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Towards Automatic Modeling of Economic Texts

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Abstract

In this paper, we present an application of perception-based logical deduction in the modeling of an economic analysis given in natural language. We use fuzzy IF-THEN rules and the theory of evaluative linguistic expressions in the frame of fuzzy type theory. We outline a description of our formal tools and discuss our methodology as well as its relations to other approaches. Finally, we present an example taken from free economic analysis on the Internet.

1 Introduction

In this paper, we will describe an application of perception-based logical deduction (PbLD) in the modeling of economic analysis given in natural language.

In paper [16], we used fuzzy type theory (FTT) [11], theory of evaluative linguistic expressions [15], and perception-based logical deduction (PbLD) [12] in the analysis of a detective story. In this paper, we will apply this method to a model of economic analysis of a macroeconomic situation. For this purpose, we use as an example a part of a free economic analysis of the Czech Savings Bank. It describes the influence of economic growth and other factors on the change in the unemployment rate in the Czech Republic. It turns out that the influence of these auxiliary factors (non-motivating social system, rigid labor market, high tax load) is important and decreases the expected positive effect of high economic growth.

Of course, the model of this particular economic analysis should be taken as an illustration of our methodology based on FTT and PbLD. Its main features are:

- It works inside well-developed and sound formal theory.
- It extensively uses formal theory of the so-called *trichotomous evaluative linguistic expressions* (evaluative expressions, for short), which are natural-language expressions describing positions on an ordered scale. Examples of evaluating linguistic expressions are *small*, *more or less big*, etc. These expressions are used incessantly by people in everyday speech. Therefore, a

sound formal model of them behaving according to human intuition is very important.

- Evaluative linguistic expressions are used in fuzzy IF-THEN rules and sets of them which we call *linguistic descriptions*. The model of the meaning of linguistic descriptions is constructed using formal tools of FTT. It allows us to model the role of context (possible world) in accordance with intuition using standard notions of intension and extension (see Section 3).
- Due to its formal nature, our methodology is open to various kinds of improvements in the direction of a higher proportion of automated extraction of linguistic descriptions, automated deduction, etc.

Our approach relates to the so-called precisiated natural language (PNL) introduced by L. A. Zadeh in [20]. It aims to formalize natural language sentences or texts using tools from the field of soft computing. See e. g. [3], where authors showed some application of PNL in the analysis of a simple economic sentence.

There are also connections to *commonsense reasoning* [18, 10], originated in large part by John McCarthy [7]. It includes formalization of reasonings performed by humans, taking into account their nonmonotonicity and other features.

Economics is, in our opinion, a very promising field of application of our methodology. Economic statements are usually expressed in natural language; they inherently contain vague notions, and vagueness contained in these notions cannot be effectively and beneficially removed. Evaluative expressions are extensively used, and automated parsing and deductions could be quite useful for practitioners.

Our methodology uses quite powerful formal tools based on the fuzzy type theory. Formalisms commonly used for commonsense reasoning are usually based on extensions of classical predicate logic. This means that these formalisms are usually easy to understand, and they can use some standard methods for theorem proving, etc. However, their expressive power is limited. Because of extreme expressive power of FTT, it can capture many fine points of the semantics of natural language, as well as that of complex economic notions which play important role in the modeling of economic texts. For example, we can express notions which involve generalized quantifiers (*many*, *about half*, etc.). These quantifiers are important, e.g., for modeling of complex notions like *difficult order* [5]. To model difficult order, we decompose this complex notion into several criteria (small amount, large discount, small delivery time, etc.) and then use a generalized quantifier to the effect that *many of these criteria should be fulfilled.* Another important feature of our system is that it naturally distinguishes *intension* and *extension* of evaluative expressions, IF-THEN rules, etc. (see Section 3 of this paper, [15]). The price we must pay is higher complexity. However, we cannot expect that complex problems can be solved using simple means.

2 Methodology

The fundamental accepted classification of mathematical fuzzy logic is *fuzzy logic in narrow sense* (FLn) and *fuzzy logic in broader sense* (FLb). The former is mathematical fuzzy logic (see [6, 14]), which is a generalization of classical mathematical logic, i.e., it has clearly distinguished syntax and semantics that is always many-valued. The syntax consists of precise definitions of formula, proof, formal theory, model, provability, etc. There are many formalisms falling into the realm of FLn. They usually differ from each other on the basis of the assumed structure of truth values, which then determines all of the properties of the given calculus. It is argued in [14] that the most distinguished calculi that are important also for the development of FLb are IMTL-, BL-, Łukasiewicz- and ŁΠ-fuzzy logics. All these calculi have been formally developed up to higher-order.

