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NONLOCAL PROBLEMS FOR QUASILINEAR FUNCTIONAL PARTIAL DIFFERENTIAL EQUATIONS OF FIRST ORDER

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Abstract

Existence and uniqueness of almost everywhere solutions of nonlocal problems to functional partial differential systems in diagonal form are investigated. The proof is based on the characteristics and fixed point methods.

1. Introduction

For any metric spaces X and Y we denote by $C(X,Y)$ the set of all continuous functions from X to Y. Let $a_0 > 0$ be a given constant and $I_{a_0} = [0,a_0] \times \mathbb{R}^m$. Write $D = [-\tau, 0] \times [-b, b]$, where $\tau \in \mathbb{R}_+ = [0,+\infty)$ and $b = (b_1, \dots, b_m) \in \mathbb{R}^m_+$. For $z : [-\tau, a_0] \times \mathbb{R}^m \to \mathbb{R}^n$ and $(x,y) = (x, y_1, \dots, y_m) \in I_{a_0}$, we define $z_{(x,y)} : D \to \mathbb{R}^n$ by $z_{(x,y)}(s,t) = z(x+s, y+t)$, $(s,t) \in D$. Thus, we see that $z_{(x,y)}$ is a restriction of z to the rectangle $[x-\tau, x] \times [y-b, y+b]$. Put $\Omega = [0,a_0] \times \mathbb{R}^m \times C(D, \mathbb{R}^m)$ and $I = [-\tau, 0] \times \mathbb{R}^m$.

We assume that $\varrho = [\varrho_{ij}] : \Omega \to \mathbb{R}^{nm}$, $i = 1, \dots, n$, $j = 1, \dots, m$, $f = (f_1, \dots, f_n) : \Omega \to \mathbb{R}^n$, $h_k = [h_{kij}] : I \to \mathbb{R}^{nn}$, $i,j = 1, \dots, n$, $k = 1, \dots, r$, $\varphi = (\varphi_1, \dots, \varphi_n) : I \to \mathbb{R}^n$ are given functions.

We consider quasilinear hyperbolic systems of functional partial differential equations

$$(1) \qquad D_x z_i(x,y) + \sum_{j=1}^{m} \varrho_{ij}(x,y,z_{(x,y)}) D_{y_j} z_i(x,y) = f_i(x,y,z_{(x,y)}),$$

$i = 1, \dots, n$, $(x,y) \in I_{a_0}$, with nonlocal condition

$$(2) \qquad z_{(0,y)} + \sum_{k=1}^{r} (h_k)_{(0,y)} z_{(a_k,y)} = \varphi_{(0,y)}, \quad y \in \mathbb{R}^m,$$

where a_k, $k = 1, \dots , r$, are finite numbers such that $0 < a_1 < a_2 < \dots < a_r \le a_0$.

The nonlocal condition (2) may be also written in the form

$$z(x,y) + \sum_{k=1}^{r} h_k(x,y) z(a_k + x, y) = \varphi(x,y), \quad (x,y) \in I. \tag{3}$$

For $r = n$, $\tau = 0$ and $h_{kij} = h_{ij}\delta_{ki}$ (δ_{ki} is the Kronecker symbol) nonlocal boundary condition (2) reduces to the nonlocal condition "á la Cesari" [**8**], [**1**]. If $h_{kij} = \delta_{ki}\delta_{ij}$ then (2) reduces to the Nicoletti condition [**10**], [**12**]. Furthermore, if all $a_k = 0$, $k = 1, \dots , r$ then we get the usual Cauchy condition.

Nonlocal condition was considered for parabolic problems in [**4**], [**5**], [**8**], and for hyperbolic problems in [**2**], [**3**], [**6**], [**9**]. Mixed problems for system (1) in two independent variables were investigated in [**10**].

System (1) contains as particular cases the system of differential equations with a retarded argument, differential-integral systems and differential-functional equations with operators of the Volterra type (see Section 4).

In this paper, we consider the local existence and uniqueness of generalized solutions of nonlocal problem (1), (2). The method used in the paper is based on characteristics theory and the fixed point theorem.

