^1(B_{R+5}) \quad \varepsilon \to 0. \tag{7}$$

We study the properties of w_0 in the region $R < |x| < R+5$ where, $\forall\varepsilon$, w^ε satisfies (see (4))

$$-\Delta w^\varepsilon - \omega^2 w^\varepsilon = 0.$$

By the interior regularity theory for elliptic equations and by the normalization condition we obtain

$$\|w_\varepsilon\|_{H^2(B_{R+4}\setminus\overline{B_{R+2}})} \leq c(1 + |\omega|^2)$$

with the constant c depending only on R. By the fact that $\nabla w_\varepsilon \to \nabla w_0$ strongly in $L^2(B_{R+5} \setminus \overline{B_R})$ and by the trace theorem, we have

$$w_\varepsilon|_{|x|=R+2} \to w_0|_{|x|=R+2} \qquad \text{strongly} \qquad \text{in } H^{3/2}(\partial B_{R+2}) \tag{8}$$

$$\frac{\partial w_\varepsilon}{\partial n} \to \frac{\partial w_0}{\partial n} \qquad \text{strongly} \qquad \text{in } H^{1/2}(\partial B_{R+2}), \tag{9}$$

where n is the outer unit normal. By multiplying formula (3) by $\frac{1}{A_\varepsilon}$ and by taking the limit as $\varepsilon \to 0$, using the formulae (8) and (9) we get the radiation condition for w_0 in $\{R < |x| < R+5\}$. By analytical continuation, the same radiation condition is satisfied in the exterior domain.

We now study the properties of w_0 for $\{|x| < R+2\}$, we have:

$$-\Delta w^\varepsilon = \frac{f}{A_\varepsilon} + \omega^2 w^\varepsilon \qquad \text{in} \qquad \mathcal{D}'(\Omega^\varepsilon) \tag{10}$$

By the results of Cioranescu and Murat [2] the limit equation, as $\varepsilon \to 0$ becomes:

$$-\Delta w^0 + \chi_B \mu w^0 - \omega^2 w^0 = 0 \qquad \text{in} \qquad \mathcal{D}'(B_{R+5}) \tag{11}$$

where χ_B is the characteristic function of B and μ is a positive constant depending only on the set **T**.

Finally, by equation (11) and by the radiation condition and the uniqueness theorem (see [10] cap.XVI, Theorem 1.1) we get $w^0 = 0$.

To get a contradiction it is enough to notice that ∇w^ε converges to 0 even strongly in $L^2(B_{R+5})$. This is due to the convergence of the energies

$$\begin{aligned}\int_{B(R+5)} |\nabla w^\varepsilon|^2 &= \int_{B(R+5)} f w^\varepsilon - \omega^2 \int_{B(R+5)} |w^\varepsilon|^2 \longrightarrow \int_{B(R+5)} f w^0 - \omega^2 \int_{B(R+5)} |w^0|^2 \\ &= \int_{B(R+5)} |\nabla w^0|^2 + \int_{B(R+5)} \chi_B \mu |w^0|^2.\end{aligned}$$

(see [2], proof of Theorem 3.4). ■

Proposition 2 *The extended solutions $\tilde{u}^\varepsilon$ of problems (P_ε) given by equations (1),(2), and (3) converge, in the distribution sense, to the solution of the following problem (P_0):*

$$-\Delta u^0 + \chi_B \mu u^0 - \omega^2 u^0 = f \qquad \textit{in} \qquad \mathcal{D}'(\mathbb{R}^3) \tag{12}$$

$$u^0(x) = \frac{1}{4\pi}\int_{|y|=R} \left(-u^0 \frac{\partial}{\partial |y|} \left(\frac{\mathrm{e}^{\,\mathrm{i}\,\omega|x-y|}}{|x-y|} \right) + \frac{\partial u^0(y)}{\partial |y|} \, \frac{\mathrm{e}^{\,\mathrm{i}\,\omega|x-y|}}{|x-y|} \right) \mathrm{d}\, S_y \tag{13}$$

$$\textit{with} \qquad |x| > R$$

PROOF. By Lemma 1, we can extract a subsequence, still denoted by u^ε, that converges to u^0 weakly in $H^1_{loc}(\mathbf{R}^3)$. Reasoning as for w^ε in the preceding lemma we obtain that u^0 satisfies problem (P_0). ■

4. Convergence of the scattering frequencies

We may consider the problem (P_ε) and the solution u^ε as functions depending on the complex parameter ω and on the function f (see (1)):

$$P_\varepsilon(f,\omega) \qquad u^\varepsilon(f,\omega)$$

and the same notation we use for P_0 and u^0 (see(12)):

$$P_0(f,\omega) \qquad u^0(f,\omega).$$

When $f = 0$ and ω is complex, we will use the symbols v^ε, v^0 and s in place of u^ε, u^0 and ω. In the preceding sections we have recalled (see Remark 5) that $v^\varepsilon(0,s)$ is a meromorphic function of $s \in \mathbb{C}$ with values in $H^1_{loc}(\mathbb{R}^3)$. The poles of $v^\varepsilon(0,s)$ are the scattering frequencies of problem $P_\varepsilon(0,s)$ and the corresponding solutions $v^\varepsilon(0,s) \neq 0$ are the scattering solutions. In the analogous way we can define the scattering frequencies and the scattering solutions of problem $P_0(0,s)$. A problem related to the perturbation of scattering frequencies and solutions, in the framework of homogenization theory, is studied in [3] and [4].

