Fuzzy Short-Run Control Charts

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Abstract

Statistical control charts are useful tools in monitoring the state of a manufacturing process. Control charts are used to plot process data and compare it to the limits set for the process. Points plotting outside these limits indicate an out-of-control condition. Standard control charting procedures, however, are limited in that they cannot take into account the case when data is of a fuzzy nature. Another limitation of standard charting methods is when the data produced by the process is short-run data. Often, the situation where the data is short-run occurs in conjunction with data that is considered fuzzy. This paper dicusses the development of a fuzzy control chartting technique, called short-Run α-cut p Control Chart, to account for fuzzy data in a short-run situation. The developed chart parameters accounted for the fuzzy nature of the data in a short-run situation. The parameters were validated by comparing the false alarm rates for various combinations of subgroup numbers (m) and subgroup sizes (n). It was shown that for every combination of m and n, the Short-Run α-cut p Control Chart limits produced a lower false alarm rate than that of the standard fuzzy α-cut control chart.

Keywords: Fuzzy Sets, Statistical Quality Control, Short Run.

1. Introduction

One of the primary tools used in the statistical control of a process is the control chart. Created by Walter Shewhart in 1924, the Shewhart control chart gives a crisp picture of the state of a process by plotting the data produced by a process on a chart bound by upper and lower specification limits [3]. The main function of a control chart is to

monitor a process in order to identify whether or not the process is in control. "In-control" conditions mean that a process is producing parts that are close to the target value with little variation. "Out-of-control" conditions mean that some type of assignable cause has occurred, and the process is, therefore, yielding products at either an unacceptable distance from the target value, with an unacceptable amount of variation, or both. The control chart consists of three lines: an upper control limit (UCL), a lower control limit (LCL), and a center line (CL) (refer to Figure 1). The upper and lower control limits are the maximum and minimum values for a process characteristic to be considered in-control while the center line is the mean value for the process. For Shewhart charts, 3-sigma control limits are used. Three sigma (3σ) control limits establish bounds on the data that extend above and below the mean of the process by 3 times the standard deviation of the process statistic being plotted. Data to be plotted on control charts are obtained directly from the process. Data points falling outside the set limits indicate a possible out-of-control condition in the process [3].
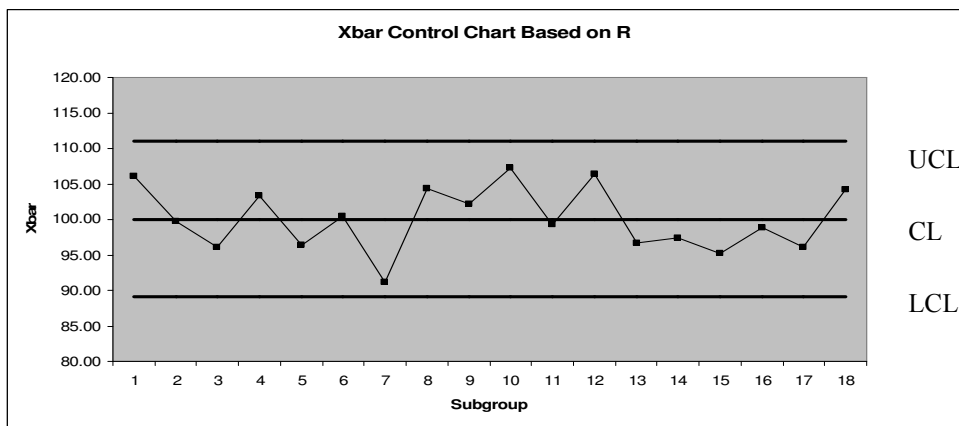

Figure 1: $\overline{X}$ Control Chart Using $\overline{R}$ [1]

The information plotted on control charts consists of either variable or attribute data. Variable data represent measurable characteristics. Examples of variable data are dimensions such as diameters, volumes, or lengths. Attribute data are data that refer to either a pass or a fail situation. In other words, if a product passes inspection, it is considered a pass, and thus, it conforms to the standards outlined for the product. If a product fails, it is considered nonconforming to the standards outlined for the product.

2. The *p*-Chart

In Statistical Quality Control the *p*-chart is used to monitor the fraction of nonconforming units for a process. It models the ratio of nonconforming items in relation to the entire population of process data [3]. Nonconformities, also known as

defects, are attributes that are either absent from the product such as a missing hole or switch, or appear on the product when they should not, such as a scratch, indention, or tear [5]. The CL of the *p* chart is the average of the individual sample nonconforming ratios. The CL is given in Equation 1.

$$CL = \bar{p} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m} \qquad (1)$$

where,

$$\hat{p}_i = \frac{D_i}{n} \qquad (2)$$

The LCL and UCL of the *p* chart are then defined as:

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad (3)$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad (4)$$

3. A *p*- chart Example

To illustrate the characteristics of the p-chart, an example is presented here. The example involves the manufacture of cardboard cans used to pack frozen orange juice. Manufacturers of the cardboard cans consider a can nonconforming if it leaks along one of its seams. The objective is to minimize the fraction nonconforming of cans produced. In order to evaluate the process, 30 samples of 50 cans (*n* = 50) each were obtained. The data produced by the process is given in Table 1 [3].

From this data, the fraction nonconforming ($\hat{p}_i$) for each sample was obtained by dividing the number of nonconforming cans for each sample (D_i) over the total sample size (*n* = 50) (Equation 2).

$$\hat{p}_i = \frac{D_i}{n}$$

The resulting values for each sample are given in Table 2. Next, the center line (CL) for the chart can be obtained by using Equation 1.

$$CL = \bar{p} = \frac{\sum_{i=1}^{m} \hat{p}_i}{m} = \frac{6.94}{30} = 0.2313$$

From the center line calculation, the lower and upper control limits were computed using equations 3 and 4, respectively:

$$LCL = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.2313 - 3\sqrt{\frac{0.2313(0.7687)}{50}} = 0.0524$$

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = 0.2313 + 3\sqrt{\frac{0.2313(0.7687)}{50}} = 0.4102$$

The fraction of nonconforming cans for each sample was then plotted on the control limits. The resulting chart is shown in Figure 2. The chart shows that samples 15 and 23 plot outside the control limits indicating that an out-of-control condition is occurring within the process. The process should then be stopped, and assignable cause should be determined for each out-of-control point. The process should then be returned to statistical control according to the causes found.