2. Assumptions and Lemma

For $\eta = (\eta_1, \dots , \eta_k) \in \mathbb{R}^k$ we write $|\eta|_k = \max\{|\eta_i| : 1 \le i \le k\}$. For the matrix $U = [u_{ij}]$, $i = 1, \dots , n$, $j = 1, \dots , m$, we define $||U|| = \max\{\sum_{j=1}^{m} |u_{ij}| : 1 \le i \le n\}$. Let $||v||$ denote the supremum norm of $v \in C(D, \mathbb{R}^n)$ and $C(D, \mathbb{R}^n; p) = \{v \in C(D, \mathbb{R}^n) : ||v|| \le p\}$, $p \in \mathbb{R}_+$. Let $L([\alpha, \beta], \mathbb{R})$ be the set of all integrable functions $l : [\alpha, \beta] \to \mathbb{R}$. We denote by $C_L(D, \mathbb{R}^n)$ the class of all functions $v \in C(D, \mathbb{R}^n)$ satisfying the condition

$$|v(s,t) - v(\bar{s}, \bar{t})|_n \le \left| \int_s^{\bar{s}} \omega(\xi)\, d\xi \right| + q|t - \bar{t}|_m, \quad (s,t), (\bar{s}, \bar{t}) \in D, \tag{4}$$

where $\omega \in L([-\tau, 0], \mathbb{R}_+)$, $q \in \mathbb{R}_+$ (ω and q depend on v). For $v \in C_L(D, \mathbb{R}^n)$ we define $||v||_L = ||v||_* + ||v||$, where

$$||v||_* = \inf \left\{ q + \int_{-\tau}^{0} \omega(\xi)\, d\xi : q \text{ and } \omega \text{ satisfy (4)} \right\}.$$

Let $C_L(D,\mathbb{R}^n;p) = \{v \in C_L(D,\mathbb{R}^n) : ||v||_L \le p\}$, $p \in \mathbb{R}_+$. Denote by Λ the set of all functions $\lambda : [0,a_0] \times \mathbb{R}_+ \to \mathbb{R}_+$ such that $\lambda(\cdot,t) \in L([0,a_0],\mathbb{R}_+)$ for each $t \in \mathbb{R}_+$ and $\lambda(s,\cdot)$ is nondecreasing on $\mathbb{R}_+$ for almost every (a.e.) $s \in [0,a_0]$.

Assumption H_1**.** Suppose that

1. the matrix-valued function $\varrho(\cdot,y,v) : [0,a_0] \to \mathbb{R}^{nm}$ is measurable for every $(y,v) \in \mathbb{R}^m \times C(D,\mathbb{R}^n)$ and $\varrho(x,\cdot) : \mathbb{R}^m \times C(D,\mathbb{R}^n) \to \mathbb{R}^{nm}$ is continuous for a.e. $x \in [0,a_0]$;
2. there exists $d \in \Lambda$ such that
$$||\varrho(x,y,v)|| \le d(x,p)$$
for all $(y,v) \in \mathbb{R}^m \times C(D,\mathbb{R}^n;p)$, a.e. $x \in [0,a_0]$;
3. there exists $l \in \Lambda$ such that
$$||\varrho(x,y,v) - \varrho(x,\bar{y},\bar{v})|| \le l(x,p)[|y-\bar{y}|_m + ||v-\bar{v}||]$$
for all $(y,v),(\bar{y},\bar{v}) \in \mathbb{R}^m \times C_L(D,\mathbb{R}^n;p)$ and a.e. $x \in [0,a_0]$.

Assumption H_2**.** Suppose that

1. there exist constants $\bar{p} \in \left(0,\frac{1}{2}\right]$, $\bar{q}_0 \in \mathbb{R}_+$ and a function $\bar{\omega}_0 \in L([-\tau,0],\mathbb{R}_+)$ such that
$$\sum_{k=1}^{r} ||h_k(x,y)|| \le \bar{p},$$
$$\sum_{k=1}^{r} ||h_k(x,y) - h_k(\bar{x},\bar{y})|| \le |\int_x^{\bar{x}} \bar{\omega}_0(s)\,ds| + \bar{q}_0|y-\bar{y}|_m;$$
2. there exist constants $p_0, q_0 \in \mathbb{R}_+$ and a function $\omega_0 \in L([-\tau,0],\mathbb{R}_+)$ such that
$$|\varphi(x,y)|_n \le p_0,$$
$$|\varphi(x,y) - \varphi(\bar{x},\bar{y})|_n \le \left|\int_x^{\bar{x}} \omega_0(s)\,ds\right| + q_0|y-\bar{y}|_m$$
for all $(x,y),(\bar{x},\bar{y}) \in I$.

