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Existence of Density for The Solution to the Three-Dimensional Stochastic Wave Equation

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Abstract. We prove existence of density for the real-valued solution to a 3-dimensional stochastic wave equation. The noise is white in time and with a spatially homogeneous correlation whose spectral measure μ satisfies that $\int_{\mathbb{R}^3} \mu(d\xi)(1+|\xi|^2)^{-\eta} < \infty$, for some $\eta \in (0, \frac{1}{2})$. Our appoach is based on the mild formulation of the equation given by means of Dalang's extended version of Walsh's stochastic integration. We use the tools of Malliavin calculus and a comparison procedure with respect to smooth approximations of the distribution-valued fundamental solution, obtained by convolution with an approximation to the identity.

Existencia de densidad para la ecuación de ondas estocástica en dimensión tres

Resumen. Demostramos la existencia de densidad para la solución de la ecuación de ondas estocástica en dimensión tres. Se considera un ruido blanco en tiempo y con correlacin espacialmente homogénea, cuya medida espectral μ satisface la condición $\int_{\mathbb{R}^3} \mu(d\xi)(1+|\xi|^2)^{-\eta} < \infty$, para algún $\eta \in (0, \frac{1}{2})$. Abordamos el problema a partir de la formulación *mild* de la ecuación, basada en la extensión de la integral estocástica introducida por Dalang; utilizamos el cálculo de Malliavin y un procedimiento de comparación respecto de aproximaciones regulares de la solución fundamental de la ecuación obtenida mediante convolución con una aproximación de la identidad.

1. Introduction

We present new results regarding the existence of density of the real-valued solution to the stochastic wave equation

$$
\begin{aligned}
&(\frac{\partial^2}{\partial t^2} - \Delta_3)u(t,x) = \sigma(u(t,x))\dot{F}(t,x) + b(u(t,x)), \\
&u(0,x) = \frac{\partial u}{\partial t}(0,x) = 0,
\end{aligned} \tag{1}
$$

where $(t,x) \in (0,T] \times \mathbb{R}^3$ and Δ_3 denotes the Laplacian operator on $\mathbb{R}^3$. We assume that the coefficients σ and b are real Lipschitz functions; the noise F is a mean-zero $L^2(\Omega, \mathcal{F}, P)-$valued Gaussian process indexed by the space $\mathcal{D}(\mathbb{R}^4)$ of test functions with covariance functional given by $J(\varphi, \psi) =$

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$\int_{\mathbb{R}_+} ds \int_{\mathbb{R}^3} \Gamma(dx)(\varphi(s) * \tilde{\psi}(s))(x)$, where $\tilde{\psi}(s,x) = \psi(s,-x)$ and Γ is a non-negative, non-negative definite tempered measure. Let μ be the spectral measure of F, that means the non-negative tempered measure $\mathcal{F}^{-1}\Gamma$, where $\mathcal{F}$ denotes the Fourier transform operator.

We follow the extension of Walsh's approach developed in [3] and give a rigurous meaning to equation (1) in the *mild form*, as follows. Let S_3 be the fundamental solution of the wave equation in dimension $d = 3$; it is well-known that for any $t > 0$, $S_3(t) = \frac{1}{4\pi}\sigma_t$, where σ_t is the uniform measure on the 3-dimensional sphere of radius t. Let $M = \{M_t(A), t \in [0,T], A \in \mathcal{B}_b(\mathbb{R}^d)\}$ be the martingale measure extension of F and let $\mathcal{F}_t$ be the σ-field generated by the random variables $M_s(A)$, $s \in [0,t]$, $A \in \mathcal{B}_b(\mathbb{R}^d)$, for any $t \in [0,T]$. Then, a solution to (1) is a *real-valued* progressively measurable stochastic processes $u = \{u(t,x), (t,x) \in [0,T] \times \mathbb{R}^d\}$ satisfying

$$u(t,x) = \int_0^t \int_{\mathbb{R}^3} S_3(t-s, x-y)\sigma(u(s,y))M(ds,dy) + \int_0^t \int_{\mathbb{R}^3} b(u(t-s, x-y))S_3(s,dy). \qquad (2)$$

From Theorem 13 in [3] it follows that such a solution exists whenever the measure μ satisfies the integrability condition

$$\int_{\mathbb{R}^3} \frac{\mu(d\xi)}{1+|\xi|^2} < \infty. \qquad (3)$$

We refer the reader to [10] for results related to [3] on the stochastic wave equation.

