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Formalization of Provenes Fuzzy Functional Dependancy in Fuzzy Databases

Nedžad Dukić, Zikrija Avdagić
Fac. Science, Dept. Mathematic, Fac. Electrical Engineering,
University of Sarajevo, Bosnia and Herzegovina
ndukic@pmf.unsa.ba

Abstract

In this paper we establish equivalence between a theory of fuzzy functional dependences and a fragment of fuzzy logic. We give a way to interpret fuzzy functional dependences as formulas in fuzzy logic. This goal is realized in a few steps. Truth assignment of attributes is defined in terms of closeness between two tuples in a fuzzy relation. A corresponding fuzzy formula is associated to a fuzzy functional dependence. It is proved that if a relation satisfies a fuzzy functional dependence, then the corresponding fuzzy formula is satisfied and vice verse. Finally, equivalence of a fuzzy formulas and a set fuzzy functional dependence is demonstrated. Thus we are in position to apply the rule of resolution from fuzzy logic, while calculating fuzzy functional dependences.

Keywords: Fuzzy relation database; Fuzzy functional dependency; Fuzzy logic; Resolution.

1 Introduction

According to the classic relation database all the information in it, have to involve precisely defined values (atomic). So in a case that those values are not defined precisely then the imprecise values could be involved as one value, so called NULL.

Codd [3] considers the NULL value in a meaning 'completely unknown' i.e. some values of attribute domain could have this meaning.

Lipski [9] extended the of Codd's null value by considering that a value though unknown is in a specific subset of the attribute domain.

In some other study extension, variety of null values have been introduced to model unknow or not-applicable data values. As an alternative approach is the usage of first order predicate calculus where Skolem functions are used to represent null values.

The other way of considering this imprecise information is the involving of fuzzy value to the domain of attribute. These imprecise information have been focused on Zadeh's fuzzy set theory and fuzzy logic. The fuzzy set theory and fuzzy logic provide mathematical framework to deal with the imprecise information in a fuzzy relational databases.

Approaches to representation of inexact information in relation database theory, include *fuzzy membership values* [1,4,18], *similarity relationships* [2,15] and possibility distributions. This paper takes the similarity-based fuzzy relational database approach.

In a fuzzy set, each element of the set has an associated degree of membership. The degree of membership is a real number between zero and one and measure the extent to which an element is in a fuzzy set [18].

As an extension of the degree of membership concept for sets elements, we have similarity relationship. Here the domain elements are considered as having varying degrees of similarity, replacing the idea of exact equality / inequality.

To deal with fuzzy data constraint, Zadeh has introduced the concept of particularization (restriction) of fuzzy relation due to a fuzzy proposition. The formed formulas of first order calculus can be used to represent integrity constraints in a classical relational databases [3,16], fuzzy integrity constraint can be represented by suitable fuzzy propositions. The particularization of fuzzy relational database due to a set of fuzzy integrity constraints can be computed by combining the fuzzy propositions associated with these integrity constraints according to the rules of fuzzy calculus.

Our primary aim in this paper is to establish a connection between theory of fuzzy functional dependence and one fragment of fuzzy logic. So it will be shown that if relation r satisfies fuzzy functional dependence then is truth value of the belonging fuzzy formula is greater or is equal to 0.5 and vice verse.

If we have some set of fuzzy functional dependences it will be possible to show whether or not some other fuzzy dependences will follow, in a way of using, the correponding axioms and inferences rules for fuzzy functional dependence. However such deduction could be very complicited, because it is not obvious which axioms has be to selected in its phase, and there isn't some globaly strategy for valid results. But in classic logic as in fuzzy logic there is effective procedure, which from its starting set of formulas as well as its logic consequence shows validity of given formula. Such a procedure is known as the rule of resolution [6,7,12].

Therefore it will be here established the equivalence of calculation of one part of fuzzy logic and fuzzy functional dependence. After establishing this equivalence, then is possible to apply the rules of deduction in fuzzy logic, on calculus of fuzzy functional dependence.

2 Similirarity-based fuzzy relational database

As in an ordinary relational database, the constituent parts of a fuzzy relational database are a set of relations comprised of tuples. Although tuples are not ordered with respect to a relation , for convenience, let t_i represent the i - th tuple. Tuple t_i takes the form $(d_{i1}, \ldots, d_{im})$ where d_{ij}, a domain value, is selected from a given domain set , D_j. In an ordinary relational databas, $d_{ij} \in D_j$. In the fuzzy relational databas, d_{ij} is not constrained to be a singleton, that is $d_{ij} \subseteq D_j$ (but $d_{ij} \neq \emptyset$).