Sample Number	Number of Nonconforming Cans, Di	Sample Number	Number of Nonconforming Cans, Di
1	12	16	8
2	15	17	10
3	8	18	5
4	10	19	13
5	4	20	11
6	7	21	20
7	16	22	18
8	9	23	24
9	14	24	15
10	10	25	9
11	5	26	12
12	6	27	7
13	17	28	13
14	12	29	9
15	22	30	6

Table 1: Number Nonconforming for Can Production

4. Short-Run Control Charts

Short-Run Control Charts are used in any situation where very little (i.e., less than 20 subgroups) or no data exists about the process, and therefore, chart parameters cannot be estimated [1]. According to Elam and Case [1], there are three situations in which a short-run control chart should be used instead of a standard control chart. The first situation is when a process has just been initiated, and thus, no data exists about it. The

second is when a process is being monitored that has just been brought into statistical control after having been previously deemed out of control. The third application area is to a process which produces very little data. In each case, traditional charting parameters and constants become insufficient, and a new method is needed.

Sample Number	Sample Fraction Nonconforming, pi	Sample Number	Sample Fraction Nonconforming, pi
1	0.24	16	0.16
2	0.3	17	0.2
3	0.16	18	0.1
4	0.2	19	0.26
5	0.08	20	0.22
6	0.14	21	0.4
7	0.32	22	0.36
8	0.18	23	0.48
9	0.28	24	0.3
10	0.2	25	0.18
11	0.1	26	0.24
12	0.12	27	0.14
13	0.34	28	0.26
14	0.24	29	0.18
15	0.44	30	0.12

Table 2: Fraction Nonconforming for Can Production

The short-run charting method used in this research has a two stage process. In the first stage, a set of data with *m* subgroups (i.e., *m* < 20) of size *n* is collected from the process, and used to estimate the initial parameters of the particular control chart of interest ($\overline{X}, R, p, c$). For the first stage, a set of factors are utilized to account for the small size of the data set being used to estimate the parameters of the process. Therefore, the first stage calculations include the point to be plotted. In the second stage, the subgroups that were not deleted from the data set, after review of the stage 1 results, are used to set the parameters for the process, given the points plotted during this stage do not indicate an out-of-control condition.
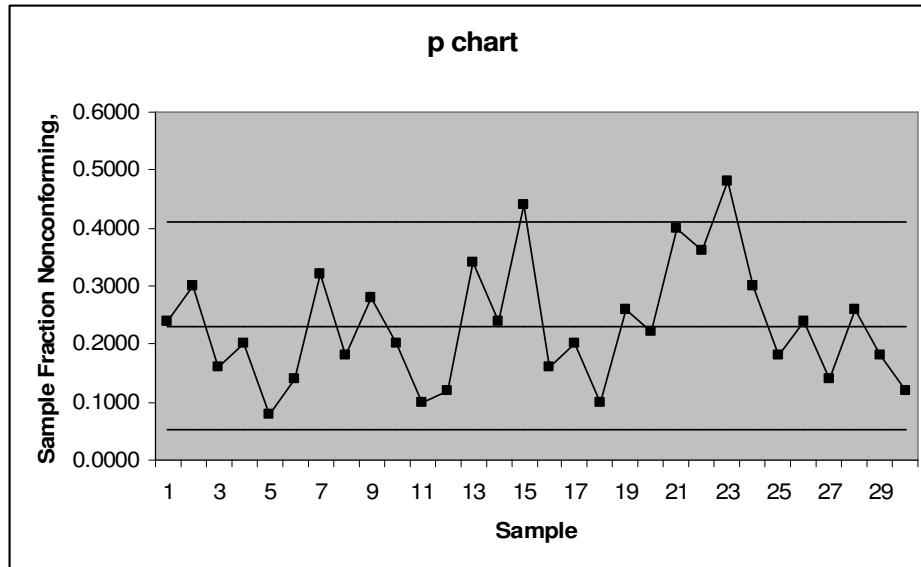
Figure 2: *p* chart for Can Production

The second stage plots a future value of the process and therefore the point being plotted is no longer included in the chart calculations. A set of factors also exist for the second stage to account for the small data set being used to estimate chart parameters [1].

5. Short-run *p* charts

Short-run *p* charts were introduced by Nedumaran and Leon [4]. They proposed that the standard *p* charts could be modified to obtain a chart which would appropriately represent processes where little or no data was available. They suggested that factors k_1 and k_2, for the first and second stages, respectively, could be substituted for the 3-sigma limits in standard *p* charts to obtain charts sensitive to the small amount of data available from the process. Each factor was based on an α value which specified the desired probability of a Type I error, or false alarm. These values were denoted as α_1 and α_2 for the first and second stage factors, respectively. For an α_1 (or α_2) value of 0.0027, both factors approach 3, as the subgroup size *m* increases.   The factors were obtained using knowledge of the probability that a point will plot outside the control limits when the process is in-control [4]. These factors are generally derived in equations 5 through 11.

    1.   Let equations 5 and 6 represent the fraction nonconforming for subgroup *i* and the average fraction nonconforming for the data set,

$$p_i = \frac{D_i}{n} \tag{5}$$

$$\overline{p} = \frac{\sum_{i=1}^{m} p_i}{m} \tag{6}$$

2. Let $\hat{\sigma}_p$ represent the estimate of the standard deviation of the fraction nonconforming for the data set.

$$\hat{\sigma}_p = \sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \tag{7}$$

3. Let α_1 represent the probability of a false alarm, or the probability that a point will plot outside of the control limits when the process is actually in-control.