For $a \in (0, a_0]$ we denote by $C_{\varphi,a}[p,\omega,q]$ the set of all functions $v \in C([-\tau, a] \times \mathbb{R}^m, \mathbb{R}^n)$ such that

$$|z(x,y)|_n \leq p,$$
$$|z(x,y) - z(\bar{x},\bar{y})|_n \leq \left| \int_x^{\bar{x}} \omega(s)\, ds \right| + q|y - \bar{y}|_m$$

for $(x,y), (\bar{x},\bar{y}) \in I_a$ and v satisfies condition (3) on I.

We consider a Carathéodory solution of (1), (2). More precisely, a function z is called a Carathéodory solution of problem (1), (2) provided the following conditions hold:

(i) $z \in C_{\varphi,a}[p,\omega,q]$;

(ii) z satisfies (1) almost everywhere in I_a and (2) everywhere in $\mathbb{R}^n$.

For $z \in C_{\varphi,a}[p,\omega,q]$ we consider the following problem

$$\eta'(t) = \varrho_i(t, \eta(t), z_{(t,\eta(t))}), \quad \eta(x) = y, \quad i = 1, \ldots, n. \tag{5}$$

Note that (5) is an ordinary differential equation. If Assumption H_1 is satisfied then for every $z \in C_{\varphi,a}[p,\omega,q]$ the right hand side of (5) satisfies Carathéodory assumptions and the following Lipschitz condition

$$|\varrho_i(t,\xi,z_{(t,\xi)}) - \varrho_i(t,\bar{\xi},z_{(t,\bar{\xi})})| \leq l(t,r_a)(1-q)|\xi - \bar{\xi}|_m,$$

holds, where $r_a = p + q + \int_{-\tau}^{a} \omega(s)\, ds$. Thus, the existence and uniqueness of the solution $g_i[z](\cdot; x, y) : [0,a] \to \mathbb{R}$ of (5) follows from classical theorems.

Lemma 1. *If Assumption H_1 is satisfied and $z, \bar{z} \in C_{\varphi,a}[p,\omega,q]$ then*

$$\begin{aligned} &|g_i[z](t;x,y) - g_i[z](t;\bar{x},\bar{y})|_m \\ &\qquad \leq \exp\left[(1+q)\int_t^x l(s,r_a)\, ds\right] \left[\left|\int_x^{\bar{x}} d(s,p)\, ds\right| + |y-\bar{y}|_m\right] \end{aligned} \tag{6}$$

$t \in [0, \min(x,\bar{x})]$, $i = 1, \ldots, n$, and

$$\begin{aligned} &|g_i[z](t;x,y) - g_i[\bar{z}](t;x,y)|_m \\ &\leq \int_t^x l(s,r_a)\, ds \exp\left[(1+q)\int_t^x l(s,r_a)\, ds\right] ||z - \bar{z}||_a, \quad t \in [0,x], \end{aligned} \tag{7}$$

where $||\cdot||_a$ *denotes the supremum norm in the space* $C(I_a, \mathbb{R}^n)$.

Proof: We will consider the case where $x \le \bar{x}$. We have, by Assumption H_1,

$$|g_i[z](t;x,y) - g_i[z](t;\bar{x},\bar{y})|_m \le |y-\bar{y}|_m + \int_x^{\bar{x}} d(s,p)\,ds$$
$$+ \left| \int_x^t l(s,r_a)(1+q)|g_i[z](s;x,y) - g_i[z](s;\bar{x},\bar{y})|_m\,ds \right|, \quad t \in [0,x].$$

Hence,by the above inequality and by Gronwall's inequality we get (6). We consider the case $x > \bar{x}$ analogously to the case $x \le \bar{x}$.