A second feature of the fuzzy relational database [1,2,4] is that for each domen set, D_j, a similarity relation, s_j, is defined over the set of elements : $s_j : D_j \times D_j \rightarrow [0,1]$. This relations is a generalization of equivalence relations in that if a,b,c $\in D_j$ then s_j is

1) reflexive : $s_j(a,a) = 1$,
2) symmetric : $s_j(a,b) = s_j(b,a)$ and
3) transitive : $s_j(a,c) \geqslant \max[\min_{\forall b \in D_j} s_j(a,b), s_j(b,c))]$.

Clearly, the identity relation is a special case of the similarity relations.

In the fuzzy relational database domain values need not to be atomic. A domain value, d_{ij} is defined to be a subset of its domain base set, D_j. That is, any member of the powerset, 2^{D_j}, may be a domain value except the null set.

A fuzzy relation instance, **r**, in the fuzzy database model is defined as a subset of the set cross product of the power sets ($2^{D1} \times \ldots \times 2^{Dm}$) of the domains attributes [1-2].

A fuzzy tuple, t, is any member of both fuzzy relation r and $2^{D1} \times \ldots \times 2^{Dm}$. An arbitrary tuple, t_i, is of form $t_i = (d_{i1}, \ldots, d_{im})$, where d_{ij} is either a nonempty subset of D_j or an element such $d_{ij} \subseteq D_j$.

3 Introduction in fuzzy functional dependency (FFD)

In the classic relation database functional dependency [3,16] is a statement that describes a semantic constraint on data.

Let r be any relation instance on scheme $R(A_1, \ldots, A_n)$, U be the universal set of attributes $A_1, \ldots, A_n$, and both X and Y be subset of U. Relation instance r is said to satisfy the functional dependency $X \rightarrow Y$ if, for ever pair of tuples t_1 and t_2 in r, $t_1[X] = t_2[X]$ implies $t_1[Y] = t_2[Y]$.

But the definition of functional dependency is not directly applicable to fuzzy relational database because it is based on the concept of equality. Functional dependency $X \rightarrow Y$, in classical database states that if $t_1[X] = t_2[X]$ then must be $t_1[Y] = t_2[Y]$. There is no clear way of checking whether two imprecise values are equal. Therefore the definiton of functional dependency have to be extended namely to be generalized and this generalization version of functional dependency is said to be the fuzzy functional dependency (FFD).

There are several way in corrected definition of fuzzy functional dependency [8,13-15,17]. One of the important definition for fuzzy functional dependences was presented in paper [15]. In that paper firstly was defined conformance of two tuples in relation.

Definition 3.1. The conformance of attribute A_k defined on domain D_k for any two tuples t_i and t_j present in relation instance r and denoted by $\varphi(A_k[t_i,t_j])$ is given as

$$\varphi(A_k[t_i,t_j]) = \min\left\{\min_{x\in d_i}\left\{\max_{y\in d_j}\{s(x,y)\}\right\}, \min_{x\in d_j}\left\{\max_{y\in d_i}\{s(x,y)\}\right\}\right\}$$

where d_i is the value of attribute A_k for tuple t_i, d_j is the value of attribute A_k for tuple t_j, s(x,y) is a similarity relation for values x and y, and s is mapping of every pair of elements in the domain D_k onto interval [0,1].

The definition of conformance is also extended to describe the closeness of two tuples on set of attributes.

Definition 3.2. The conformance of attribute set X for any two tuples t_i and t_j present in relation instance r and denote by $\varphi(X[t_i,t_j])$ is given as

$$\varphi(X[t_i,t_j]) = \min_{A_k \in X}\{\varphi(A_k[t_i,t_j])\}$$

3.1 Properties of conformance

Proposition 3.1.1. If$X \supseteq Y$,then $\varphi(Y[t_i,t_j]) \geqslant \varphi(X[t_i,t_j])$ for any t_i and t_j in r.

Proposition 3.1.2. If X = $\{A_1,\ldots,A_n\}$ and $\varphi(A_k[t_i,t_j]) \geqslant \theta$,for all k , $1 \leqslant k \leqslant n$, then $\varphi(X[t_i,t_j]) \geqslant \theta$ for any t_i and t_j in r, $\theta \in$[0,1].

Proposition 3.1.3. If $\varphi(X[t_i,t_j]) \geqslant \theta$, and $\varphi(X[t_j,t_k]) \geqslant \theta$, then $\varphi(X[t_i,t_k]) \geqslant \theta$, for any t_i, t_j and t_k in r, $\theta \in$[0,1].