The approach to our problem is based on Malliavin calculus, which provides a useful tool for the analysis of densities of functionals of Gaussian families indexed by a real separable Hilbert space. Application of this technique to spde's extend to the heat equation and different examples of hyperbolic spde's, including the stochastic wave equation in dimension $d \in \{1,2\}$ (see for instance [2], [5], [6], [8], [9], [11]). In all these works the fundamental solution of the underlying partial differential equation is a real-valued function, while in our case it is a distribution. The main issues derived from this new situation concern the equation satisfied by the Malliavin derivative and the control of the norm of the principal term of this equation.

2. Main result

Let $\mathcal{E}$ be the inner-product space consisting of functions φ in $\mathcal{S}(\mathbb{R}^3)$ -the space of rapidly decreasing $\mathcal{C}^\infty$ test functions- endowed with the inner-product $\langle \varphi, \psi \rangle_{\mathcal{E}} := \int_{\mathbb{R}^d} \Gamma(dx)(\varphi * \tilde{\psi})(x)$, where $\tilde{\psi}(x) = \psi(-x)$. Notice that $\langle \varphi, \psi \rangle_{\mathcal{E}} = \int_{\mathbb{R}^3} \mu(d\xi)\mathcal{F}\varphi(\xi)\overline{\mathcal{F}\psi(\xi)}$. Let $\mathcal{H}$ denote the completion of $(\mathcal{E}, \langle\cdot,\cdot\rangle_{\mathcal{E}})$. Set $\mathcal{H}_T = L^2([0,T];\mathcal{H})$; the spaces $\mathcal{H}$ and $\mathcal{H}_T$ may contain not only functions but also distributions. The space $\mathcal{H}_T$ is a real Hilbert separable space. For $h \in \mathcal{H}_T$, set $W(h) = \int_0^t \int_{\mathbb{R}^3} h(s,x)M(ds,dx)$ where the stochastic integral can be interpreted in Dalang's sense (see [3]). Then $\{W(h), h \in \mathcal{H}_T\}$ is a Gaussian process and we can use the differential Malliavin calculus based on it (see for instance [7]). Our main result is as follows.

Theorem 1 *Assume that:*

(i) the coefficients σ and b are $\mathcal{C}^1$ functions with bounded Lipschitz continuous derivatives;

(ii) there exists $\sigma_0 > 0$ such that $\inf\{|\sigma(z)|; z \in \mathbb{R}\} \geq \sigma_0$;

(iii) there exists $\eta \in (0, \frac{1}{2})$ such that

$$\sup_{y \in \mathbb{R}^3} \int_{\mathbb{R}^3} \Gamma(dx)\mathcal{F}^{-1}\Big(\frac{1}{(1+|\xi|^2)^\eta}\Big)(x-y) < \infty.$$

Then, for any fixed $(t,x) \in (0,T] \times \mathbb{R}^3$, the random variable $u(t,x)$ has a density.

Owning to Bouleau's and Hirsch's criterium (see [1]) the preceding Theorem is a consequence of the next Propositions.