Fuzzy logic in broader sense is an extension of FLn that aims to develop a *formal theory of human reasoning that would include a mathematical model of meaning of special expressions of natural language and generalized quantifiers with regard to their vagueness.* One can see that it overlaps with two other paradigms proposed in the literature, namely *commonsense reasoning* and *precisiated natural language.* The main drawback of the up-to-date formalizations of commonsense reasoning, in our opinion, is that it neglects vagueness present in a meaning of natural language expressions.

As mentioned, our theory can be classified as a part of the methodology introduced by L. A. Zadeh in his papers [19, 20] and called *Precisiated Natural Language* (PNL). The latter is an attempt at developing a unified formalism for various tasks involving natural language propositions.

Two main premises of PNL are the following:

(a) Much of world knowledge is perception-based.

(b) Perception-based information is intrinsically fuzzy.

It is important to stress that the term "precisiated natural language" especially means a reasonable working formalization of semantics of natural language without the pretension of capturing it in detail and fineness. Its goal is to provide an acceptable and applicable technical solution. It should also be noted that the term "perception" is not considered here as a psychological term, but instead as a result of human and intrinsically imprecise measurement. In our formal theory, we technically identify perceptions with evaluative expressions of natural language characterizing certain values.

The PNL methodology requires presence of the so-called *World Knowledge Database* (WKDB), which contains all the necessary information (including perception-based propositions that describe the knowledge acquired by direct human experience) that can be used in the deduction process. A *multiagent, modular deduction database* (MDE) contains various rules of deduction. However, no exact formalization of PNL has been developed until now and thus should be taken mainly as a reasonable methodology.

Our concept of FLb is thus a glue between both paradigms described above that should combine the best of each. So far, FLb consists of the following theories:

(a) Formal theory of evaluative linguistic expressions,

(b) formal theory of fuzzy IF-THEN rules,

(c) formal theory of perception-based logical deduction,

(d) formal theory of intermediate quantifiers.

FLn-mathematical basis for all these theories is *fuzzy type theory.*

Our version of PNL incorporates logical machinery. The translation from natural language to fuzzy IF-THEN rules is done manually so far. The formal frame is Łukasiewicz fuzzy type theory (Ł-FTT) and theories from the above list. Moreover, we apply some principles of non-monotonic reasoning. However, we will not go into details of its role in this paper.

3 Formal tools

3.1 Fuzzy type theory

As stated, the main tool for the construction of a model[1] of our economic analysis is fuzzy type theory. In this section, we will very briefly overview some of its main points. A detailed explanation of FTT can be found in [11]. Classical type theory is in detail described in [1].

The *Types* is a set of types constructed iteratively from the atomic types ϵ (elements) and o (truth values). $Form_\alpha$ denotes a set of formulas of type $\alpha \in Types$. If $A \in Form_\alpha$ is a formula of type $\alpha \in Types$ then we write A_α.

Formulas of type o (truth value) can be joined by the following connectives (derived formulas): $\vee$ (disjunction), $\wedge$ (conjunction), **&** (strong conjunction), ∇ (strong disjunction), $\Rightarrow$ (implication). General ($\forall$) and existential ($\exists$) quantifiers are defined as special formulas. For the details on their definition and semantics — see [11].

If $A \in Form_{o\alpha}$, then A represents a property of elements of the type α. By abuse of language, we will often say "A is a property" (of elements of type α) and similarly, $A_{(o\alpha)\alpha}$ is a relation (between elements of type α). We will freely write or omit the type when no misunderstanding may occur.

A theory T is a set of formulas of type o (determined by a subset of special axioms, as usual). Provability is defined as usual.

The operator

$$\imath z_\alpha A_o := \iota_{\alpha(o\alpha)}(\lambda z_\alpha A_o)$$

picks up an element of type α such that the formula A_o is true for it in the degree 1.

Semantics. The structure of truth values in this paper is the Łukasiewicz$_\Delta$ algebra and so, the corresponding FTT is Łukasiewicz (Ł-FTT). Δ is the Baaz delta [6]. Let J be a language of Ł-FTT. A *frame* for J is a tuple $\mathcal{M} = \langle (M_\alpha, =_\alpha)_{\alpha \in Types}, \mathcal{L}_\Delta \rangle$ where $\mathcal{L}_\Delta$ is Łukasiewicz$_\Delta$ algebra of truth values, $=_\alpha$ is a fuzzy equality on M_α.

[1]We are using the term *model* in two meanings: as a general description of some situation, system etc., and as a formal model in the sense of mathematical logic.

Recall that if $\beta\alpha$ is a type then the corresponding set $M_{\beta\alpha}$ contains (not necessarily all) functions $f : M_\alpha \to M_\beta$.