3.2 Fuzzy functional dependencis (FFD)

Definition 3.2.1. Let r be any fuzzy relation instance on scheme R($A_1,\ldots,A_n$), U be the universal set of attributes $A_1,\ldots,A_n$, and both X and Y be subsets of U. Fuzzy relation instance r is said to satisfy the fuzzy functional depedency (FFD)$X \xrightarrow[F]{\theta} Y$ if , for every pair of tuples t_1 and t_2 in r,

$$\varphi(Y[t_1,t_2]) \geqslant \min(\theta, \varphi(X[t_1,t_2])).$$

Here , θ is a real number within the range [0,1], describing the linguistic strength [15,17].

3.3 Inference rules for fuzzy functional dependency

IR1 ***Inclusive*** rule for fuzzy functional dependency :
If $X \xrightarrow[F]{\theta_1} Y$ and $\theta_1 \geqslant \theta_2$, then $X \xrightarrow[F]{\theta_2} Y$ holds.
IR2 ***Reflexive*** rule for fuzzy functional dependency :
If $X \supseteq Y$, then $X \xrightarrow[F]{} Y$ holds.
IR3 ***Augmentation*** rule for fuzzy functional dependency:

$$\{X \xrightarrow[F]{\theta} Y\} \Rightarrow XZ \xrightarrow[F]{\theta} YZ$$

IR4 ***Transitivity*** rule for fuzzy functional dependency:

$$\{X \underset{F}{\overset{\theta_1}{\rightarrow}} Y, Y \underset{F}{\overset{\theta_2}{\rightarrow}} Z\} \Rightarrow X \underset{F}{\overset{\min(\theta_1,\theta_2)}{\rightarrow}} Z.$$

3.4. Additional inference rules for fuzzy functional dependency

IR5 *Union* rule for fuzzy functional dependency :

$$\{X \underset{F}{\overset{\theta_1}{\rightarrow}} Y, X \underset{F}{\overset{\theta_2}{\rightarrow}} Z\} \Rightarrow X \underset{F}{\overset{\min(\theta_1,\theta_2)}{\rightarrow}} YZ$$

IR6 *Pseudotransitivity* rule for fuzzy functional dependency :

$$\{X \underset{F}{\overset{\theta_1}{\rightarrow}} Y, WY \underset{F}{\overset{\theta_2}{\rightarrow}} Z\} \Rightarrow WX \underset{F}{\overset{\min(\theta_1,\theta_2)}{\rightarrow}} Z$$

IR7 *Decomposition* rule for fuzzy functional dependency :
If $X \underset{F}{\overset{\theta}{\rightarrow}} Y$ holds and $Z \subseteq Y$, then $X \underset{F}{\overset{\theta}{\rightarrow}} Z$ holds.

4 Fuzzy Logic and Resolution Principle

Fuzzy logic is based on the concepts of fuzzy sets and symbolic logic. Logic operators of conjuction, disjunction and negation are defined as folows,
a) $x_1 \wedge x_2 = \min(x_1, x_2)$
b) $x_1 \vee x_2 = \max(x_1, x_2)$
c) $\neg x = 1 - x$,
where x_i (i=1,2,...,n) variable in [0,1] [5-7,10-12].

In fuzzy logic, the truth value of a formula, can assume any value in the interval [0,1] and is used to indicate the degree of truth represented by the formula.

4.1. Satisfiability in Fuzzy Logic

Definition 4.1.1. A formula f ∈ S , where is S set of a fuzzy formulas, is said to satisfy in interpretation I, if truth value of a formula T(f) $\geqslant$ 0.5 under I. An interpretation I is said to falsity S if T(f) $\leqslant$ 0.5

A formula is said to be unsatisfiable if it is falsified by every interpretation of it [7].

Definition 4.1.2. Let D_1 : $L_1 \vee D_1$‘ and D_2 : $L_2 \vee D_2$‘ be two disjuncts, and L_1 and L_2, contra pair of literals i.e. L_2 : ¬ L_1 and let D_1‘ and D_2‘ do not contain any such pair. Then, disjunct D_1‘$\vee D_2$‘ is said to be resolvent disjuncts D_1 and D_2 with the key word L_1.