Proposition 1 *Assume that the coefficients σ and b satisfy the assumption (i) of Theorem 1. Then,*

(a) for any $(t,x) \in [0,T] \times \mathbb{R}^3$, $u(t,x)$ belongs to $\mathbb{D}^{1,2}$;

(b) there exists an $\mathcal{H}_T$-valued stochastic process $\{Z(t,x), (t,x) \in [0,T] \times \mathbb{R}^3\}$ satisfying

$$\sup_{(t,x)\in[0,T]\times\mathbb{R}^3} ||Z(t,x)||_{L^2(\Omega,\mathcal{H}_T)} < \infty$$

such that

$$\begin{aligned} Du(t,x) &= Z(t,x) + \int_0^t \int_{\mathbb{R}^3} S_3(t-s,x-z)\sigma'(u(s,z))Du(s,z)M(ds,dz) \\ &+ \int_0^t \int_{\mathbb{R}^3} b'(u(t-s,x-z))Du(t-s,x-z)S_3(s,dz). \end{aligned} \tag{4}$$

Moreover, for any $(t,x) \in [0,T] \times \mathbb{R}^3$,

$$E\big(\|Z(t,x)\|^2_{\mathcal{H}_T}\big) = E\Big(\int_0^t \int_{\mathbb{R}^3} S_3(t-s,x-z)\sigma(u(s,z))M(ds,dz)\Big)^2. \tag{5}$$

Proposition 2 *Assume that the coefficients σ and b are $\mathcal{C}^1$ functions with bounded derivatives and moreover, the assumptions (ii) and (iii) of Theorem 1 are satisfied. Then $\|Du(t,x)\|_{\mathcal{H}_T} > 0$, a.s.*

Let us make some comments on these statements. (1) The hypothesis (iii) in Theorem 1 implies the following strengthening of (3):
(H_η) There exists $\eta \in (0,1)$ such that $\int_{\mathbb{R}^3} \frac{\mu(d\xi)}{(1+|\xi|^2)^\eta} < \infty$.
In fact both conditions are nearly equivalent (see [4] for further details).
(2) Giving a precise meaning to Equation (4) requires to set up an extension of Dalang's stochastic integral with respect to Hilbert-valued integrators. With this ingredient, existence and uniqueness of solution to Equation (4) can be proved by a fixed point argument.
(3) The $\mathcal{H}_T$-valued stochastic process

$$Z(t,x) + \int_0^t \int_{\mathbb{R}^3} S_3(t-s,x-z)\sigma'(u(s,z))Du(s,z)M(ds,dz),$$

$(t,x) \in [0,T] \times \mathbb{R}^3$, is the Malliavin derivative of the stochastic integral

$$\int_0^t \int_{\mathbb{R}^3} S_3(t-s,x-z)\sigma(u(s,z))M(ds,dz).$$

Naively $Z(t,x) = S_3(t-\cdot,x-*)\sigma(u(\cdot,*))$.
(4) Proposition 1 can be extended to more general spde's defined by a differential operator L such that the fundamental solution of $Lu = 0$ is a function in time with values on the space of non-negative distributions with rapid decrease. Moreover, it holds that $u(t,x)$ belongs to $\mathbb{D}^{1,p}$ for any $p \in [2,\infty)$.

3. Sketch of the proofs

PROOF OF PROPOSITION 1: Let ψ be a non-negative function in $\mathcal{C}^\infty(\mathbb{R}^3)$ with support contained in the unit ball of $\mathbb{R}^3$ and such that $\int_{\mathbb{R}^d} \psi(x)dx = 1$. Set $\psi_n = n^d\psi(nx)$, $n \geq 1$. Define $S_3^n(t) = \psi_n * S_3(t)$. Consider the real-valued process $\{u_n(t,x), (t,x) \in [0,T] \times \mathbb{R}^d\}$ solution to the integral stochastic equation

$$u_n(t,x) = \int_0^t \int_{\mathbb{R}^d} S_3^n(t-s,x-z)\sigma(u_n(s,z))M(ds,dz) + \int_0^t ds \int_{\mathbb{R}^d} b(u_n(t-s,x-z))S_3(s,dz). \tag{6}$$