Let p be an assignment of elements from $\mathcal{M}$ to variables. An interpretation $\mathcal{I}^{\mathcal{M}}$ is a function that assigns every formula A_α, $\alpha \in \mathit{Types}$ and every assignment p a corresponding element, that is, a function of the type α. A general model is a frame $\mathcal{M}$ such that $\mathcal{I}_p^{\mathcal{M}}(A_\alpha) \in M_\alpha$ holds true.

The following is a special formula representing a non-zero truth value:

$$\Upsilon_{oo} := \lambda z_o \cdot \neg\mathbf{\Delta}(\neg z_o).$$

3.2 Trichotomous evaluative linguistic expressions

Trichotomous evaluative linguistic expressions (or, simply, evaluative expressions) are expressions of natural language, for example, *small, medium, big, about twenty five, roughly one hundred, very short, more or less deep, not very tall, roughly warm or medium hot, quite roughly strong, roughly medium size*, and many others. They form a small but very important part of natural language and they are present in its everyday use any time. The reason is that people very often need to evaluate phenomena around them. Moreover, they often make important decisions based on them, learn how to control using them, and apply them in many other activities.

Due to lack of space and quite complicated formalism, we will only touch on this theory and refer to the contribution [13]. All details can be found in [15].

An important role in the theory of evaluative expressions is played by the concept of the *context*. It characterizes a range of possible values for (numerical) variables and is represented by a special type ω. For simplicity, we will suppose that in each model, the context is given by a triple $\langle v_L, v_M, v_R\rangle$, where $v_L, v_M, v_R \in \mathbb{R}$ and $v_L < v_M < v_R$. The values v_L, v_M, v_R characterize minimal, middle, and maximal values of the given context, respectively. On a syntactical level, the context is denoted by a variable $w \in \mathit{Form}_\omega$ and is represented by three constants $\perp_w, \dagger_w$ and $\top_w$.

In applications, we often also need a more general type of context that allows both positive as well as negative values. We will call this type of context *two-sided*. It is given by $\langle v_{NL}, v_{NM}, v_Z, v_{PM}, v_{PR}\rangle$, where $v_{NL} < v_{NM} < v_Z < v_{PM} < v_{PR}$. In this context, the sign is used before evaluative linguistic expressions, e.g. *negative small*, *positive big* (abbreviations: $-\,\mathit{Sm}$, $+\,\mathit{Bi}$), etc. There is also special evaluative linguistic expression *zero*. Contexts from the previous paragraph will be called *simple*.

Evaluative expressions are denoted by letters $\mathcal{A}, \mathcal{B}, \ldots$. *Intension* of $\mathcal{A}$ is a formula $\mathrm{Int}(\mathcal{A})$. Recall that intension means *a property* which is denoted by $\mathcal{A}$. It is important to note that intension does not depend on the context. For example, *very small* is a name of a property of being "very small" which may mean about 150 cm (and less) when speaking about people, about 3 mm when speaking about beetles, etc. The type of $\mathrm{Int}(\mathcal{A})$ is $(o\alpha)\omega$. The latter will often be denoted by φ. The formal theory of evaluative expressions is denoted by T^{Ev}. This theory provides means by which the above concepts of context, intension, and others can be effectively formalized.

3.3 Fuzzy IF-THEN rules and perception-based logical deduction

The perception-based logical deduction in the frame of FTT has been described in [12]. Though the method is more general, we will suppose that all considered linguistic expressions are evaluative ones.

A fuzzy IF-THEN rule is a linguistic expression of the form

$$\mathcal{R} := \ \mathsf{IF}\ X\ \mathsf{is}\ \mathcal{A}\ \mathsf{THEN}\ Y\ \mathsf{is}\ \mathcal{B}. \tag{1}$$

where $\mathcal{A}, \mathcal{B}$ are evaluative expressions. The linguistic predication 'X is $\mathcal{A}$' is called *antecedent* and 'Y is $\mathcal{B}$' is called *consequent*.

Intension of a fuzzy IF-THEN rule $\mathcal{R}$ is (represented by) a formula

$$\mathrm{Int}(\mathcal{R}) := \lambda w\, \lambda w' \cdot \lambda x\, \lambda y \cdot Ev^A wx \Rightarrow Ev^C w'y. \tag{2}$$

The symbols Ev^A, Ev^C denote intensions of evaluative expressions in the antecedent and consequent, respectively. We denote by ρ a special (meta-)type for formulas that are intensions of fuzzy IF-THEN rules of the form (1).