Let S be a set of clauses. The resolution of S, denoted Res(S), is the set consisting of members of S together with all the resolvents of the pairs of members of S. The *n*th resoluton of S, denoted $\text{Res}^n(S)$, is defined for n $\geqslant$0 as follows :

$\text{Res}^0(S) = S$ and $\text{Res}^{n+1}(S) = \text{Res}(\text{Res}^n(S))$.

5 Main results : Fuzzy functional dependency and fuzzy formulas

In this section we establish a connection between fuzzy logic and the theory of fuzzy functional dependencies. We give a way to interpret fuzzy functional dependencies as formulas in fuzzy logic. For a set of fuzzy dependencies **F** and single fuzzy functional dependency **f** , we show that **F** implies **f** as fuzzy functional dependencies if and only if **F** implies **f** under the logic interpretation.

The correspondence between fuzzy functional dependencies and fuzzy formulas is direct. Let $X \xrightarrow[F]{\theta} Y$ be an fuzzy functional dependencies where $X = A_1A_2 \ldots A_m$ and $Y = B_1B_2 \ldots B_n$. The corresponding logical formula is $(A_1 \wedge A_2 \wedge \ldots \wedge A_m) \rightarrow (B_1 \wedge B_2 \wedge \ldots \wedge B_n)$

For determination of truth assignment attribute in relation r, we take definition of *conformance* the two tuples on attribute.

Let r be a fuzzy relation over schema R with exactly two tuples . Fuzzy relation r can be used to define a truth assignment, for attributes in R when they are considered as fuzzy variables.

Definition 5.1. Let $R = \{A_1, A_2, \ldots, A_m\}$ be a relation schema and let $r = \{t_1, t_2\}$ be a two tuple relation on R. The truth assignment for r, denoted i_r, is the function from R to [0,1] defined by

$$i_r(A_k)\begin{cases} > 0.5 & if \min\left\{\min\limits_{x\in d_i}\left\{\max\limits_{y\in d_j}\{s(x,y)\}\right\},\right. \\ & \left.\min\limits_{x\in d_j}\left\{\max\limits_{y\in d_i}\{s(x,y)\}\right\}\right\} \geqslant \theta \in [0,1]; \\ \leqslant 0.5 & if\ \varphi(A_k[t_i,t_j]) \prec \theta. \end{cases}$$

where d_i is the value of attribute A_k for tuples t_i, d_j is the value of attribute A_k for tuple t_j, s(x,y) is a similarity relation for values x and y, s is mapping of every pair of elements in the domain D_k onto interval [0,1] and θ is strenght of the dependency.

The following theorem enables equivalence between fuzzy functional dependence and fuzzy formulas. So by that theorem will be proved the mentioned equivalence when for the fuzzy formulas are taken the following

$X \rightarrow Y = \max(1-X, Y)$(Kleen-Dienes)

$X \rightarrow Y = \max(\min(X,Y), 1-X)$ (Zadeh).

Theorem 5.1. Let $X \xrightarrow[F]{\theta} Y$ be a FFD over relation scheme R and let r be relation on R with two typles. A FFD $X \xrightarrow[F]{\theta} Y$ is satisfied by relation r if and only if $X \rightarrow Y$ is satisfy under the truth assignments $\mathbf{i}_r$.

Proof.

a) For Kleens-Diens implication $X \rightarrow Y = max(1\text{-}X, Y)$.

Let assume, as first , that relation r satisfies FFD $X \xrightarrow[F]{\theta} Y$ i.e. let be hold

$\varphi(Y[t_1,t_2]) \geqslant \min(\theta, \varphi(X[t_1,t_2])$

where is X = $\{A_1, A_2, \ldots, A_m\}$ and Y = $\{B_1, B_2, \ldots, B_n\}$.

Let assume contra to theorem assertion that assigments

F : $(A_1 \wedge A_2 \wedge, \ldots, \wedge A_m) \rightarrow (B_1 \wedge B_2 \wedge, \ldots, \wedge B_n)$

is falsify in interpretation $i_{r'}$.