The existence and uniqueness of such process can be easily deduced from the arguments used in the proof of Theorem 13 of [3]. By an easy adaptation of the proof of Proposition 2.4 of [5], taking into account that $|\mathcal{F}S_3^n(t)(\xi)| \le |\mathcal{F}S_3(t)(\xi)|$, for each $t \ge 0, \xi \in \mathbb{R}^d$, we show that $u_n(t,x) \in \mathbb{D}^{1,2}$ and that the derivative of u_n satisfies the equation in $\mathcal{H}_T$

$$\begin{aligned} Du_n(t,x) &= S_3^n(t-\cdot, x-*)\sigma(u_n(\cdot,*)) \\ &\quad + \int_0^t \int_{\mathbb{R}^d} S_3^n(t-s,x-z)\sigma'(u_n(s,z))Du_n(s,z)M(ds,dz) \\ &\quad + \int_0^t ds \int_{\mathbb{R}^d} S_3(s,dz)b'(u_n(t-s,x-z))Du_n(t-s,x-z). \end{aligned} \tag{7}$$

Using the properties of the stochastic integral we prove that

$$\lim_{n\to\infty} \big(\sup_{(t,x)\in[0,T]\times\mathbb{R}^3} E|u_n(t,x)-u(t,x)|^2 \big) = 0, \tag{8}$$

$$\sup_{n\ge 1} \sup_{(t,x)\in[0,T]\times\mathbb{R}^3} E\big(\|Du_n(t,x)\|^2_{\mathcal{H}_T}\big) < \infty. \tag{9}$$

This yields the statement (a) and in addition that the sequence $\{Du_n(t,x), n \ge 1, (t,x) \in [0,T]\times\mathbb{R}^3\}$ converges in the weak topology of $L^2(\Omega, \mathcal{H}_T)$.

We next prove that $\{Z_n(t,x) = S_3^n(t-\cdot, x-*)\sigma(u_n(\cdot,*)), n \ge 1\}$ is a Cauchy sequence in $L^2(\Omega;\mathcal{H}_T)$. Let $Z(t,x)$ be its limit. Then, by a Gronwall's type argument we show the covergence in $L^2(\Omega;\mathcal{H}_T)$ of $\{Du_n(t,x), n \ge 1\}$ as $n \to \infty$ to the process $Du(t,x), (t,x) \in [0,T]\times\mathbb{R}^3$, solution to Equation (4). The identity (5) follows from the construction of $Z(t,x)$ and the isometry property of Dalang's stochastic integral. ∎

Remark 1 With a little more effort one can prove that (9) holds with the L^2-norm replaced by the L^p-norm, and that the sequence of processes $u_n, n \ge 1$, is Cauchy in $L^p(\Omega)$ for any $p \in [2,\infty)$. Hence (8) can be extended to an L^p-convergence and consequently $u \in \mathbb{D}^{1,p}$, for any $p \in [2,\infty)$. ∎

PROOF OF PROPOSITION 2: We will check that $E(\|Du(t,x)\|^{-p}_{\mathcal{H}_T}) < \infty$ for some $p > 0$. Equivalently, for $\eta_0 > 0$ small enough and for some $p > 0$,

$$\int_0^{\eta_0} \epsilon^{-\frac{p}{2}-1} P\{\|Du(t,x)\|^2_{\mathcal{H}_T} < \epsilon\}\, d\epsilon < \infty.$$

Owning to the expression of $Du(t,x)$ given in (4) we consider, as in [6], the decomposition $\|D_{r,*}u(t,x)\|^2_{\mathcal{H}} = \|Z_{r,*}(t,x)\|^2_{\mathcal{H}} + U(t,r,x)$.