A *linguistic description* LD is a set of fuzzy IF-THEN rules. Its *topic* is a set of linguistic expressions $\{\mathrm{Int}(\mathcal{A}_j) \mid j = 1, \ldots, m\}$ and its focus is $\{\mathrm{Int}(\mathcal{B}_j) \mid j = 1, \ldots, m\}$, where m is a number of IF-THEN rules in the linguistic description LD.

We can formally represent linguistic description, its topic, and focus by special crisp formulas of FTT:

$$LD \equiv \lambda z_\rho \cdot \bigvee_{j=1}^{m} \boldsymbol{\Delta}(z_\rho \equiv \mathrm{Int}(\mathcal{R}_j)), \tag{3}$$

$$Topic^{LD} \equiv \lambda z_\varphi \cdot \bigvee_{j=1}^{m} \boldsymbol{\Delta}(z_\varphi \equiv Ev_j^A), \tag{4}$$

$$Focus^{LD} \equiv \lambda z_\varphi \cdot \bigvee_{j=1}^{m} \boldsymbol{\Delta}(z_\varphi \equiv Ev_j^C). \tag{5}$$

In practice, we usually need more antecedent variables. In this case, the antecedent variables are usually connected by the linguistic connective AND which is interpreted by a logical connective $\wedge$.

In perception-based logical deduction, we must introduce several special formulas. For some, we will give only their informal description and refer to [12] for their precise definitions. The formula $\prec$ denotes the relation of *sharpness* between (intensions of) evaluative expressions. For example, if x is, at least partly, "very big" in all contexts then it is also "big" in all of them, i.e., $\mathrm{Int}(\textit{very big}) \prec \mathrm{Int}(\textit{big})$. The formula $Ev_1|Ev_2$ expresses that both evaluative expressions are incomparable.

There is also a useful evaluative expression *undefined*, which allows us to express *default value* of some proposition. It has the property that its intension is less sharp than the intension of any other evaluative expression, i.e.,

$$\mathrm{Int}(\mathcal{A}) \prec \mathrm{Int}(\textit{undefined}) \tag{6}$$

for all $\mathcal{A}$ different from *undefined*.

We will also introduce formula $Eval_{o(\varphi\alpha\omega)}$ ($Eval\ wx\,\mathrm{Int}(\mathcal{A})$ expresses that an element x in context w is evaluated by $\mathcal{A}$).

One of principal paradigms of the concept of precisiated natural language is that world knowledge, i.e., the knowledge accumulated by people during their life, is *perception based.* We will formalize the concept of perception using a formula $Perc \in Form_{(o\varphi)\alpha}$ which expresses that an intension z_φ is a *perception* of $x_\alpha \in Form_\alpha$ (for definition see [16]).

We will also introduce a *local perception* $LPerc_{o(\varphi\alpha\omega)}$ relative to a specific context w. Let *Topic* be a formula (4) representing some topic. Then we may formalize the concept of local perception using a formula $LPerc \in Form_{o(\varphi\alpha\omega)}$ defined as follows:

$$LPerc_{o(\varphi\alpha\omega)} := \lambda w\, \lambda x\, \lambda z_\varphi \cdot Topic z_\varphi \,\&\, (\exists w)(Eval\, wxz_\varphi\, \&$$
$$(\forall z'_\varphi)((Eval\, wxz'_\varphi \,\&\, Topic z'_\varphi) \Rightarrow ((z'_\varphi wx < z_\varphi wx) \vee (z_\varphi \prec z'_\varphi) \vee (z'_\varphi | z_\varphi)))). \quad (7)$$

$LPerc\ wx_\alpha z_\varphi$ expresses that the intension z_φ is a *local perception* of $x_\alpha \in Form_\alpha$ with respect to the given set *Topic* of linguistic expressions. The meaning of (7) is the following: the intension z_φ is a local perception of $x \in Form_\alpha$ with respect to the topic of the given linguistic description if x_α is evaluated by z_α in the given context w, and for every other z'_φ which also evaluates x_α in w, either the truth value of $z'_\varphi wx$ is smaller than $z_\varphi wx$, or z_φ is sharper than z'_φ, or it is incomparable with the latter.

Let *LD* be a linguistic description. Then we may consider a formula $LPerc^{LD}$ obtained from (7) by inserting $Topic^{LD}$ for *Topic*. For example, for a given linguistic description *LD* the formula

$$LPerc^{LD} wxEv$$

means that the *evaluative expression Ev* from the topic of *LD* is a *local perception* of x in the context w. More precisely, x is evaluated in w by the sharpest *Ev* in the best way (among evaluative expressions belonging to the topic of *LD*).

Lemma 1
(a) Let $Ev_1 \prec Ev_2$*. Then* $T^{Ev} \vdash LPerc\ wxEv_1 \Rightarrow LPerc\ wxEv_2$.