Definition 1 A complex number s_0 is an accumulation point of scattering frequencies if, for every neighbourhood $U(s_0,\delta)$ of s_0 of radius δ, there exists ε and a complex number s_ε such that s_ε is a scattering frequency of the problem $P_\varepsilon(0,s_\varepsilon)$.

Remark 6 Let s_0 be an accumulation point of scattering frequencies, i.e., there exists s_ε, scattering frequency of problem P_ε, such that $s_\varepsilon \to s_0$. Let v^ε be the corresponding scattering solution that we may assume normalized, i.e.,

$$\|v_\varepsilon\|_{L_2(B_{R+5})} = 1. \quad ■ \tag{14}$$

Then, the following Lemma holds.

Lemma 2 *The normalized scattering solution introduced in Remark 6 are bounded in $H^1(B_{R+5})$, i.e., there exists a constant k such that*

$$\|v^\varepsilon\|_{H^1(B_{R+5})} \leq k \tag{15}$$

for all $\varepsilon > 0$.

PROOF. We obtain the statement integrating by parts, on B_{R+5}, the equation of problem $P_\varepsilon(0,s)$, multiplied by u^ε, and using trace theorems and interior estimates for elliptic equations. ■

Lemma 3 *Under the assumptions of Lemma 2,*

$$v^\varepsilon \longrightarrow v_0 \qquad\qquad \textit{weakly in} \quad H^1(B_{R+3}) \tag{16}$$

as $\varepsilon \to 0$, where u_0 is a solution of the problem $P_0(0,s)$.

PROOF. By formula (15) we can extract a subsequence such that (16) is verified. As in Proposition 2 we obtain that u_0 satisfies the problem $P_0(0,s)$. ■

Proposition 3 *Let s_0 be an accumulation point of scattering frequencies of the problems $P_\varepsilon(0,s)$, then s_0 is a scattering frequency of the limit problem $P_0(0,s)$.*

PROOF. The lemmas 2 and 3 show that v_0 solves problem $P_0(0,s)$, then, to prove that s_0 is a scattering frequency, we have only to verify $v_0 \neq 0$. Taking the limit, as $\varepsilon \searrow 0$, in the equation (13) for the problem $P_\varepsilon(0,s)$, we have that the convergence is uniform in every annulus $\{R < |x| < R+S\}$ for all $S > 0$, then $v_\varepsilon \longrightarrow v_0$ in $L^2(B_R \setminus \overline{B_{R+S}})$ strongly. Then, by (16), we deduce that $v_\varepsilon \longrightarrow v_0$ in $L^2(B_{R+5})$ strongly, as $\varepsilon \searrow 0$. But, by the hypothesis (14), this means:

$$v_0 \neq 0$$

and the proof is achieved. ■

Proposition 4 *Let s_0 be a scattering frequency of the limit problem $P_0(0,s)$, then, for every $\varepsilon > 0$, there exists at least a frequency s_ε of the problem $P_\varepsilon(0,s)$ such that $s_\varepsilon \longrightarrow s_0$ as $\varepsilon \to 0$.*

PROOF. We take $s \in \mathbb{C}$ different from a scattering frequency of the problem $P_0(f,s)$, with f fixed element of $L^2(|x| < R)$, then a unique solution $u_0(s)$ exists for problem $P_0(f,s)$. But, if s_0 is a scattering frequency for the problem P_0, $u_0(s)$ has an isolated singularity (pole) for $s = s_0$. Let Γ be a circle centered at s_0 and D its interior; we assume Γ be sufficiently small such that no further scattering frequencies, except s_0, belongs to the closure $\overline{D}$ of D. If the statement is not true, for all ε, the corresponding problem $P_\varepsilon(f,s)$ has not scattering frequencies in $\overline{D} = D \cup \Gamma$. Then we can consider the unique solution $u_\varepsilon(s)$ of the problem $P_\varepsilon(f,s)$, with $s \in \Gamma$. Reasoning as in Lemma 2, we can prove that $u_\varepsilon(s)$, with $s \in \Gamma$, are bounded in $L^2(B_{R+5})$, independently with respect to ε. But the solution $u_0(s)$ of the problem $P_0(f,s)$ has an isolated singularity (pole) for $s = s_0$. We can take the the Laurent's series and we obtain that there is an entire $m > 0$ such that $(s-s_0)^m u_0(s)$ has a residue $R_0 = 0$ in s_0. Moreover we can calculate:

$$\int_\Gamma (s-s_0)^m u_\varepsilon(s)\,\mathrm{d}s = R_\varepsilon \tag{17}$$

and by the hypothesis that $s \in \Gamma$ and that $u_\varepsilon(s)$ has not singularity in $\overline{D}$, $\forall \varepsilon$, we have:

$$R_\varepsilon = 0 \qquad \forall \varepsilon. \tag{18}$$

Since u_ε are bounded in $L^2(B_{R+5})$, then for any fixed $s \in \Gamma$, we take the limit, as $\varepsilon \searrow 0$, and following Lemma 3 we obtain that $u_\varepsilon \to u_0$ strongly in $L^2(B_{R+5})$, where $u_0(s)$ is solution of the limit problem $P_0(f,s)$. Moreover, by Lebesgue dominated convergence theorem, we can take the limit in equation (17):

$$\int_\Gamma (s-s_0)^m u_\varepsilon(s)\,\mathrm{d}s = R_\varepsilon \longrightarrow \int_\Gamma (s-s_0)^m u_0(s)\,\mathrm{d}s = R_0 \tag{19}$$

then $R_\varepsilon \to R_0 \neq 0$ and we have a contradiction with relation (18) and the statement is achieved. ■

Remark 7 With the same methods, one obtains the convergence of the scattering solutions (see [4]). ■

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