4. Then, the probability that a point (p_i) will plot within the control limits is $1 - \alpha_1$ or:

$$\Pr\left\{ -k_1 \leq \left(\frac{p_i - \overline{p}}{\hat{\sigma}_p} \right) \leq k_1 \right\} = 1 - \alpha_1 \tag{8}$$

5. Solving for k_1 results in Equation 9.

$$k_1 = z_{\alpha_1/2} \sqrt{\frac{m-1}{m}} \tag{9}$$

The second-stage factor, k_2, was obtained similarly. The probability of a false alarm for the k_2 derivation is α_2. The probability of a point plotting within the control limits is then represented by Equation 10 with p_f representing the fraction nonconforming for a future subgroup f.

$$\Pr\left\{ -k_2 \leq \left(\frac{p_f - \overline{p}}{\hat{\sigma}_p} \right) \leq k_2 \right\} = 1 - \alpha_2 \tag{10}$$

Solving for the factor k_2, Equation 11 is obtained.

$$k_2 = z_{\alpha_2/2} \sqrt{\frac{m+1}{m}} \tag{11}$$

The resulting control limits, after substituting the first and second stage factors, are given in equations 12 through 17.

First-Stage:

$$LCL = \overline{p} - k_1 \sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \tag{12}$$

$$CL = \overline{p} \tag{13}$$

$$UCL = \overline{p} + k_1 \sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \tag{14}$$

Second-Stage:

$$LCL = \overline{p} - k_2 \sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \tag{15}$$

$$CL = \overline{p} \tag{16}$$

$$UCL = \overline{p} + k_2 \sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \tag{17}$$

6. Fuzzification of the *p* Chart

For a *p chart*, the sample mean (M_j) and center line (CL) are defined as:

$$M_j = \frac{\sum_i k_{ij} r_i}{n_j} \qquad j = 1,\ldots,m \tag{18}$$

$$CL = \overline{M}_j = \frac{\sum_{j=1}^{m} M_j}{m} \tag{19}$$

The sample mean, M_j, is essentially a weighted average of the data according to the categories in which the data values are placed. For each of the specified categories, a membership value exists. This membership value serves as the weight for the category in the M_j calculation. In Equation 18, k_{ij} is the number of data values within the category i for the sample j, r_i is the membership value for the particular category i, and n_j is the sample size of sample j.

The equation for the center line (Equation 19) is an average of the sample means. This average results in a fuzzy set where this fuzzy set can be represented by triangular fuzzy numbers (TFNs). Using this representation, the $L_j(\alpha)$ and $R_j(\alpha)$, the left and right plot values, can be defined as in equations 20 and 21.

$$L_j(\alpha) = M_j \alpha \qquad (20)$$

$$R_j(\alpha) = 1 - [(1 - M_j)\alpha] \qquad (21)$$

The resulting membership function of the mean $\overline{M}$, or CL is given in Equation 22.

$$\mu_{M_j}(x) = \begin{cases} 0, & if \ x \le 0 \\ \dfrac{x}{\overline{M}}, & if \ 0 \le x \le \overline{M} \\ \dfrac{1-x}{1-\overline{M}}, & if \ \overline{M} \le x \le 1 \\ 0, & if \ x \ge 1 \end{cases} \qquad (22)$$

The representation of the mean $\overline{M}$ and the sample mean M_j as triangular fuzzy numbers is shown in Figure 3.
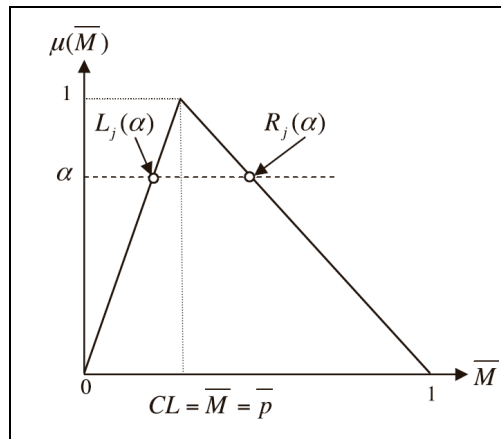

Figure 3: TFN Representation of $\overline{M}$ and M_j [2]

The membership function of the *CL* is divided into two parts to represent the left and right portions of the TFN. Therefore, a *CL*, *UCL*, and *LCL* exist for each half of the *CL*. The resulting control limits are given in Equation 23.

$$ControlLimits(\alpha) = \begin{cases} \begin{cases} CL^L = \overline{M}\alpha \\ LCL^L = max\left\{ CL^L - 3\sqrt{\frac{(CL^L)(1-CL^L)}{\overline{n}}}, 0 \right\} \\ UCL^L = min\left\{ CL^L + 3\sqrt{\frac{(CL^L)(1-CL^L)}{\overline{n}}}, 1 \right\} \end{cases} \\ \begin{cases} CL^R = 1 - \left[(1-\overline{M})\alpha\right] \\ LCL^R = max\left\{ CL^R - 3\sqrt{\frac{(CL^R)(1-CL^R)}{\overline{n}}}, 0 \right\} \\ UCL^R = min\left\{ CL^R + 3\sqrt{\frac{(CL^R)(1-CL^R)}{\overline{n}}}, 1 \right\} \end{cases} \end{cases} \tag{23}$$

The α-cut is represented by the horizontal dotted line shown in Figure 4. This value can be adjusted according to the inspection tightness desired by the quality controller or management personnel. As the value of α approaches 1, the inspection of the data becomes tighter. Similarly, as the value of α approaches 0, the tightness of the inspection limits loosens. In the former case, as the inspection limits tighten, the probability of a point falling outside the control limits increases; however, in the latter, the inspection limits are loosening and the probability of a point falling outside the control limits decreases. With the fuzzy control chart, the basis for deciding whether or not a process is in control is much the same as with traditional charts. The process is considered in-control if both the left and right plot values plot inside the control limits. If either plot value is found outside its respective control limits, the process is considered out-of-control.

7. Fuzzy Short-Run α-cut *p* Control Chart

The parameters of the Fuzzy Short-Run α-cut *p* Control Chart were derived from the principles of the short-run control charts for the fraction nonconforming as well as the reported application of the α-cut method for fraction nonconforming charts derived by Gulbay et al. [2]. The resulting Short-Run α-cut *p* Control Chart provides a method of monitoring processes that are new, have been recently brought back into control, or

contain too little data to monitor by standard methods. With the properties of the α-cut method included, the chart also has the advantage of allowing practitioners to specify differing levels of inspection ranging from 0 to 1, with 0 used for less precise processes, and values closer to 1 used for more intricate processes.

