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Weak entropic solution to a scalar hyperbolic-parabolic conservation law

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Abstract. In this paper we are interested in the Dirichlet problem of a hyperbolic-parabolic degenerate equation. Thanks to a global entropic formulation in the sense of F. Otto, we propose a result of existence and uniqueness of the entropic measure valued solution and of the entropic weak solution in the space DM_2.

Solución débil entrópica de una ley de conservación escalar hyperbólica-parabólica

Resumen. Abordamos en este trabajo el problema de Dirichlet en el caso de una ecuación degenerada hyperbólica-parabólica mediante una formulación debil. Utilizando una formulación entrópica en el sentido de F. Otto, proponemos un doble resultado de existencia y unicidad de la solución a valor medida y de la solución en el espacio DM_2.

1. Introduction

After having studied, in a previous paper ([14]), a nonlinear problem of first order hyperbolic type, the aim of this paper is to adapt the same tools to the following hyperbolic-parabolic nonlinear degenerate problem

$$\frac{\partial u}{\partial t} - \Delta\phi(u) + div[f(t,x,u)] + g(t,x,u) = 0 \quad \text{in } Q =]0,T[\times\Omega, \tag{1}$$

where ϕ is a, non null, non decreasing Lipschitzian function with $\phi(0) = 0$.

We consider the formal Dirichlet boundary condition

$$u = 0 \quad \text{on } \Sigma =]0,T[\times\Gamma \quad \text{where } \Gamma = \partial\Omega, \tag{2}$$

and the initial condition

$$u(0,.) = u_0 \quad \text{in } \Omega, \tag{3}$$

where u_0 is a bounded non negative measurable function, in order to obtain non negative solutions.

In this note, we present result of existence and of uniqueness for the *weak entropic solution* to the above Cauchy-Dirichlet problem. For that type of problems, it is well-known that the Dirichlet condition $u = 0$ on Σ is impossible, even if the condition $\phi(u) = 0$ on Σ is available since it is under-determined. Thus,

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we decide to consider that the admissible physical solution comes from the convergence of the sequence $(u_\varepsilon)_{\varepsilon>0}$ of *evanescent viscosity solutions*. Therefore, the asymptotic behaviour of $(u_\varepsilon)_\varepsilon$ on the parabolic boundary would led to the weak entropic boundary condition.

One may find many papers on this subject. Without any intention to be exhaustive, let us give some approaches. The main reference concerning uniqueness Kruzhkov's method for that kind of problem is Carrillo [3]. Moreover, a small modification of this technique allows us to generate the missing boundary term of Otto [11]'s entropic formulation, needed for the uniqueness method. Therefore, one is able to propose a global entropic formulation, without having to separate interior entropic formulation and boundary formulation, as stated in Mascia *et al.* [9].

Contrary to Rouvre [12] , where a Bardos *et al.* [1] type global entropic formulation is proposed in the context of $BV(Q)$ functions, we are interested in a [11]'s type one (see Vallet [14]), in the context of Young measure solutions and then of bounded measurable functions. Therefore, the problem of the boundary conditions has to be set, more precisely, the existence of a trace and of the normal flux on $\Sigma =]0,T[\times\partial\Omega$ (see A. [16] for comments about this). Rouvre [12] propose to use a result of Ph. Bénilan, written in Dautray et al [5], to show that the normal derivative belongs to $L^1(\Sigma)$. Opposed to these strong solutions, Michel [10] presents Young measure solutions, where the boundary flux is obtained via the limit in an integral on Q (following the ideas of [11] or Szepessy [13], see [14] too), using a particular sequence. To avoid this, Chen *et al.* [4] propose the functional context of DM_2, the space of $L^2(Q)^{N+1}$ vectors such that the divergence is a bounded Radon measure on Q. One may find in Burger *et al.* [2] some applications of this space to hyperbolic-parabolic problem in one dimensional space; and in Mascia *et al.* [9], the use of the same space for a non homogeneous Dirichlet hyperbolic-parabolic problem in $BV(Q)$.

The aim of our work is to prove the existence and the uniqueness of the solution u in $DM_2(Q)\cap L^\infty(Q)$ (and of the Young measures solution) to the problem (1)-(2)-(3), without any $BV(Q)$ assumption.