Then follows that in interpretation $i_{r'}$ truth validness of $i_{r'}(F) \leqslant 0.5$ respectively

$i_{r'}(F) = i_{r'}((A_1 \wedge A_2 \wedge, \ldots, \wedge A_m) \rightarrow (B_1 \wedge B_2 \wedge, \ldots, \wedge B_n)) =$

$= \max(1 - i_{r'}(A_1), 1 - i_{r'}(A_2), \ldots, 1 - i_{r'}(A_m), \min(i_{r'}(B_1), i_{r'}(B_2), \ldots, i_{r'}(B_n)) \leqslant 0.5$

so, we have

$$i_{r'}(F) = \begin{cases} i'_r(A_i) \succ 0.5, \ \forall i = 1, 2, ..., m \ and \\ i'_r(B_j) \leqslant 0.5, \exists j = 1, 2, ..., n \end{cases}$$

If is valid $i'_r(A_i) \succ 0.5 \ \forall i = 1, 2, ..., m$

then according to *definition* 5.1 is $\varphi(A_i\,[t_1,t_2]) \geqslant \theta$

Based on the *definition 3.2* we have $\varphi(X\,[t_i, t_j]) = \min_{A_k \in X}\{\varphi\,(A_k\,[t_i, t_j])$. Now, therefore on basis of proposition 3.1.2 is also $\varphi(X\,[t_i, t_j]) \geqslant \theta$.

Because of theorem assumption that FFD is satisfied, we have

$\varphi(Y[t_1, t_2] = \min(\varphi(B_1[t_1,t_2]), \ldots, \varphi(B_n[t_1,t_2]))$

$\geqslant \min(\theta, (\varphi(X[t_1,t_2])) =$

$= \min(\theta, \min(\varphi(A_1[t_1,t_2]), \ldots, \varphi(A_m[t_1,t_2])) \geqslant \theta$

This results that $\varphi(B_j[t_1,t_2]) \geqslant \theta$ for each j =1,2,...,n.

So follows $i'_r(B_j) \succ 0.5$, what is contrary to $i'_r(B_j) \leqslant 0.5$.

Therefore the assertion is valid if relation r satisfies FFD$X \xrightarrow[F]{\theta} Y$, then its assigment fuzzy fomula is satisfy in the interpretation $i_{r'}$.

Let be proved, now, vice verse of theorem. Assume that F satisfy in interpretation $i_{r'}$. Then

$i'_r(F) = \max(1 - i'_r(A_1), 1 - i'_r(A_2), \ldots, 1 - i'_r(A_m), \min(i'_r(B_1), i'_r(B_2), \ldots, i'_r(B_n)) \succ 0.5$ what results

$i_{r'}(A_1 \wedge A_2 \wedge, \ldots, \wedge A_m) \leqslant 0.5$ or

$i_{r'}(B_1 \wedge B_2 \wedge, \ldots, \wedge B_n) \succ 0.5$

Let be valid i)

$i_{r'}(A_1 \wedge A_2 \wedge, \ldots, \wedge A_m) =$ **min** $(i_{r'}(A_1), i_{r'}(A_2), \ldots, i_{r'}(A_m))$

then $i_{r'}(A_j) \leqslant 0.5$ for some j from {1,2,...,m},

from which follow $\varphi(A_j[t_1,t_2] \prec \theta$ for some j from {1,2,...,m}. Then

$\varphi(X[t_1,t_2]) = \min\{\varphi(A_1[t_1,t_2]), \ldots, \varphi(A_m[t_1,t_2])\} \prec \theta$. From this follows that relation satisfies FFD$X \xrightarrow[F]{\theta} Y$.

Let be valid ii) i.e. $i_{r'}(B_1 \wedge B_2 \wedge, \ldots, \wedge B_n) \succ 0.5$

then for each i = 1,2,...,n

min $(i_{r'}(B_1), i_{r'}(B_2), \ldots, i_{r'}(B_n)) \succ 0.5$

respectively $i_{r'}(B_i) \succ 0.5$ for each i = 1,2,...,n

from which follow $\varphi(B_i[t_1,t_2]) \geqslant \theta$respectively

$\varphi(B_i[t_1,t_2]) \geqslant \theta \geqslant \min(\theta, \varphi(X[t_1,t_2]))$.

Hence it follows that r satisfies the FFD $X \xrightarrow[F]{\theta} Y$.

b) For Zadeh implication $X \to Y = max(1\text{-}X, min(X, Y))$.

Let assume, as first , that relation r satisfies FFD $X \xrightarrow[F]{\theta} Y$ i.e. let be hold

$\varphi(Y[t_1,t_2]) \geqslant \min(\theta, \varphi(X[t_1,t_2])$

where is $X = \{A_1, A_2, \ldots, A_m\}$ and $Y = \{B_1, B_2, \ldots, B_n\}$.

Let assume contra to theorem assertion that assigments

$F : (A_1 \wedge A_2 \wedge, \ldots, \wedge A_m) \to (B_1 \wedge B_2 \wedge, \ldots, \wedge B_n)$

is falsify in interpretation $i_{r'}$.