For the standard *p* chart, it was found by Nedumaran and Leon [4] that the factors k_1 and k_2 can be substituted for the 3-sigma limits, assumed for a false alarm probability of 0.0027, to account for the fact that the data is of a short-run nature. This modification of the traditional *p* chart limits was considered for the α-cut chart limits proposed by Gulbay et al. [2]. It was found that since the α-cut method was derived from the same traditional *p* chart limits as the short-run method, and the α-cut method also used 3-sigma limits, the modification using k_1 and k_2 could be used here as well. Therefore, the α-cut limits given in Equation 3.11 were modified for both the first and second stage short-run situations by replacing 3-sigma with the first and second stage short-run factors, respectively. The resulting limits are given in equations 24 and 25.
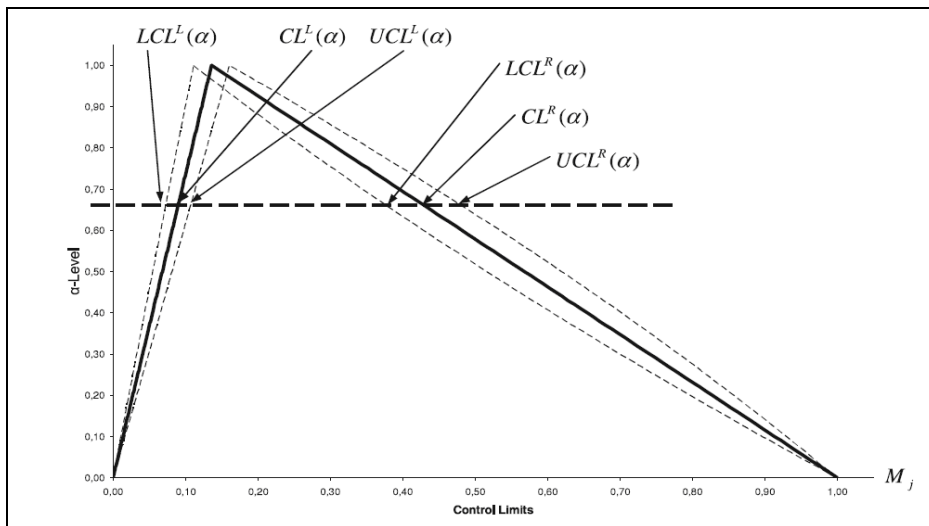

Figure 4: α-cut Control Limits Using Average Sample Size

As with the original α-cut method, the resulting limits produce control charts with both a left and a right-hand side. The interpretation of these charts is also the same as that of the original α-cut method; any points plotting outside the limits of either the left or right-hand sides are considered indications of an out-of-control condition.

First-Stage:

$$ControlLim\ its(\alpha) = \begin{cases} \begin{cases} CL^L = \overline{M}_\alpha \\ LCL^L = \max\left\{ CL^L - k_1 \sqrt{\dfrac{(CL^L)(1-CL^L)}{\overline{n}}}, 0 \right\} \\ UCL^L = \min\left\{ CL^L + k_1 \sqrt{\dfrac{(CL^L)(1-CL^L)}{\overline{n}}}, 1 \right\} \end{cases} \\ \begin{cases} CL^R = 1 - \left[(1-\overline{M})\alpha\right] \\ LCL^R = \max\left\{ CL^R - k_1 \sqrt{\dfrac{(CL^R)(1-CL^R)}{\overline{n}}}, 0 \right\} \\ UCL^R = \min\left\{ CL^R + k_1 \sqrt{\dfrac{(CL^R)(1-CL^R)}{\overline{n}}}, 1 \right\} \end{cases} \end{cases} \quad (24)$$

Second-stage:

$$ControlLim\ its(\alpha) = \begin{cases} \begin{cases} CL^L = \overline{M}_\alpha \\ LCL^L = \max\left\{ CL^L - k_2 \sqrt{\dfrac{(CL^L)(1-CL^L)}{\overline{n}}}, 0 \right\} \\ UCL^L = \min\left\{ CL^L + k_2 \sqrt{\dfrac{(CL^L)(1-CL^L)}{\overline{n}}}, 1 \right\} \end{cases} \\ \begin{cases} CL^R = 1 - \left[(1-\overline{M})\alpha\right] \\ LCL^R = \max\left\{ CL^R - k_2 \sqrt{\dfrac{(CL^R)(1-CL^R)}{\overline{n}}}, 0 \right\} \\ UCL^R = \min\left\{ CL^R + k_2 \sqrt{\dfrac{(CL^R)(1-CL^R)}{\overline{n}}}, 1 \right\} \end{cases} \end{cases} \quad (25)$$

8. Example of a Fuzzy Short-Run α-cut *p* Control Chart

Using the data provided by Gulbay et al. [2] in his example of the standard α-cut *p* Control chart (Table 3), the short-run limits were applied.

Sample	Standard	Second Choice	Third Choice	Chipped	Size (n)	M_i
1	144	46	12	5	207	0.109
2	142	50	9	5	206	0.107
3	142	35	16	6	199	0.114
4	130	70	19	10	229	0.162
5	126	60	15	10	211	0.154
6	112	47	9	8	176	0.138
7	151	28	22	9	210	0.129
8	127	43	45	30	245	0.258
9	102	79	20	3	204	0.161
10	137	64	24	5	230	0.143
11	147	59	16	6	228	0.126
12	146	30	6	6	188	0.088
13	135	51	16	8	210	0.137
14	186	82	23	7	298	0.131
15	183	53	11	9	256	0.108
16	137	65	26	4	232	0.143
17	140	70	10	3	223	0.114
18	135	48	15	9	207	0.138
19	122	52	23	10	207	0.167
20	109	42	28	9	188	0.178
21	140	31	9	4	184	0.088
22	130	22	3	8	163	0.092
23	126	29	11	8	174	0.119
24	90	23	16	2	131	0.120
25	80	29	19	8	136	0.182
26	138	55	12	12	217	0.146
27	121	35	18	10	184	0.151
28	140	35	15	6	196	0.114
29	110	15	9	1	135	0.069
30	112	37	28	11	188	0.182

Table 3: Data for Short-Run α-cut p Control Chart [2]