2. Notations and definition of a solution

Let us set in the sequel:

• Ω is a connected bounded open subset of $\mathbb{R}^N$ ($N \leq 3$) with a smooth boundary Γ, an outward unit normal η, $Q =]0,T[\times\Omega$, $\Sigma =]0,T[\times\Gamma$ and $\operatorname{div} = \sum_{i=1}^{N} \dfrac{\partial}{\partial x_i}$.

• $E = \{l \in \mathbb{R},\ \{l\} = \phi^{-1}\{\phi(l)\}\}$.

• f is continuous on $\mathbb{R}^{N+2}$ such that if one denotes $h = \dfrac{\partial f}{\partial t}$ or $h = \dfrac{\partial f}{\partial x_i}, \forall i \in \{1,..,N\}$, then:

i) $\forall M > 0,\ \exists h_M \in L^\infty(Q),\ \forall\lambda \in [-M,M],\ |h(.,.,\lambda)| \leq h_M$ a.e. in Q,

ii) $a.e.\ (x,t)$ in $Q, \forall\varepsilon > 0, \exists\eta > 0, |\alpha-\beta| \leq \eta \Rightarrow |h(t,x,\alpha) - h(t,x,\beta)| \leq \varepsilon$,

- $\operatorname{div} f$ is a Carathéodory function such that:

$\exists c_1, c_2 \in L^\infty(\mathbb{R}^{N+1}), \forall\lambda \in \mathbb{R}, (t,x)$ a.e. in $\mathbb{R}^{N+1},\ |\operatorname{div} f(t,x,\lambda)| \leq c_1(t,x)|\lambda| + c_2(t,x)$.

• g is a Carathéodory function such that: $g(t,x,0) \in L^\infty(\mathbb{R}^{N+1})$ and

$\exists c \in L^\infty(\mathbb{R}^{N+1}), \forall(\alpha,\beta) \in \mathbb{R}^2,\ |g(t,x,\alpha) - g(t,x,\beta)| \leq c(t,x)|\alpha-\beta|$, a.e. (t,x) in Q.

• $sgn(x) = -1$ if $x < 0$, 1 if $x > 0$ and 0 else. And sgn_η is a non decreasing Lipschitzian approximation of sgn, such that $sgn_\eta(0) = 0$.

• $F(t,x,u,v) = sgn(u-v)[f(t,x,u) - f(t,x,v)]$, $G(t,x,u,k) = \operatorname{div} f(t,x,k) + g(t,x,u)$ and

$\mathfrak{L}(t,x,u,k,v) = |u-k|\dfrac{\partial v}{\partial t} - \nabla|\phi(u) - \phi(k)|\nabla v + F(t,x,u,k).\nabla v - sgn(u-k)G(t,x,u,k)v$.

• One sets the spaces $V = H^1(Q) \cap L^\infty(Q)$, V^+ the non negative functions of V, $V_0^+ = \{v \in V^+,\ v(t = 0) = v(t = T) = 0\ a.e.\ in\ \Omega\}$ and, following Chen *et al.* [4], $DM_2(Q) = \{U = (u_0, u_1, ..u_N) \in [L^2(Q)]^{N+1},\ \operatorname{Div}_{(t,x)} U \in Mes_b(Q)\}$.

In order to take into account the boundary flux, these authors propose a trace theorem and the following

Gauss-Green formulae: for any U of $DM_2(Q)$ and any v of V,

$$\int_Q U\nabla_{(t,x)} v\, dxdt + \int_Q v\, d(\underset{(t,x)}{\mathrm{Div}}\, U) =< U.\eta, v > .$$

Let us give now the definition of a weak entropic solution with F. Otto's boundary conditions:

Definition 1 *A weak entropic solution to (1)-(2) and (3) is an element u of $L^\infty(Q)$ such that:*
i) $\dfrac{\partial u}{\partial t} \in L^2(0,T;H^{-1}(\Omega))$, $\phi(u) \in L^2(0,T;H_0^1(\Omega))$,
$U_k = (|u-k|, -\nabla|\phi(u)-\phi(k)| + F(t,x,u,k)) \in DM_2(Q)$,
ii) u is a weak solution to (1), that is: $\forall v \in L^2(0,T;H_0^1(\Omega))$,

$$\int_0^T < \frac{\partial u}{\partial t}, v >_{H^{-1},H_0^1} dt + \int_Q \{\nabla\phi(u)\nabla v - f(t,x,u)\nabla v + g(t,x,u)v\}\, dxdt = 0 \tag{4}$$

iii) For any real k and any v in $H_0^1(Q)$ with $v \geq 0$, $\int_Q \mathcal{L}(t,x,u,k,v)\, dxdt \geq 0$.
iv) *a)* $\lim\limits_{t\to 0^+} ess \int_\Omega |u(t,x) - u_0(x)|\, dx = 0$,
and *b)* $\forall v \in V_0^+,\ \forall k \in \mathbb{R},\quad - < U_k.\eta, v > \leq < U_0.\eta, v > - \int_\Sigma F(t,x,k,0).\eta v\, d\mathcal{H}^N$.