(b) Let $T^{Ev} \vdash z_\varphi wx \Rightarrow z'_\varphi w'y$*. Then* $T^{Ev} \vdash Eval\ \ wxz_\varphi \Rightarrow Eval\ \ w'yz'_\varphi$.

A linguistic description *LD* characterizes a certain kind of dependence (relation) between features of objects (and, consequently, the objects themselves) using natural language. People use it when they want to describe a certain situation or process but they do not know it precisely. Therefore, the most important role of the linguistic description is to provide a conclusion about consequent objects Y when information about antecedent objects X is given. Such an information has the character of *perception* of properties of the latter objects and so, the corresponding procedure is called *perception-based logical deduction*.

On the basis of the theory presented in [12], the following special inference rule of perception-based logical deduction can be introduced. Let *LD* be a linguistic description consisting of rules of the form (1) and $x \in Form_\alpha$, $y \in Form_\beta$, $w \in Form_{\alpha o}$, $w' \in Form_{\beta o}$. Then the following scheme is a valid special inference rule:

$$r_{PbLD} : \frac{LPerc^{LD} wx Ev_i^A, \quad LD}{Eval\, w'\, \hat{y}_i Ev_i^C} \tag{8}$$

where $\hat{y}_i \equiv \imath y \cdot Ev_i^A wx \Rightarrow Ev_i^C w'\, y$, $i \in \{1, \ldots, m\}$, $T \vdash Topic^{LD} Ev^A$ and $T \vdash Focus^{LD} Ev^C$.

This rule has the following interpretation: Let LD be a linguistic description consisting of fuzzy IF-THEN rules of the form (1). If we find a formula $\mathrm{Int}(\mathcal{A}_i) \equiv Ev_i^A$ of some expression from the topic $Topic^{LD}$ and an element $\mathbf{u}^0$ in the context $\mathbf{w}^0$ such that $Ev_i^A \mathbf{w}^0 \mathbf{u}^0$ has a non-zero truth degree, then (denoting $\mathbf{b}_i^0 \equiv Ev_i^A \mathbf{w}^0 \mathbf{u}^0$) we conclude that the element $\imath y \cdot \mathbf{b}_i^0 \Rightarrow Ev_i^C w'y$ which is *typical for the formula* $\mathbf{b}_i^0 \Rightarrow Ev_i^C w'y$, is evaluated by the linguistic expression $Ev_i^C \in Focus^{LD}$ in every context w'.

Note that the main result of r_{PbLD} is the element $\hat{y}_i$. Then, in every model $\mathcal{M}$ we can find a specific element $\mathcal{I}_p^{\mathcal{M}}(\hat{y}_i) = v \in M_\beta$ using the operation $\mathcal{I}_p^{\mathcal{M}}(\iota_{\beta(o\beta)})$ which in fuzzy set theory is just the defuzzification function. In our case, the DEE method (defuzzification of linguistic expressions) should be used. The detailed, less formal explanation of perception-based logical deduction including examples is presented in [17].

Human reasoning that is based on a complex of experience, observation, logical reasoning, and world knowledge is necessarily nonmonotonic. Hence, our model must include also the theory of *nonmonotonic reasoning* (cf. [2, 16]). We deal with a class of theories that themselves are consistent but when using them simultaneously, we may arrive at a contradiction or, at least, at a non-desirable result. Therefore, we consider a special preference relation that tells us which theory should be used in the given state (called *belief state* in [2]). At each state, we work in a special theory which, in our case, is determined by a linguistic description (one or more) and possibly also by some perception (recall that this is a formula representing intension of some evaluative linguistic expression).

In this paper, we are not going into details of this approach. We will suppose that our formal theories contain only those parts of the theory of evaluative expressions T^{Ev}, that are necessary for deductions based on used linguistic descriptions and perceptions.

4 Example of a model of economic analysis

We present a model of a macroeconomic situation using PbLD. We use a section from free economic analysis of Czech Savings Bank for the fourth quarter of 2006:[2]

> Our computations show that acceleration of economy by one per cent causes the unemployment rate to decrease only by 0.3 per cent. A non-motivating social system, rigid labor market, and high tax rate on labor

[2]http://www.csas.cz/banka/content/inet/internet/cs/treasury_ie.xml, in Czech.

> expenses are, in our opinion, the main culprits of a structurally high rate of unemployment while the economy grows at a high rate.

We will present an analysis of this quotation by means of the above outlined theory.