As with the example of short-run control charts, 10 subgroups were examined initially with 10 additional subgroups added to each succeeding examination of the data. The examination of the first 10 subgroups was conducted, and the chart parameters determined. Since the subgroup size differed over the subgroups, the chart parameters were not constant but varied with each subgroup. As an example of the computations used to obtain the chart parameters, the calculation of the parameters for the first subgroup are given below. The value for inspection tightness, α, was assumed, as in the original Gulbay et al. [2] example, as α = 0.30.

$$CL = \overline{M} = \frac{\sum_{i=1}^{10} M_j}{10} = \frac{1.474}{10} = 0.147$$

Left-Hand Limits:

$$CL^L = \overline{M}\alpha = 0.147 * 0.30 = 0.044$$

$$LCL^L = \max\left\{ CL^L - k_2 \sqrt{\frac{(CL^L)(1-CL^L)}{n}}, 0 \right\}$$

$$= \max\left\{ 0.044 - 3.146\sqrt{\frac{0.044(1-0.044)}{207}}, 0 \right\}$$

$$= 0.000$$

$$UCL^L = \min\left\{ CL^L + k_2 \sqrt{\frac{(CL^L)(1-CL^L)}{n}}, 1 \right\} = \max\left\{ 0.044 + 3.146\sqrt{\frac{0.044(1-0.044)}{207}}, 1 \right\}$$

$$= 0.089$$

Right-Hand Limits:

$$CL^R = 1 - [(1-\overline{M})\alpha] = 1 - [(1-0.147) * 0.30] = 0.744$$

$$LCL^R = \max\left\{ CL^R - k_2 \sqrt{\frac{(CL^R)(1-CL^R)}{n}}, 0 \right\}$$

$$= \max\left\{ 0.744 - 3.146\sqrt{\frac{0.744(1-0.744)}{207}}, 0 \right\}$$

$$= 0.648$$

$$UCL^R = \min\left\{ CL^R + k_2 \sqrt{\frac{(CL^R)(1-CL^R)}{n}}, 1 \right\}$$

$$= \max\left\{ 0.744 + 3.146\sqrt{\frac{0.744(1-0.744)}{207}}, 1 \right\}$$

$$= 0.834$$

The second-stage factor, k_2, was calculated as:

$$k_2 = z_{\alpha_2/2}\sqrt{\frac{m+1}{m}} = z_{.0027/2}\sqrt{\frac{10+1}{10}} = 3\sqrt{\frac{11}{10}} = 3.146427$$

The resulting chart for the first 10 subgroups is given in Figure 5. Since the chart, in its standard form, is difficult to interpret because of the wide gap between the left and right hand side portions, the graph was split into its respective components for closer examination. These charts are shown in Figures 6 and 7. The charts for the first 10 subgroups do not indicate an out-of-control condition, and therefore, the next 10 subgroups are combined with the first 10 for a second evaluation. Therefore, the chart parameters for the second evaluation were based on an average of all 20 subgroups. The results are given in Figure 8 with the left and right-hand sides displayed in figures 9 and 10.
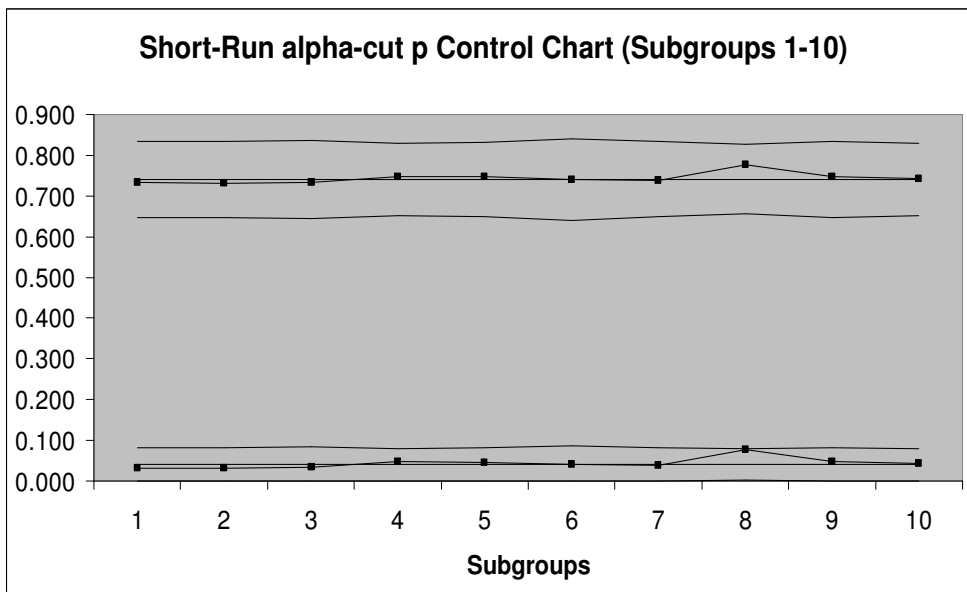
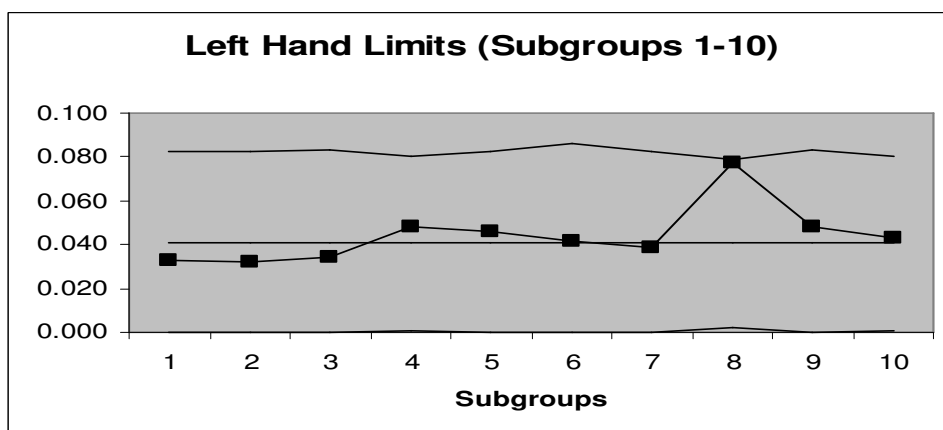Figure 5: Short-Run α-cut *p* Control Chart (Subgroups 1-10)