Then follows

$i_{r'}(F) = i_{r'}(A_1 \wedge, \ldots, \wedge A_m) \to (B_1 \wedge, \ldots, \wedge B_n)$

$= max(1\text{-}\ i_{r'}(A_1), \ldots,\ 1\text{-}\ i_{r'}(A_m),\ min(i_{r'}(A_1), \ldots,$

$i_{r'}(A_m), i_{r'}(B_1), i_{r'}(B_2), \ldots, i_{r'}(B_n)) \leqslant 0.5$

so, we have

$1 - i_{r'}(A_i) \leqslant 0.5,\ i = 1,2, \ldots, m$, and

$min(i_{r'}(A_1), \ldots, i_{r'}(A_m), i_{r'}(B_1), \ldots, i_{r'}(B_n)) \leqslant 0.5$

so follows

$i_{r'}(A_i) \succ 0.5,\ i = 1,2, \ldots, m$

and $i_{r'}(B_j) \leqslant 0.5, \exists\, j = 1,2, \ldots, n$

then according to *definition* 5.1 is $\varphi(A_i\,[t_1,t_2]) \geqslant \theta$

Based on the *definition 3.2* we have $\varphi(X\,[t_i,t_j]) = \min_{A_k \in X}\{\varphi\,(A_k\,[t_i,t_j])$. Now, is also $\varphi(X\,[t_i,t_j]) \geqslant \theta$.

Because of theorem assumption that FFD is satisfied ,we have

$\varphi(Y[t_1,t_2] = \min(\varphi(B_1[t_1,t_2]), \ldots, \varphi(B_n[t_1,t_2]))$

$\geqslant \min(\theta, (\varphi(X[t_1,t_2]))$

$= \min(\theta, \min(\varphi(A_1[t_1,t_2]), \ldots, \varphi(A_m[t_1,t_2])) \geqslant \theta$

This results that $\varphi(B_j[t_1,t_2]) \geqslant \theta$ for each $j = 1,2, \ldots, n$.

So follows $i'_r(B_j) \succ 0.5$, what is contrary to $i'_r(B_j) \leqslant 0.5$.

Let be proved, now, vice verse of theorem. Assume that F satisfy in interpretation $i_{r'}$. Then

$i_{r'}(F) = i_{r'}((A_1 \wedge A_2 \wedge, \ldots, \wedge A_m) \to (B_1 \wedge B_2$

$\wedge, \ldots, \wedge B_n))$

$= max(1\text{-}\ i_{r'}(A_1), 1 - i_{r'}(A_2), \ldots,\ 1\text{-}\ i_{r'}(A_m),$

$min(i_{r'}(A_1), i_{r'}(A_2), \ldots, i_{r'}(A_m),$

$i_{r'}(B_1), i_{r'}(B_2), \ldots, i_{r'}(B_n)) \succ 0.5$

Then $\exists\, j = 1,2, \ldots, m$ for which hold $i'_r(A_j) \leqslant 0.5$

Then according to definition 5.1

$\varphi(A_j[t_1,t_2]) \prec \theta$, $j = 1,2, \ldots, m$

Based on the definition 3.2 we have

$\varphi(X[t_1,t_2]) = \min(A_1[t_1,t_2]), \varphi(A_2[t_1,t_2]), \ldots, \varphi(A_m[t_1,t_2]) \prec \theta$.

From this follows that relation satisfies FFD$X \xrightarrow[F]{\theta} Y$.

If hold $i_{r'}(A_i) \succ 0.5$ then $i_{r'}(B_1 \wedge B_2 \wedge, \ldots, \wedge B_n) \succ 0.5$ then for each $i = 1,2, \ldots, n$,i.e.

min $(i_{r'}(B_1), i_{r'}(B_2), \ldots, i_{r'}(B_n)) \succ 0.5$

respectively $i_{r'}(B_i) \succ 0.5$ for each $i = 1,2, \ldots, n$

from which follow $\varphi(B_i[t_1,t_2]) \geqslant \theta$respectively

$\varphi(B_i[t_1,t_2]) \geqslant \theta \geqslant \min(\theta, \varphi(X[t_1,t_2]))$.

Hence it follows that r satisfies the FFD $X \underset{F}{\overset{\theta}{\rightarrow}} Y$.

By this is proved the theorem.

In the following theorem we are going to show that if relation r satisfies a set of fuzzy functional dependece F and does not satisfy dependency $X \underset{F}{\overset{\theta}{\rightarrow}} Y$ then exists two tuples subrelation, of relation r, which satisfies all the fuzzy functional dependece from set F, and does not satisfy dependency $X \underset{F}{\overset{\theta}{\rightarrow}} Y$.