It follows, from Assumption H_1, that

$$|g_i[z](t;x,y) - g_i[\bar{z}](t;x,y)|_m \le \left| \int_x^t l(s,r_a)[|g_i[z](s;x,y) - g_i[\bar{z}](s;x,y)\right|_m$$
$$+ ||z_{(s,g_i[z](s;x,y))} - \bar{z}_{(s,g_i[\bar{z}](s;x,y))}||]\,ds|, \quad t \in [0,x].$$

Since

$$||z_{(s,g_i[z](s;x,y))} - \bar{z}_{(s,g_i[\bar{z}](s;x,y))}||$$
$$\le ||z-\bar{z}||_a + q|g_i[z](s;x,y) - g_i[\bar{z}](s;x,y)|_m,$$

we get

$$|g_i[z](t;x,y) - g_i[\bar{z}](t;x,y)|_m \le \left| \int_x^t l(s,r_a)\,ds \right| ||z-\bar{z}||_a$$
$$+ \left| \int_x^t l(s,r_a)(1+q)|g_i[z](s;x,y) - g_i[\bar{z}](s;x,y)|\,ds \right|.$$

Utilizing the Gronwall's inequality, we obtain (7). The proof of Lemma 1 is complete. ∎

Assumption H_3**.** Suppose that

1. the vector-valued function $f(\cdot,y,v) : [0,a_0] \to \mathbb{R}^n$ is measurable for every $(y,v) \in \mathbb{R}^m \times C(D,\mathbb{R}^n)$ and $f(x,\cdot) : \mathbb{R}^m \times C(D,\mathbb{R}^n) \to \mathbb{R}^n$ is continuous for a.e. $x \in [0,a_0]$;
2. there exists $d_1 \in \Lambda$ such that
$$|f(x,y,v)|_n \le d_1(x,p)$$
for all $(y,v) \in \mathbb{R}^m \times C(D,\mathbb{R}^n;p)$, a.e. $x \in [0,a_0]$;
3. there exists $l_1 \in \Lambda$ such that
$$|f(x,y,v) - f(x,\bar{y},\bar{v})|| \le l_1(x,p)[|y-\bar{y}|_m + ||v-\bar{v}||]$$
for all $(y,v),(\bar{y},\bar{v}) \in \mathbb{R}^m \times C_L(D,\mathbb{R}^n;p)$ and a.e. $x \in [0,a_0]$.

3. The main theorem

Theorem 1. *Suppose that Assumptions H_1-H_3 are satisfied. Then there exist $a \in (0, a_0]$, $p, q \in \mathbb{R}_+$ and $\omega \in L([-\tau, a], \mathbb{R}_+)$ such that problem (1), (2) has a unique solution z in the class $C_{\varphi,a}[p, \omega, q]$.*

Proof: First, we take a so small that

$$\tag{8} \begin{gathered} \int_0^a d_1(t,p)\,dt \le 1, \quad K_a = \exp[L_a(1+q)] \le 2, \\ L_{1a}(1+q) \le 1, \quad R_a L_a K_a + L_{1a} < \frac{1}{2}, \end{gathered}$$

where $L_a = \int_0^a l(t,p)\,dt$, $L_{1a} = \int_0^a l_1(t,p)\,dt$ and $R_a = q_0 + \bar{q}_0 p + \bar{p} q + L_{1a}(1+q)$.

Let us choose constants p, q with

$$\tag{9} \begin{aligned} &p \ge (1-\bar{p})(1+p_0), \\ &q \ge 2(1-2\bar{p})^{-1}(q_0 + \bar{q}_0 p + 1) \end{aligned}$$

and function

$$\tag{10} \omega(t) = \max\{q d(t,p) + d_1(t,p), (1-\bar{p})^{-1}[\omega_0(t) + p\bar{\omega}(t)]\}.$$

We define the following operator

$$\begin{aligned} (Tz)_i(x,y) &= \varphi_i(0, g_i(0;x,y)) - \sum_{k=1}^{r} h_{ki}(0, g_i(0;x,y)) z_i(a_k, g_i(0;x,y)) \\ &\quad + \int_0^x f_i(t, g_i(t;x,y), z_{(t, g_i(t;x,y))})\,dt, \quad (x,y) \in I_a, \\ (Tz)_i(x,y) &= \varphi_i(x,y) - \sum_{k=1}^{r} h_{ki}(x,y) z_i(a_k + x, y), \quad (x,y) \in I. \end{aligned}$$