The definition of an entropic measure valued solution is the following

Definition 2 *An entropic measure valued solution to (1)-(2) and (3) is a bounded Young measure $\mathbf{u}$ (of variables $(t,x) \in Q$ and $\alpha \in]0,1[$, if one uses Th. Gallouët's notations in Eymard et al. [6]) such that if one notes $u(t,x) = \int_0^1 \mathbf{u}(t,x,\alpha)\ d\alpha$:*
i) $u \in W^{1,2}(0,T;L^2(\Omega),H^{-1}(\Omega))$, $\phi(u) \in L^2(0,T;H_0^1(\Omega))$,
$\mathbf{U}_k = (\int_{]0,1[} |\mathbf{u}-k|\, d\alpha,\ -\nabla|\phi(u)-\phi(k)| + \int_{]0,1[} F(t,x,\mathbf{u},k)\, d\alpha) \in DM_2(Q)$,
ii) $\forall v \in L^2(0,T;H_0^1(\Omega))$,

$$\int_0^T < \frac{\partial u}{\partial t}, v >_{H^{-1},H_0^1} dt + \int_Q \nabla\phi(u)\nabla v - \int_0^1 f(t,x,\mathbf{u})\nabla v + g(t,x,\mathbf{u})v\, d\alpha\, dxdt = 0$$

iii) For any real k and for any non negative v in $H_0^1(Q)$, $\int_{Q\times]0,1[} \mathcal{L}(t,x,\mathbf{u},k,v)\, d\alpha dxdt \geq 0$.
iv) *a)* $\lim\limits_{t\to 0^+} \mathrm{ess} \int_{\Omega\times]0,1[} |\mathbf{u}(t,x,\alpha) - u_0(x)|\, dxd\alpha = 0$,
and *b)* $\forall v \in V_0^+,\ \forall k \in \mathbb{R},\quad - < \mathbf{U}_k.\eta, v > \leq < \mathbf{U}_0.\eta, v > - \int_\Sigma F(t,x,k,0).\eta v\, d\mathcal{H}^N$.

Remark 1 Thanks to the regularity of $\phi(u)$, for a.e. (t,x) in Q, for a.e. α in $]0,1[$, $\phi(u) = \phi(\mathbf{u})$. ■

This definition leads to the following "global" entropic formulation in $]0,T[\times\overline{\Omega}$:

Proposition 1 *(F. Otto entropic formulation) Let u be a weak entropic solution and $\mathbf{u}$ an entropic measure valued solution to (1)-(2) and (3). Then,*

$$\forall v \in V_0^+,\ \forall k \in \mathbb{R}, \qquad -\int_Q \mathcal{L}(t,x,u,k,v)\, dxdt \leq < U_0.\eta, v > - \int_\Sigma F(t,x,k,0).\eta\, v\ d\mathcal{H}^N,$$

$$-\int_{Q\times]0,1[} \mathcal{L}(t,x,\mathbf{u},k,v)\, d\alpha dxdt \leq < \mathbf{U}_0.\eta, v > - \int_\Sigma F(t,x,k,0).\eta\, v\ d\mathcal{H}^N.$$

PROOF. One has to use $v\omega_n$ as a test-function in definition 1 & 2 iii) and pass to the limit when n goes to infinity, where $(\omega_n)_n$ is a sequence in $\mathcal{D}(Q)$, $0 \leq \omega_n \leq 1$, that converges everywhere in Q towards 1. ■

3. Existence of a measure valued solution

The existence of a measure valued solution is obtained by passing to the limit in a sequence of solution to a viscous regular problem, as presented in the introduction. Let us denote by $\phi_\varepsilon(x) = \phi(x) + \varepsilon x$ and f_ε and g_ε regular approximations of f and g (cf. [14]), then:

Proposition 2 *For any positive ε, a unique u_ε exists in $H^1(Q) \cap L^2(0,T;H^1_0(\Omega))$ such that for any v in $H^1_0(\Omega)$ and a.e. t in $]0,T[$,*

$$\int_\Omega \left\{ \frac{\partial u_\varepsilon}{\partial t} v + \nabla\phi_\varepsilon(u_\varepsilon)\nabla v - f_\varepsilon(t,x,u_\varepsilon)\nabla v + g_\varepsilon(t,x,u_\varepsilon)v \right\} dx = 0 \tag{5}$$

with $u_\varepsilon(0,.) = u_0^\varepsilon$ a.e. in Ω where u_0^ε converges towards u_0 in $L^1(\Omega)$, $u_0^\varepsilon \in H^1_0(\Omega) \cap L^\infty(\Omega)$ and $-||u_0||_\infty \leq u_0^\varepsilon \leq ||u_0||_\infty$.
Moreover, the boundary condition of Definition 1 iv-b) is satisfied.

PROOF. Results of existence and uniqueness of u_ε are classical, and the boundary condition comes from the properties of $H^1_0(\Omega))$ functions w such that Δw is a bounded Radon measure (cf. Dautray *et al.* [5], prop. 9 pp. 580–581). ■

Let us indicate some *a priori* estimates of the generalised sequence $(u_\varepsilon)_{\varepsilon>0}$:

Proposition 3 *independently from ε, $(u_\varepsilon)_{\varepsilon>0}$ is bounded in $L^\infty(Q)$, $(\phi_\varepsilon(u_\varepsilon))_{\varepsilon>0}$ and $(\sqrt{\varepsilon}u_\varepsilon)_{\varepsilon>0}$ are bounded in $L^2(0,T;H^1_0(\Omega))$, $(\dfrac{\partial u_\varepsilon}{\partial t})_{\varepsilon>0}$ is bounded in $L^2(0,T,H^{-1}(\Omega))$ and $(t,x) \mapsto \sqrt{t}\phi_\varepsilon(u_\varepsilon)(t,x)$ is bounded in $H^1(Q)$.*
For any real k, $\{U_{\varepsilon,k}\}_\varepsilon = \{(|u_\varepsilon - k|, -\nabla|\phi_\varepsilon(u_\varepsilon) - \phi_\varepsilon(k)| + F_\varepsilon(t,x,u_\varepsilon,k))\}_\varepsilon$ is bounded in $DM_2(Q)$.

PROOF. Most of these estimates are classical (cf. Gagneux *et al* [7]). Then, for the last one, following Burger *et al.* [2]'s idea, for any v in $\mathcal{D}^+(Q)$, one has to prove that, uniformely with respect to ε, $v \mapsto \mu_\varepsilon(v) = \lim_{\eta\to 0^+} \int_Q v\, sgn'_\eta(\phi_\varepsilon(u_\varepsilon) - \phi_\varepsilon(k))|\nabla\phi_\varepsilon(u_\varepsilon)|^2\, dxdt$ is a bounded non negative measure on Q. ■

Therefore, a sequence can be extracted from $(u_\varepsilon)_{\varepsilon>0}$, still denoted by $(u_\varepsilon)_{\varepsilon>0}$, such that:

Proposition 4 *There exists:*
$i)$ u in $L^\infty(Q) \cap W^{1,2}(0,T,L^2(\Omega),H^{-1}(\Omega)) \cap C_s([0,T],L^2(\Omega))$ such that u_ε converges towards u in $L^\infty(Q)$ weak$-*$; and, if we denote by $\mathbf{u}$ (of variables $(t,x) \in Q$ and $\alpha \in]0,1[$) the Young measure generated by $(u_\varepsilon)_{\varepsilon>0}$: i.e. for any bounded Carathéodory function $(t,x,\lambda) \mapsto h(t,x,\lambda)$, one has: $\lim_{\varepsilon\to 0} \int_Q h(t,x,u_\varepsilon(t,x))\, dxdt = \int_{Q\times]0,1[} h(t,x,\mathbf{u}(t,x,\alpha))\, d\alpha dxdt$.
$ii)$ $\Phi(t,x) = \int_{]0,1[} \phi(\mathbf{u}(t,x,\alpha))\, d\alpha$ the limit in $L^\infty(Q)$ weak$-*$ and weakly in $L^2(0,T,H^1_0(\Omega))$ of $\phi_\varepsilon(u_\varepsilon(t,x))$. Moreover, since $\{(t,x) \mapsto \sqrt{t}\phi_\varepsilon(u_\varepsilon)\}_{\varepsilon>0}$ is a bounded sequence in $H^1(Q)$ and as ϕ is non decreasing, one gets: $\Phi(t,x) = \phi\left(\int_{]0,1[} \mathbf{u}(t,x,\alpha)\, d\alpha\right) = \phi(u(t,x))$.
$iii)$ for any real k, $\mathbf{U}_k$ belongs to $DM_2(Q)$ and is the weak limit in $L^2(Q)^{N+1}$ of $U_{\varepsilon,k}$. □