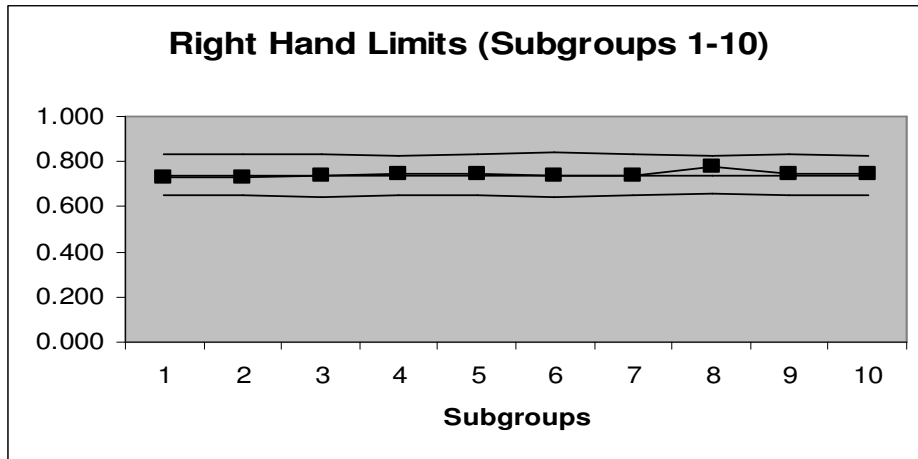Figure 6: Left-Hand Portion, Subgroups 1-10

Right Hand Limits (Subgroups 1-10)

Figure 7: Right-Hand Portion, Subgroups 1-10

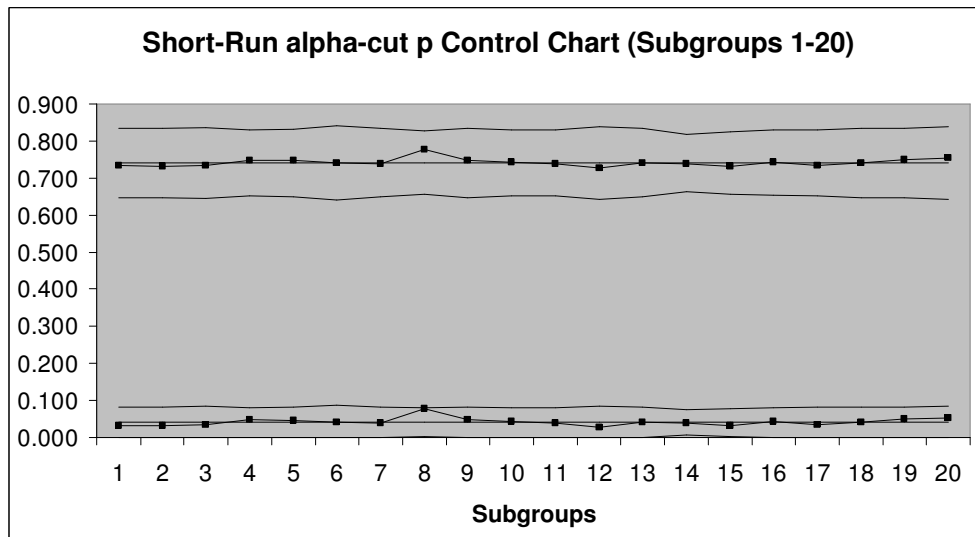**Short-Run alpha-cut p Control Chart (Subgroups 1-20)**

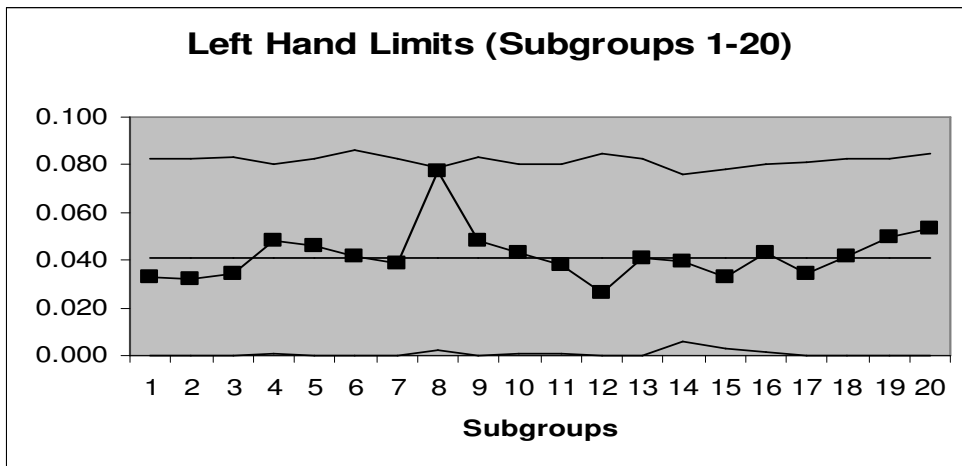Figure 8: Short-Run α-cut *p* Control Chart (Subgroups 1-20)
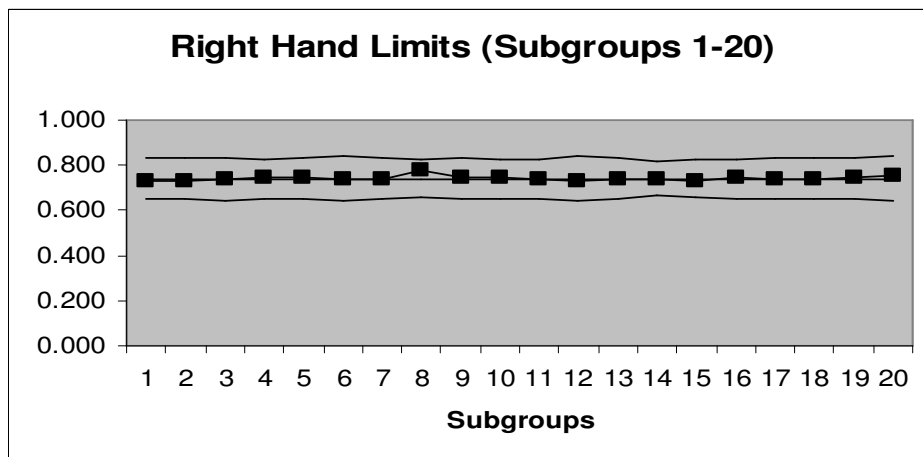
Figure 9: Left-Hand Portion, Subgroups 1-20