4.1 Analysis of the example

We model the above natural-language macroeconomic analysis using two linguistic descriptions with hierarchic structure. We will use an intermediate variable *strength of auxiliary factors* for the overall influence of the non-motivating social system, rigid labor market, and high tax rate. Hence, we will introduce a linguistic description LD_{Aux} with three antecedent variables and one consequent variable, the above mentioned *strength of auxiliary factors*. This variable is then used as an antecedent variable to the second linguistic description LD_{Un}, with the second antecedent variable *rate of economy acceleration*. The consequent variable in this linguistic description is *rate of unemployment change* – the result.

Naturally, it would be unreasonable to expect that automatic derivation of this hierarchical structure would be possible without some human assistance. However, it is more readable and transparent for human agents in this form.

Let us remark that, using principles of nonmonotonic reasoning, we could understand a rule "IF the economy grows THEN the unemployment rate decreases" as a *default rule*. This means that it holds, unless additional information or information to the contrary is available. The presence of auxiliary factors can modify this default rule to another one with greater preference. It is possible to model it by means of an *epistemic state* [2] which corresponds to this problem, see also the model of Columbo's case in [16].

4.2 World knowledge

World knowledge in this example is all the needed knowledge that is not present in the quotation above. It includes proper determination of types, special formulas and constants, contexts of variables, and possibly also *default rules* (see Remark 1). In our case, the default rule has the (informal) form "If the economy grows, unemployment rate decreases."

(i) $\vartheta \in \mathit{Types}$ represents a general feature of objects that can be characterized using grades. In the model, a set of this type can be, e.g., a subset of the real numbers.

(ii) $\beta \in \mathit{Types}$ represent objects of type (national) state.[3]

(iii) $\gamma \in Type$ represents general abstract objects.

(iv) $Ss \in \mathit{Form}_{\vartheta\beta}, Lm \in \mathit{Form}_{\vartheta\beta}, Tl \in \mathit{Form}_{\vartheta\beta}, Uc \in \mathit{Form}_{\vartheta\beta}, Ea \in \mathit{Form}_{\vartheta\beta}, Af \in \mathit{Form}_{\vartheta\gamma}$, are special formulas characterizing state of social system, rigidity of

[3] This type could also represent other territorial objects, e.g., regions, European Union etc.

labor market, level of tax load, unemployment change, economy acceleration (of objects of type β), and auxiliary factors (of object of type γ), respectively.

(v) In correspondence with the previous item, we also need to consider contexts w_{Ss}, w_{Lm}, w_{Tl}, w_{Uc}, w_{Ea}, $w_{Af} \in Form_{\vartheta o}$.

(vi) Special constant concerning the story, namely: $\mathbf{c}_{CR} \in Form_\beta$ representing in our case *the Czech Republic*. However, linguistic descriptions below should also be valid for other countries.

4.3 Formalization and reasoning

(i) *Contexts.* Context can be either *simple* (items (a), (b), (c), (e)) or *two-sided* (items (d), (f)), see Section 3.2. Two-sided contexts are used for variables which can take on both positive and negative values.

(a) Motivation of social system: $w_{Ss} = \langle 0, 0.5, 1\rangle$ (abstract degrees).

(b) Rigidity of labor market: $w_{Lm} = \langle 0, 0.5, 1\rangle$ (abstract degrees).

(c) Strength of tax load: $w_{Tl} = \langle 0, 15, 40\rangle$ (%).

(d) Rate of unemployment change: $w_{Uc} = \langle -1, -0.5, 0, 0.5, 1\rangle$ (%).

(e) Strength of auxiliary factors: $w_{Af} = \langle 0, 0.5, 1\rangle$ (abstract degrees).

(f) Rate of economy acceleration per year: $w_{Ea} = \langle -6, -3, 0, 5, 6\rangle$ (%).

(ii) *Linguistic descriptions*

(a) Linguistic description LD_{Aux} for the influence of auxiliary factors:[4]

$$\begin{array}{c}\mathsf{IF}\ X_{Ss}\ \text{is}\ \mathit{Sm}\ \mathsf{AND}\ X_{Lm}\ \text{is}\ \mathit{Bi}\ \mathsf{AND}\ X_{Tl}\ \text{is}\ \mathit{Bi}\\ \mathsf{THEN}\ X_{Af}\ \text{is}\ \mathit{Bi}\end{array} \tag{9}$$

. . . .