From Assumptions H_2, H_3 and inequalities (8), (9), we obtain

$$\begin{aligned} |(Tz)_i(x,y)| &\le |\varphi_i(0, g_i(0;x,y))| \\ &\quad + \left| \sum_{k=1}^{r} h_{ki}(0, g_i(0;x,y)) z_i(a_k, g_i(0;x,y)) \right| \\ &\quad + \left| \int_0^x f_i(t, g_i(t;x,y), z_{(t, g_i(t;x,y))})\,dt \right| \\ &\le p_0 + \bar{p} p + \int_0^a d_1(t,p) dt \le p, \quad (x,y) \in I_a, \end{aligned}$$

and

$$|(Tz)_i(x,y)| \leq |\varphi_i(x,y)| + \left|\sum_{k=1}^{r} h_{ki}(x,y) z_i(a_k + x, y)\right|$$
$$\leq p_0 + \bar{p}p \leq p, \quad (x,y) \in I.$$

For any $(x,y), (\bar{x},\bar{y}) \in I_a$, from Assumptions H_1-H_3 and Lemma 1, we get

$$|(Tz)_i(x,y) - (Tz)_i(\bar{x},\bar{y})| \leq R_a |g_i(t;x,y) - g_i(t;\bar{x},\bar{y})| + \left|\int_x^{\bar{x}} d_1(t,p)\,dt\right|$$
$$\leq R_a K_a \left|\int_x^{\bar{x}} d(t,p)\,dt\right| + \left|\int_x^{\bar{x}} d_1(t,p)\,dt\right| + R_a K_a |y - \bar{y}|_m.$$

For $(x,y), (\bar{x},\bar{y}) \in I$, we have

$$|(Tz)_i(x,y) - (Tz)_i(\bar{x},\bar{y})| \leq \left|\int_x^{\bar{x}} \omega_0(t)\,dt\right| + \bar{p}\left|\int_x^{\bar{x}} \omega(t)\,dt\right|$$
$$+ p\left|\int_x^{\bar{x}} \bar{\omega}_0(t)\,dt\right| + (q_0 + \bar{p}q + p\bar{q}_0)|y - \bar{y}|_m.$$

Thus, from (8)-(10), we obtain

$$|(Tz)_i(x,y) - (Tz)_i(\bar{x},\bar{y})| \leq \left|\int_x^{\bar{x}} \omega(t)\,dt\right| + q|y - \bar{y}|_m, \quad (x,y) \in I_a.$$

We see that $Tz \in C_{\varphi,a}[p,\omega,q]$.

Now, we prove that T is a contraction. Indeed, for any $z, \bar{z} \in C_{\varphi,a}[p,\omega,q]$ we have

$$|(Tz)_i(x,y) - (T\bar{z})_i(x,y)| \leq |\varphi_i(0, g[z](0;x,y)) - \varphi_i(0, g[\bar{z}](0;x,y))|$$

$$+ \left|\sum_{k=1}^{r} [h_{ki}(0, g_i[z](0;x,y)) - h_{ki}(0, g_i[\bar{z}](0;x,y))] z_i(a_k, g_i[z](0;x,y))\right|$$

$$+ \left|\sum_{k=1}^{r} [h_{ki}(0, g_i[\bar{z}](0;x,y))[z_i(a_k, g_i[z](0;x,y)) - \bar{z}_i(a_k, g_i[\bar{z}](0;x,y))]\right|$$

$$+ \left|\int_0^x f_i(t, g_i[z](t;x,y), z_{(t,g_i[z](t;x,y))}) - f_i(t, g_i[\bar{z}](t;x,y), \bar{z}_{(t,g_i[\bar{z}](t;x,y))})]\,dt\right|$$

$$\leq (R_a L_a K_a + L_{1a} + \bar{p})||z - \bar{z}||_a, \quad (x,y) \in I_a,$$

and

$$|(Tz)_i(x,y)-(T\bar{z})_i(x,y)| \le \left|\sum_{k=1}^{r} h_{ki}(x,y)[z_i(a_k+x,y)-\bar{z}_i(a_k+x,y)\right|$$
$$\le \bar{p}||z-\bar{z}||_a, \quad (x,y)\in I.$$

Thus, it follows from (8) that T is a contraction.