Figure 10: Right-Hand Portion, Subgroups 1-20

Again, no data points plotted outside of the control limits, and hence, the next set of subgroups is added to the evaluation. The results for the examination of all 30 subgroups are given in figures 11 through 13. The charts for all 30 subgroups show that no data points fall outside the control limits. Since greater than 20 subgroups were examined, with no data points plotting outside the control limits, the parameters are considered to be an accurate measure of the condition of the process.
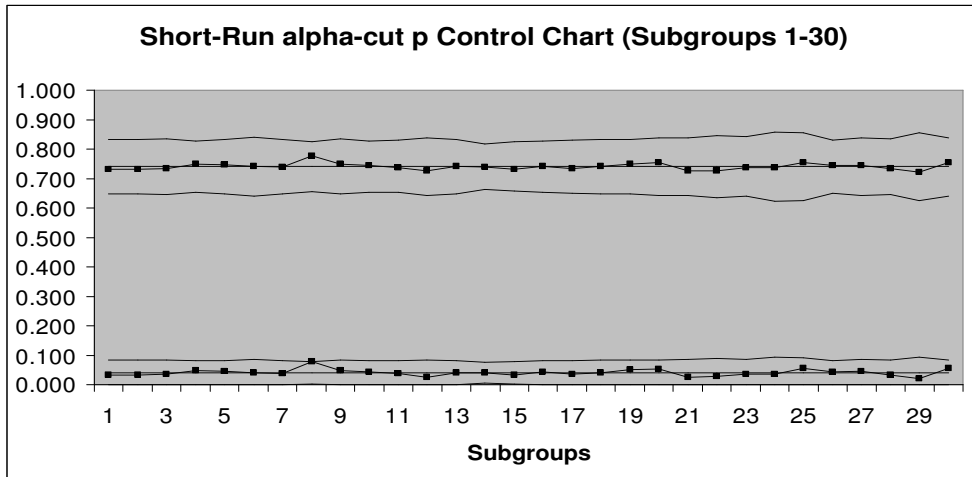
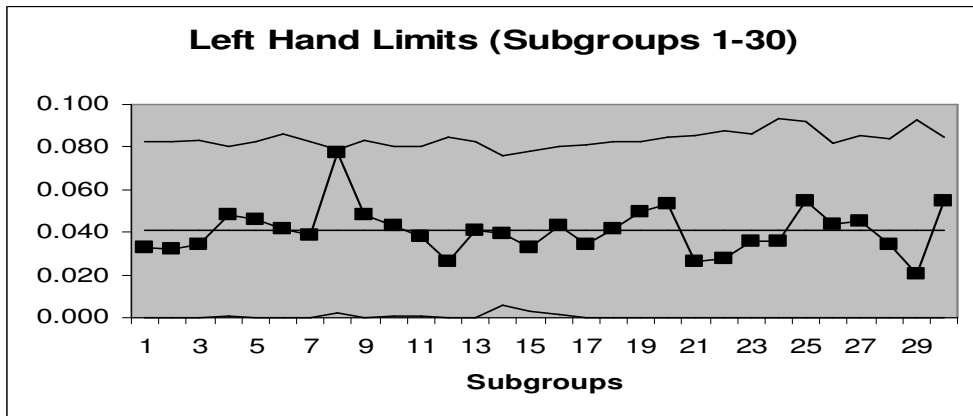Figure 11: Short-Run α-cut *p* Control Chart (Subgroups 1-30)
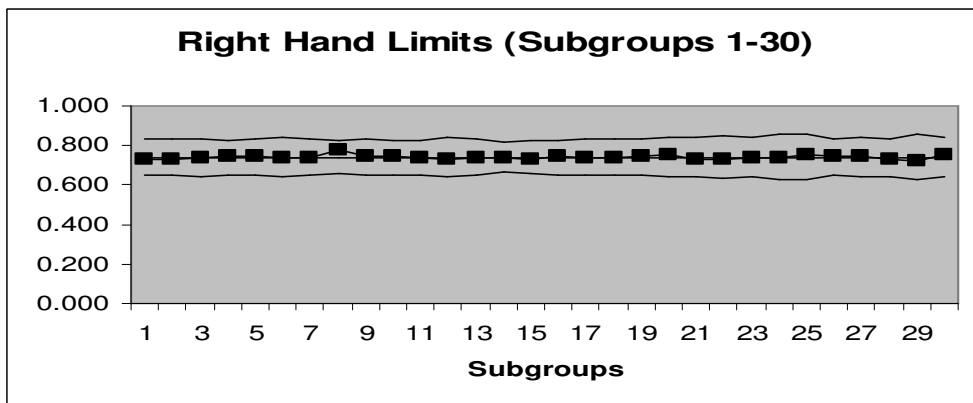
Figure 12: Left-Hand Portion, Subgroups 1-30

Figure 13: Right-Hand Portion, Subgroups 1-30

9. Validation of the Fuzzy Short-Run α-cut p Control Chart

The validation of the Short-Run α-cut p Control Chart was executed by testing the false alarm rate produced by simulated data when the algorithms for the Short-Run α-cut p Control Chart limits were used. The false alarm rate is the Type I error for the process. The resulting false alarm rates were then compared to the false alarm rate of the standard α-cut control chart for the same combination of m and n. The goal of this comparison was to prove that for any combination of m and n, the false alarm rate of the Short-Run α-cut p Control Chart would be lower than that of the standard α-cut p control chart. For the purposes of this research, the second-stage calculations, where future data values are plotted, is only of concern. First, the fuzzy short-run control limit calculations were used to construct control limits for combinations of m subgroups and n subgroup sizes. In order to test the limits specifically for the short-run case (i.e., $m < 20$), the combinations were restricted to be less than or just above 20 subgroups. Control limits were constructed for all combinations of $m = 5, 6, 7, 8, 9, 10, 15, 20,$ and 25, and $n = 5, 6, 7, 8, 9, 10, 15, 20,$ and 25.