Theorem 5.2. Let $X \underset{F}{\overset{\theta}{\rightarrow}} Y$ be an FFD over scheme R, and $\{ A_1, A_2, \ldots, A_m \} = X \subseteq R$, and $\{ B_1, B_2, \ldots, B_n \} \subseteq R$, and let **F** be a set of FFDs over R Then hold ,

1) $\mathbf{F} \Rightarrow X \underset{F}{\overset{\theta}{\rightarrow}} Y$ if and only if

2) $\mathbf{F} \Rightarrow X \underset{F}{\overset{\theta}{\rightarrow}} Y$ in the world of two tuple relations.

Proof . Obviously 1) implies 2).

Let prove the reverse of theorem 2) implies 1).

Let assumed a contra to the theorem that is not valid $F \Rightarrow X \underset{F}{\overset{\theta}{\rightarrow}} Y$ in relation r.

In that case some relation r satisfied all the fuzzy functional dependencies from F, and do not satisfy dependency $X \underset{F}{\overset{\theta}{\rightarrow}} Y$.

This means that exists the elements t_1 and t_2 from r , for which hold $\varphi(Y[t_1,t_2]) \prec \min(\theta, \varphi(X[t_1,t_2]))$.

Let be $r^* = \{t_1, t_2\}$. It is obvioes that r* satisfies all the FFDs from F , but does not satisfty this dependency $X \underset{F}{\overset{\theta}{\rightarrow}} Y$. By this is shown that following

Lema 5.1. Let r be a relation, let F be set of FFDs on R, and let $X \underset{F}{\overset{\theta}{\rightarrow}} Y$ be a single FFD on R. If relation r satisfies all the FFDs from set F and violates fuzzy dependency $X \underset{F}{\overset{\theta}{\rightarrow}} Y$, then some two tuple subrelation r* of r satisfies F and violates $X \underset{F}{\overset{\theta}{\rightarrow}} Y$.

The opposite to contraposition of this claim is the claim that 2) implies 1).

Theorem 5.3. Let $X \underset{F}{\overset{\theta}{\rightarrow}} Y$ be an FFD over relation scheme **R** and let **F** be a set of FFDs over **R**. Then holds, **F** implies $X \underset{F}{\overset{\theta}{\rightarrow}} Y$ in the world of two tuple relations, if and only if **F** implies $\mathbf{X} \rightarrow \mathbf{Y}$ when FFDs are interpreted as fuzzy formulas.

Proof. Let assume that $i_r : \mathbf{R} \rightarrow [0,1]$ be such interpretation where every formulas are satisfied, which are generated FFDs from set **F** , at let formula which is generated by dependency $X \underset{F}{\overset{\theta}{\rightarrow}} Y$ be falsify. Let we consider that

$Z = \{ A \in \mathbf{R} : i_r (A) \succ 0.5\}$

Let r_z be fuzzy relation instance with two tuples t_1 and $t_2$as shown in Fig.1. We choose the set **{a,b}** as the domain of each attributes in **R**, where $a = a_1,\dots,a_p$, and $b = b_1,\dots,b_q$ ($p \geqslant 1$, $q \geqslant 1$). Let $s(a_i,a_j) = \theta$, (which implies that $\varphi(A[t_1,t_2]) \geqslant \theta$, for any attribute set A in r_z), and where s is similiraty relation.

Attributes of Z other attributes

t_1	a,...,a	a,...,a
t_2	a,...,a	b,...,b

Fig.1. The fuzzy relation instance r_z.

Namely $r_z = \{ t_1 , t_2\}$ where $t_1 = a,\dots,a$ for each attribute **A** from **R**, and let t_2 be defined as

$$t_2 = \begin{cases} a,\dots,a & R \in Z \\ b,\dots,b & R \notin Z \end{cases}$$

Let prove that relation, r_z defined in such way is satisfying each fuzzy functional dependencies from F. To be able to prove this, let $U \underset{F}{\overset{\theta}{\rightarrow}} V$, any fuzzy functional dependency from F for which then holds

$\varphi(U[t_1,t_2]) \geqslant \theta$

Due to the definition t_1, now it have to be and $t_2 = a,\dots,a$ for each attribute **A** from U, namely $\varphi(A[t_1,t_2]) \geqslant \theta$. This means that i_r (A) $\succ$0.5 , for each A from U. From this hold $U \subseteq Z$, i.e.