Let $\epsilon_1, \delta > 0$ be such that for any $\epsilon \in (0,\epsilon_1]$, $t - \epsilon^\delta > 0$. Then we obviously have $P\{\|Du(t,x)\|^2_{\mathcal{H}_T} < \epsilon\} \le P^1(\epsilon,\delta) + P^2(\epsilon,\delta)$, with

$$\begin{aligned} P^1(\epsilon,\delta) &= P\big\{\big|\int_{t-\epsilon^\delta}^t dr\, U(t,r,x)\big| \ge \epsilon\big\}, \\ P^2(\epsilon,\delta) &= P\big\{\int_{t-\epsilon^\delta}^t dr \|Z_{r,*}(t,x)\|^2_{\mathcal{H}} < 2\epsilon\big\}. \end{aligned}$$

Let $I_1(\epsilon,\delta) = \int_0^{\epsilon^\delta} ds \int_{\mathbb{R}^3} \mu(d\xi)|\mathcal{F}S_3(s)(\xi)|^2$, $I_2(\epsilon,\delta) = \int_0^{\epsilon^\delta} ds \int_{\mathbb{R}^3} S_3(s,dx)$. Chebyhev's inequality applied to the function $g(x) = |x|$, Schwarz's inequality and L^2 estimates of stochastic and pathwise integrals involved in the term $U(t,r,x)$ yield that

$$P^1(\epsilon,\delta) \le C\epsilon^{-1}\big(I_1(\epsilon,\delta)^{3/2} + I_1(\epsilon,\delta)I_2(\epsilon,\delta)\big) \le C\epsilon^{-1}\big(\epsilon^{\frac{3}{2}\delta(3-2\eta)} + \epsilon^{\delta(5-2\eta)}\big),$$

where the very last inequality follows from the estimates (12), (15). Thus, $\int_0^{\eta_0} \epsilon^{-\frac{p}{2}-1} P^1(\epsilon,\delta) d\epsilon < \infty$ if and only if $\frac{2}{p+2}\big(\frac{3}{2}(3-2\eta) \wedge (5-2\eta)\big) > \frac{1}{\delta}$.

The triangle inequality implies $P^2(\epsilon,\delta) \le P^{21}(\epsilon,\delta) + P^{22}(\epsilon,\delta)$, with

$$P^{21}(\epsilon,\delta) = P\{\|\Lambda_{\epsilon^{-1}}(\cdot, x-*)\sigma(u(t-\cdot,*))\|^2_{\mathcal{H}_{\epsilon^\delta}} < 6\epsilon\},$$
$$P^{22}(\epsilon,\delta) = P\{\|Z_{t-\cdot,*}(t,x) - \Lambda_{\epsilon^{-1}}(\cdot, x-*)\sigma(u(t-\cdot,*))\|^2_{\mathcal{H}_{\epsilon^\delta}} \ge \epsilon\},$$

where $\Lambda_{\epsilon^{-1}}(t) = \psi_{\epsilon^{-1}} * S_3(t)$.
By assumption (ii) there exist positive universal constants C_1, C_2 such that

$$\begin{aligned}
&\|\Lambda_{\epsilon^{-1}}(\cdot, x-*)\sigma(u(t-\cdot,*))\|^2_{\mathcal{H}_{\epsilon^\delta}} \ge \sigma_0^2 \int_0^{\epsilon^\delta} dr \int_{\mathbb{R}^3} \mu(d\xi) |\mathcal{F}\Lambda_{\epsilon^{-1}}(r)(\xi)|^2 \\
&\ge \sigma_0^2 \Big(\frac{1}{2}\int_0^{\epsilon^\delta} dr \int_{\mathbb{R}^3} \mu(d\xi)|\mathcal{F}S_3(r)(\xi)|^2 - \int_{\mathbb{R}^3}\mu(d\xi)|\mathcal{F}S_3(r)(\xi)|^2 |\mathcal{F}\psi_{\epsilon^{-1}}(\xi) - 1|^2\Big) \\
&\ge \sigma_0^2 \Big(\frac{C_1}{2}\epsilon^{3\delta} - C_2 \epsilon^{\delta(2-2\eta)+1}\Big), \qquad (10)
\end{aligned}$$

where the very last lower bound follows from the inequatlity $|\mathcal{F}\psi_{\epsilon^{-1}}(\xi) - 1|^2 \le 4\pi|\xi|\epsilon$ and the estimate (13). Assume that $1 + 2\eta < \frac{1}{\delta}$ and $3\delta < 1$. Then, for ϵ sufficiently small the set $\{\|\Lambda_{\epsilon^{-1}}(\cdot, x-*)\sigma(u(t-\cdot,*))\|^2_{\mathcal{H}_{\epsilon^\delta}} < 6\epsilon\}$ is empty and therefore $P^{21}(\epsilon,\delta) = 0$.