   Once the second-stage limits were obtained for each of the subgroup and subgroup size combinations, a simulation was performed to test the performance of the limits. The simulation was conducted using FORTRAN 90. Several data elements used in the simulation were initialized. The initialized values included the 3 p values used for data classification, and the membership values for each category. Also, before each simulation was conducted, the FORTRAN prompt screen queried the user for the right-side LCL, right-side UCL, left-side LCL, left-side UCL, the subgroup size n, and the α value to be used. For all charts and simulations, an α value of 0.30, used in the Gulbay et al. [2] example, was used. For the simulation, random data from a Uniform (0,1) distribution was produced. Once the data was generated, it was evaluated by a set of conditional statements. The conditional statements used the p values employed in the construction of the control limits to determine in which category to place the data. The p values were $p_1 = 0.70$, $p_2 = 0.95$, and $p_3 = 0.985$. From these three p values, four categories were produced. These categories were again obtained from an example given by Gulbay et al. [2], and were labeled "Standard", "Second Choice", "Third Choice", and "Chipped". If a p value produced by the random number generator was less than 0.7, it was then classified as a "Standard" product. If the p value was between 0.7 and 0.95, it was classified as "Second Choice". If the p value was between 0.95 and 0.985, it was classified as "Third Choice". Finally, if the p value was greater than 0.985, it was classified as "Chipped". Counters were used to separate each p value into its respective group, with one counter for each of the four categories. The creation and classification of random uniform values was repeated n times. The n data values produced and classified in the first step represented one subgroup. These data values were then used to calculate right and left side plot values for each subgroup. The plot values were calculated as in the Gulbay et al. [2]'s example by first computing the weighted average (M_j) of the count for each data category, with the membership values for each category serving as the respective weights. From this weighted average, the left side plot value

was obtained by multiplying m times the provided α value. The right side plot value was obtained by a slightly more complicated equation. The calculation of the M_j value, and the left and right side plot values is given in equations 26 through 28.

$$M_j = \frac{\sum_i k_{ij} r_i}{n_j} \qquad j = 1,\ldots,m \qquad (26)$$

In Equation 26, k_{ij} is the number of data values in category i for the particular sample j, r_i is the membership value representing the particular category, and n_j is the subgroup size of subgroup j. The value of n_j can also be replaced by $\bar{n}$, the average subgroup size for all m subgroups, to obtain a constant upper and lower control limit. The average sample size approach is used in the validation of the data.

$$Left - Side\ PlotValue = M * \alpha \qquad (27)$$
$$Right - Side\ PlotValue = 1 - [(1 - M) * \alpha] \qquad (28)$$

Once the left and right side plot values were calculated, they were compared to the lower and upper control limits entered for the particular subgroup size, and if the values plotted outside these limits, a separate counter was increased by 1. This creation of n data values was repeated for 10,000 subgroups. After the conclusion of these repetitions, the number of points plotting outside the control limits was totaled. This process was repeated 1,000 times and the false alarm rate was calculated by dividing the sum of the points plotting outside the control limits by the number of subgroups (10,000) times, the number of repetitions of the entire program (1,000). This value represented the false alarm rate of the process for the given subgroup size.

The false alarm rate for each combination of m and n was obtained. The false alarm rates for both the Short-Run α-cut p Control Chart, and the standard α-cut p Control Chart [2] were obtained. For some combinations, the false alarm rate equaled 0.00. Only the false alarm rates which were greater than 0.00 are shown in Table 4. The false alarm rates in Table 4 show that for every combination of m and n, the rate for the short-run control limits was lower than the rate for the standard α-cut control limits. Therefore, for every combination of m and n, the short-run control limits produced a lower false alarm rate than the α-cut limits suggested by Gulbay et al. [2].

m	*n*	Short-Run	Gulbay	*m*	*n*	Short-Run	Gulbay
5	5	0.0000194	0.0000553	8	7	0.0000000	0.0000006
5	9	0.0000147	0.0000470	9	5	0.0000000	0.0000002
5	10	0.0000043	0.0000145	9	8	0.0000000	0.0000002
5	25	0.0000050	0.0000050	9	9	0.0000001	0.0000005
6	5	0.0000194	0.0000553	10	5	0.0000002	0.0000002
6	6	0.0000024	0.0000065	10	7	0.0000000	0.0000006
6	8	0.0000178	0.0000518	10	8	0.0000003	0.0000019

7	6	0.0000065	0.0000206	15	10	0.0000001	0.0000001
7	7	0.0000000	0.0000006	20	5	0.0000000	0.0000002
8	5	0.0000000	0.0000002	20	10	0.0000001	0.0000001
8	6	0.0000000	0.0000001	25	6	0.0000008	0.0000024

Table 4: False Alarm Rates for Short-Run and Standard α-cut Control Charts

10. Conclusion

The objective of this research was to modify the α-cut procedure for fraction nonconforming control charts to account for the case when data collected from the process is of a short-run nature. The results of the study included algorithms for all chart parameters involving the center line, plot values, and upper and lower control limits. The Short-Run α-cut p Control Chart parameters were validated and verified, and have been proven successful in producing results that exceed that of the standard α-cut p control chart by Gulbay et al. [2].

References

[1] Elam, Matthew E. and Case Kenneth E. "Two-stage short-run ($\overline{X}$,s) control charts." *Quality Engineering* Vol.17, 2005, pp.95-107.

[2] Gulbay, Murat, Kahraman, Cengiz and Ruan, Da. "α-cut fuzzy control charts for linguistic data." *International Journal of Intelligent Systems* Vol.19, 2004, pp.1173-1195.

[3] Montgomery, Douglas C. *Introduction to Statistical Quality Control*. New York: John Wiley & Sons, Inc., 2001.

[4] Nedumaran, Gunabushanam and Leon, Jorge V. "*p*-chart control limits based on a small number of subgroups." *Quality Engineering* Vol.11, No.1, 1998, pp.1-9.

[5] "Quality Advisor: Attributes Data." PQ Systems. http://www.qualityadviosor.com/sqc/attributes_data.php (accessed 5 May 2006).