The corresponding intensions of such rules are

$$\begin{aligned}&\lambda w_{Ss}\,\lambda w_{Lm}\,\lambda w_{Tl}\,\lambda w_{Af}\,\cdot\lambda x_{1,\vartheta}\,\lambda x_{2,\vartheta}\,\lambda x_{3,\vartheta}\,\lambda y_\vartheta\\ &\qquad\cdot(Ev_1^A w_{Ss}\,x_{1,\vartheta})\wedge(Ev_2^A w_{Lm}\,x_{2,\vartheta})\wedge(Ev_3^A w_{Tl}\,x_{3,\vartheta})\\ &\qquad\qquad\Rightarrow Ev^C w_{Af}\,y_\vartheta.\end{aligned} \tag{10}$$

(b) Linguistic description LD_{Un} for the computation of unemployment change.

$$\mathsf{IF}\ X_{Ea}\ \text{is}\ +\mathit{Bi}\ \ \mathsf{AND}\ X_{Af}\ \text{is}\ \mathit{Bi}\ \mathsf{THEN}\ X_{Uc}\ \text{is}\ -\mathit{Sm} \tag{11}$$

$$\mathsf{IF}\ X_{Ea}\ \text{is}\ +\mathit{Bi}\ \ \mathsf{AND}\ X_{Af}\ \text{is}\ \mathit{undefined}\ \mathsf{THEN}\ X_{Uc}\ \text{is}\ -\mathit{Bi} \tag{12}$$

. . . .

[4]Only one rule based on natural-language macroeconomic analysis is given here. However, several similar rules can be present, possibly using evaluative expression *undefined*.

The corresponding intensions of such rules are

$$\lambda w_{Ea}\, \lambda w_{Af}\, \lambda w_{Uc} \cdot \lambda x_{1,\vartheta}\, \lambda x_{2,\vartheta}\, \lambda y_{\vartheta} \cdot (Ev_1^A w_{Ea}\, x_{1,\vartheta}) \wedge (Ev_2^A w_{Af}\, x_{2,\vartheta}) \Rightarrow Ev^C w_{Uc}\, y_{\vartheta}. \quad (13)$$

Remark 1
It is advantageous to use a special evaluative expression *undefined* (see Section 3.3) for the modeling of default rules. Consider IF-THEN rule (12) from linguistic description LD_{Un}: This rule is used, if it holds that

$$T^{Uc} \vdash LPerc^{LD_{Un}} w_{Af}(Af\mathbf{c}_{AF})\, \mathit{undefined}\,.$$

According to (6), the intension of *undefined* is less sharp that intension of any other evaluative expression. Hence, this rule is used if it is not the case that

$$T^{Uc} \vdash LPerc^{LD_{Un}} w_{Af}(Af\mathbf{c}_{AF})\, Bi$$

holds ($\mathbf{c}_{AF}$ is a new constant, see below). Informally, it means that if the strength of auxiliary factors is *big*, rule (11) is used, otherwise rule (12) is used.

(iii) Furthermore, we must construct a specific *model*. It is based on a frame

$$\mathcal{M} = \langle (M_\alpha, =_\alpha)_{\alpha \in \mathit{Types}}, \mathcal{L}_\Delta \rangle.$$

where $M_o = [0,1]$ and $M_\alpha = \mathbb{R}$ for $\alpha = \vartheta$.

(iv) *Perceptions*

(P1) The social system is non-motivating. We can say that the motivation of social system is *small*. Formally,

$$\mathcal{I}_p^{\mathcal{M}}(LPerc\, w_{Ss}(Ss\mathbf{c}_{CR})\, Sm) = 1. \quad (14)$$

(P2) The labor market is rigid. This means that the rigidity of the labor market is *big*. Formally,

$$\mathcal{I}_p^{\mathcal{M}}(LPerc\, w_{Lm}(Lm\mathbf{c}_{CR})\, Bi) = 1. \quad (15)$$

(P3) The tax load is high. Formally,

$$\mathcal{I}_p^{\mathcal{M}}(LPerc\, w_{Tl}(Ss\mathbf{c}_{CR})\, Bi) = 1. \quad (16)$$

(P4) The economic acceleration is positive big. Formally,

$$\mathcal{I}_p^{\mathcal{M}}(LPerc\, w_{Ea}(Ss\mathbf{c}_{CR}) + Bi) = 1. \quad (17)$$

(v) *Deductions* We use a formal theory T^{Uc} for the reasoning about our example. It includes appropriate parts of the theory of evaluative expressions T^{Ev}, intensions of linguistic descriptions LD_{Aux} and LD_{Un}, and perceptions (P1)–(P4).

$$T^{Uc} = \{LPerc\, w_{Ss}(Ss\mathbf{c}_{CR})\, Sm, LPerc\, w_{Lm}(Lm\mathbf{c}_{CR})\, Bi, LPerc\, w_{Tl}(Ss\mathbf{c}_{CR})\, Bi, LPerc\, w_{Ea}(Ss\mathbf{c}_{CR}) + Bi, (10), (13)\} \cup T^{Ev}\}. \quad (18)$$

Below, we give the main ideas of the proofs.