It remains to prove that the fixed point z of the operator T is the Carathéodory solution of (1), (2). By taking $y=g_i(x;0,\eta)$ in the equation $z_i(x,y)=(Tz)_i(x,y)$, we get

$$\begin{aligned}(11)\quad z_i(x,g_i(x;0,\eta)) &= \varphi_i(0,\eta)-\sum_{k=1}^{r} h_{ki}(0,\eta)z_i(a_k,\eta)\\ &\quad+\int_0^x f_i(t,g_i(t;0,\eta),z_{(t,g_i(t;0,\eta))})\,dt,\end{aligned}$$

since the solution g_i of (5) satisfies the following group property $g_i(t';t,g_i(t;x,y))=g_i(t';x,y)$. By differentiation of (11) with respect to x, using the chain rule differentiation Lemma (4.ii) of [**7**], and by putting again $y=g_i(x;0,\eta)$, we obtain that z satisfies (1) almost everywhere in I_a. It follows immediately that z satisfies (2). The proof of Theorem 1 is complete. ■

4. Special cases of system (1)

We list below a few examples of problems which can be derived from (1) by specializing the functions ϱ and f.

1) Suppose that $\bar{\varrho}: I_{a_0}\times\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^{nm}$ and $\bar{f}: I_{a_0}\times\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^n$ are given functions. Let

$$\varrho(x,y,v)=\bar{\varrho}\left(x,y,v(0,0),\int_D v(s,t)\,ds\,dt\right),$$

$$f(x,y,v)=\bar{f}\left(x,y,v(0,0),\int_D v(s,t)\,ds\,dt\right),\quad (x,y,v)\in I_a\times C(D,\mathbb{R}^n).$$

Then system (1) reduces to the differential-integral system

$$\begin{aligned}D_x z_i(x,y)&+\sum_{j=1}^{m}\bar{\varrho}_{ij}\left(x,y,z(x,y),\int_D z(x+s,y+t)\,ds\,dt\right)D_{y_j}z_i(x,y)\\ &=\bar{f}_i\left(x,y,z(x,y),\int_D z(x+s,y+t)\,ds\,dt\right),\quad i=1,\dots,n.\end{aligned}$$

2) Suppose that $\alpha = (\alpha_0, \alpha')$, $\beta = (\beta_0, \beta') : I_{a_0} \times C(D, \mathbb{R}^n) \to \mathbb{R}^{1+m}$, $\bar{\varrho} : I_{a_0} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{nm}$ and $\bar{f} : I_{a_0} \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ are given. Assume that

$$\begin{gathered}(\alpha_0(x,y,v) - x, \alpha'(x,y,v) - y) \in D, \\ (\beta_0(x,y,v) - x, \beta'(x,y,v) - y) \in D\end{gathered}$$

for $(x,y,v) \in I_{a_0} \times C(D, \mathbb{R}^n)$.

Put

$$\begin{aligned}\varrho(x,y,v) &= \bar{\varrho}(x,y,v(0,0), v(\alpha_0(x,y,v) - x, \alpha'(x,y,v) - y)), \\ f(x,y,v) &= \bar{f}(x,y,v(0,0), v(\beta_0(x,y,v) - x, \beta'(x,y,v) - y)).\end{aligned}$$

Then system (1) reduces to the differential system with a retarded argument

$$\begin{aligned}D_x z_i(x,y) + \sum_{j=1}^{m} \bar{\varrho}_{ij}(x,y,z(x,y), z(\alpha_0(x,y,z_{(x,y)}), \alpha'(x,y,z_{(x,y)}))) D_{y_j} z_i(x,y) \\ = \bar{f}_i(x,y,z(x,y), z(\beta_0(x,y,z_{(x,y)}), \beta'(x,y,z_{(x,y)}))), \quad i = 1, \dots, n.\end{aligned}$$

The functions α and β depend on the functional argument. Therefore, we cannot apply existence theorems from [**13**], [**14**] to the above system.

3) If we take

$$\begin{aligned}\varrho(x,y,v) &= \bar{\varrho}(x,y,v(0,0), (V(\mathcal{I}_{(x,y)} v))(x,y)), \\ f(x,y,v) &= \bar{f}(x,y,v(0,0), (V(\mathcal{I}_{(x,y)} v))(x,y)),\end{aligned}$$

where $(\mathcal{I}_{(x,y)} v)(s,t) = v(s-x, t-y)$ then system (1) reduces to the system of differential-functional equations ([**13**], [**14**])

$$\begin{aligned}D_x z_i(x,y) + \sum_{j=1}^{m} \bar{\varrho}_{ij}(x,y,z(x,y), (Vz)(x,y)) D_{y_j} z_i(x,y) \\ = \bar{f}_i(x,y,z(x,y),(Vz)(x,y)), \quad i = 1, \dots, n.\end{aligned}$$

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