Passing to the limit when ε goes to 0, the above limit do not allows us to obtain a function-solution but only a Young measure solution.

Proposition 5 *This bounded Young measure $\mathbf{u}$ is an entropic measure valued solution to (1)–(2) and (3).*

Let us prove that such a solution exists.
PROOF OF PROPOSITION 4. Claims $i)$ and $ii)$ of definition 2 lead directly from the limit when ε tends towards 0. Claim $iii)$ and $iv)$ are obtained using [14]'s technics. ■

4. Uniqueness of the measure valued solution, existence and uniqueness of the weak solution

The aim of this section is to prove that the measure valued solution is unique. Moreover, thanks to the entropic formulation, the demonstration of uniqueness of the solution leads to the existence, and the uniqueness, of the weak solution (see [14] and references).

In order to, for two measure valued solutions $\mathbf{u}$ and $\hat{\mathbf{u}}$, let us use the S.N. Kruzhkov uniqueness method [8], classical for first order hyperbolic problems.

Firstly, one has to adapt the proposition (see [3]) to measure valued solutions, taking into account the boundary conditions *i.e.*, for any k in E and any v in V_0^+,

$$\begin{aligned} & -\int_{Q\times]0,1[} \mathfrak{L}(t,x,\mathbf{u},k,v)\,dxdtd\alpha + \lim_{\eta\to 0^+}\int_Q sgn'_\eta(\phi(u)-\phi(k))|\nabla\phi(u)|^2\,v\,dxdt \\ \leq < \quad & U_0.\eta, v > - \int_\Sigma F(t,x,k,0).\eta\, v\, d\mathcal{H}^N. \end{aligned}$$

Then, following the method proposed by Carrillo in [3] (see [12] too), the proof given in [14] for the left hand side and Mascia *et al.* [9] for the right hand side, for any non negative function γ of $\mathcal{D}(0,T)$, one has:

$$\begin{aligned} & -\iint_{Q\times]0,1[^2} |\mathbf{u}(t,x,\alpha)-\hat{\mathbf{u}}(t,x,\beta)|\gamma'(t)\,d\alpha d\beta dxdt \\ & + \iint_{Q\times]0,1[^2} sgn(\mathbf{u}(t,x,\alpha)-\hat{\mathbf{u}}(t,x,\beta))[g(t,x,\mathbf{u}(t,x,\alpha))-g(t,x,\hat{\mathbf{u}}(t,x,\beta))]\gamma(t)\,d\alpha dy dxdt \\ \leq \quad & \frac{1}{2} < U_0.\eta,\gamma > + \frac{1}{2} < \widehat{U}_0.\eta,\gamma > - \frac{1}{2} < U_0.\eta,\gamma > - \frac{1}{2} < \widehat{U}_0.\eta,\gamma > . \end{aligned}$$

The conclusion is then the same as for first order hyperbolic problems (cf. [14]). In particular, thank the the $L^1(\Omega)-$ continuity in 0^+ (cf. initial condition), one has that

$$\iint_{Q\times]0,1[^2} |\mathbf{u}(t,x,\alpha)-\hat{\mathbf{u}}(t,x,\beta)|\,d\alpha d\beta dxdt = 0.$$

So, $\mathbf{u}(t,x,\alpha) = u(t,x) = \hat{\mathbf{u}}(t,x,\beta) = \hat{u}(t,x)$, that means:

Theorem 1
i) There exists a unique entropic measure valued solution.
ii) Any entropic measure valued solution is an entropic weak solution (i.e. a measurable function),
iii) There exists a unique entropic weak solution in the sense of definition 1. □

One may find in [15] details about the proofs.

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