(a)

$$T^{Uc} \vdash LPerc^{LD_{Aux}} w_{Ss}(Ss\mathbf{c}_{CR})\, Sm \quad (19)$$

This means that *small* is the perception of variable *motivation of social system* with respect to the linguistic description LD_{Aux}.

PROOF: Note that in IF-THEN rule (9) is in the variable X_{ss} evaluative expression *small*. If there is no other IF-THEN rule with the same evaluative expression *small* in the variable X_{ss}, then (19) follows from the definition of *LPerc*. □

(b)

$$T^{Uc} \vdash LPerc^{LD_{Aux}} w_{Lm}(Lm\mathbf{c}_{CR})\, Bi\,. \quad (20)$$

This means that *big* is the perception of variable *rigidity of labor market* with respect to linguistic description LD_{Aux}.

PROOF: In (b) and (c), the evaluative expression in perception (P2) and (P3) is equal to the evaluative expression in corresponding variables of (9), respectively. Proofs are analogous to (a). □

(c)

$$T^{Uc} \vdash LPerc^{LD_{Aux}} w_{Tl}(Tl\mathbf{c}_{CR})\, Bi\,. \quad (21)$$

It means that *big* is the perception of variable *tax load* with respect to linguistic description LD_{Aux}.

(d)

$$T^{Uc} \vdash Eval\, w_{Af}(Af\mathbf{c}_{AF})\, Bi\,. \quad (22)$$

This deduction means that $\mathbf{c}_{AF}$ is evaluated by evaluative expression *big*. It represents the result of deduction over linguistic description LD_{Aux} – *strength of auxiliary factors* is *big*.

PROOF: This follows from r_{PbLD} and (10), where $\mathbf{c}_{AF}$ is a new constant representing the result of this deduction. □

(e)
$$T^{Uc} \vdash LPerc^{LD_{Un}} w_{Af}(Af\mathbf{c}_{AF})\, Bi\,. \tag{23}$$

It means that *big* is the perception of variable *strength of auxiliary factors* with respect to the linguistic description LD_{Un}.

PROOF: Suppose that in linguistic description LD_{Un} there is no other rule which has in the variable X_{af} the evaluative expression *big*. Then (23) follows from the definition of *LPerc* and (22). □

(f)
$$T^{Uc} \vdash Eval\, w_{Uc}(Uc\, \mathbf{c}_{UC})(-\, Sm). \tag{24}$$

This deduction means that $\mathbf{c}_{UC}$ is evaluated by the evaluative expression *negative small*. It represents the result of deduction over linguistic description LD_{Un} – *rate of unemployment change* is *negative small*.

PROOF: It follows from r_{PbLD} and (13), where $\mathbf{c}_{UC}$ is a new constant. □

Hence, we can conclude that change of unemployment is *negative small*, given that economic growth is *high* and strength of auxiliary factors is *high*.

4.4 Remarks

- At first sight, it may seem that the result of our toy example (change of unemployment is *negative small*) can be read directly from IF-THEN rules and perceptions without using complex formal tools like fuzzy type theory. However, what is easy for humans can be quite difficult for computer systems, which are not able to "see" the result using intuition and experience. Moreover, more complicated texts which involve complex phenomena, generalized quantifiers, a higher number of input and output variables, etc., are usually not that transparent, even for humans. Then, our approach will certainly be quite useful and efficient.

- If, for example, the perception of variable *tax load* would be *small*, then the result of PbLD on linguistic description LD_{Aux} would not be defined, and default rule (12) would be used, resulting in the conclusion that *change of unemployment rate* is *negative big*. It is in accordance with the intuitive rule that default rules should be used whenever there is no further information available.

- An implementation of the theory of evaluative expressions and perception-based logical deduction is available as a part of the software system LFLC (Linguistic Fuzzy Logic Controller, demo version available at `http://irafm.osu.cz`) [4]. This system is designed in such a way that the user is not supposed to be aware of the complicated formal systems presented in previous sections. If he/she provides contexts, linguistic descriptions in the form of IF-THEN rules and perceptions, then deductions from subsection 4.3

are performed automatically. Hence, the method described in this paper is accessible also for users without advanced knowledge of fuzzy logic.

5 Conclusions

In this paper, a model of a quotation from a macroeconomic analysis using perception-based logical deduction was presented, and deductions based on this model were described. It shows that this method can be useful and promising, however, a lot of work is ahead if we want to be able to derive such analyses automatically. In further research, we will concentrate on:

- Methods for automated or semi-automated extraction of linguistic descriptions.
- Methods of automated deduction.
- Use of portions of world knowledge from large collections of it, e.g. Cyc [9] or ConceptNet [8].
- Incorporation of further parts of the theory of nonmonotonic reasoning.
- Modeling of more complicated and longer economic texts.

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