As for $P^{22}(\epsilon,\delta)$, we apply Chebychev's inequality to the function g specified above. The isometry property of the extended stochastic integral and the upper bound (14) with $Y(s,x) = \sigma(u(t-s,x))$ yields $P^{22}(\epsilon,\delta) \le C\epsilon^{\delta(2-2\eta)}$. Therefore, $\int_0^{\eta_0} \epsilon^{-\frac{p}{2}-1} P^{22}(\epsilon,\delta) d\epsilon < \infty$ if and only if $\delta(2-2\eta) - \frac{p}{2} > 0$. By summarising the restrictions encountered so far, it is easy to check that they match up for any $p < 1 - 2\eta$. Hence, the Proposition is completely proved. ■

4. Auxiliary results

Let S_d denote the fundamental solution of the wave equation in any dimension $d \ge 1$. We collect here some estimates used in the proof of Theorem 1; they are proved using the well-known expresion of its Fourier transform, that is, $\mathcal{F}S_d(t)(\xi) = \dfrac{\sin(2\pi t|\xi|)}{2\pi|\xi|}$, they hold for any $t \in [0,T]$.

(A) Assume that condition (3) holds. There exists a real constant $C_1 > 0$ such that

$$C_1(t \wedge t^3) \le \int_0^t ds \int_{\mathbb{R}^3} \mu(d\xi)|\mathcal{F}S_d(s)(\xi)|^2. \qquad (11)$$

This is a consequence of Lemma 5.4.3 in [4].

(B) Assume (H_η). Then there exists a real constant $C_\eta > 0$ such that

$$\int_0^t ds \int_{\mathbb{R}^3} \mu(d\xi)|\mathcal{F}S_d(s)(\xi)|^2 \le C_\eta\, t^{3-2\eta}. \qquad (12)$$

This is an extension of Lemma 3.4 in [5].

(C) Suppose that (H_η) holds for $\eta \in (0, \frac{1}{2})$. Then there exists a positive real constant $\bar{C}_\eta$ such that

$$\int_0^t ds \int_{\mathbb{R}^3} \mu(d\xi)|\xi||\mathcal{F}S_d(s)(\xi)|^2 \le \bar{C}_\eta t^{2-2\eta}, \qquad (13)$$

(D) Let $\{Y(t,x),(t,x)\in[0,T]\times\mathbb{R}^3\}$ be a predictable L^2-process with stationary covariance function, such that $\sup_{(t,x)\in[0,T]\times\mathbb{R}^3} E(|Y(t,x)|^2)<\infty$. Set $g(s,x)=E(Y(s,y)Y(s,x+y))$ and $\Gamma_s^Y(dx)=g(s,x)\Gamma(dx)$. Set $\mu_s^Y=\mathcal{F}^{-1}(\Gamma_s^Y)$. Assume that the condition (iii) of Theorem 1 is satisfied. Then there exists a positive real constant C such that

$$\int_0^t ds\int_{\mathbb{R}^3}\mu_s^Y(d\xi)|\xi||\mathcal{F}S_d(s)(\xi)|^2\le Ct^{2-2\eta}. \tag{14}$$

(E) Let $d\in\{1,2,3\}$. A direct computation based on the expression of S_d shows that

$$\int_0^t ds\int_{\mathbb{R}^3} S_d(s,dy)\le C_3t^2, \tag{15}$$

where C_3 is a positive real constant which depends on d.

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