(*) i_r (U) $\succ$0.5

If $\varphi(V[t_1,t_2]) \geqslant \theta$ would not hold, then would be $t_1 = a,\dots,a$ and $t_2 = b,\dots,b$ for some attribute A from V, namely $\varphi(A[t_1,t_2]) \prec \theta$. From this we have that A does not belong set Z, and would hold i_r (A) $\prec$0.5, and also i_r (V) $\prec$0.5.

Based on this and (*) we have that for Kleens-Diens implication and Zadeh implication hold

$i_r (U \rightarrow V) = \max (i_r (1-U), i_r (V)) \leqslant \mathbf{0.5}$

$i_r (U \rightarrow V) = \max(1-U,\min(U,V)) \leqslant \mathbf{0.5}$

and this is would be in contra to first assumption.

Let prowe that r_z not satisfy fuzzy functional dependency $X \underset{F}{\overset{\theta}{\rightarrow}} Y$ i.e.

$\varphi(Y[t_1,t_2]) \prec \min(\theta, \varphi(X[t_1,t_2])$.

As it is by assumption that the fuzzy formula is falsify in the interpretation i_r, then must be that

i_r (X) $\succ$0.5 and

(**) i_r (Y) $\leqslant$0.5

Let assume that

$\varphi(X[t_1,t_2]) \geqslant \theta$,

if would hold $\varphi(Y[t_1,t_2]) \geqslant \theta$, then would hold $Y \subseteq Z$, namely

$i_r (B_j) \succ 0.5$ for each $j = 1,2,\dots,n$, $B_j \in Y$.

This result that i_r (Y) $\succ$0.5, what is contradiction with (**).

Let we prove vice verse of Theorem. Let assume contra, i.est . that does not hold that from set of FFDs F follows and FFD $X \underset{F}{\overset{\theta}{\rightarrow}} Y$.

Then exist two tuples relation r = {t,t'} which satisfies each FFDs from F , but does not satisfy and FFD$X \xrightarrow[F]{\theta} Y$. By the above mentioned description it is defined the interpretation i_r , by the relation r , formulas $U_1 \wedge U_2 \wedge ... \wedge U_p \rightarrow V_1 \wedge V_2 \wedge ... \wedge V_q$, for $U \rightarrow V$ from F and formula $X_1 \wedge ... \wedge X_m \rightarrow Y_1 \wedge ... \wedge Y_n$.

Let prove now that hold

i) $i_r(U_1 \wedge ... \wedge U_p \rightarrow V_1 \wedge ... \wedge V_q)) \succ 0.5$ and

ii) $i_r((X_1 \wedge ... \wedge X_m) \rightarrow (Y_1 \wedge ... \wedge Y_n)) \leqslant 0.5$

When would not be i) then

i_r $(U_i) \succ 0.5$ and

i_r $(V_j) \leqslant 0.5$

namely

$\varphi(P[t,t']) \geqslant \theta$,

for each P from U and

$\varphi(Q[t,t']) \prec \theta$,

for some Q from V.

This first would mean that $\varphi(U[t,t']) \geqslant \theta$, and the second that $\varphi(V[t,t']) \prec \theta$. Therefore these together is contradiction with start assumption that r satisfies each fuzzy functional dependencies from F. By it is proved i).

If would not be ii) then would be

iii) i_r $(X_i) \leqslant 0.5$ or

iv) i_r $(Y_j) \succ 0.5$

If iii) hold , then $\varphi(A_i[t,t']) \prec \theta$, for some j = 1,2,...,m, $A_i \in X$ and from these $\varphi(X[t,t']) \prec \theta$. It is obvious that r satisfies fuzzy functional dependency

$X \xrightarrow[F]{\theta} Y$,what is contradiction with the beginning assumption.

If hold iv) then $\varphi(B_i[t,t']) \geqslant \theta$, for each j = 1,2,...,n, $B_j \in Y$ and from these $\varphi(Y[t,t']) \geqslant \theta$. From this, we would conclusion that and in this case r satisfied fuzzy functional dependency $X \xrightarrow[F]{\theta} Y$, what is also contradiction with the beginning assumption.

The right proved theorems enable the application of resolution rules in fuzzy logic as the rule of inference on calculation of fuzzy functional dependencies.

Example 5.1. Let R = {*Name,Intelligence,Capability,Job,Success*} be a relation scheme, and let

$\Gamma = \{A_1A_2 \xrightarrow[F]{\theta_1} A_3, A_2 \xrightarrow[F]{\theta_2} A_4, A_3A_4 \xrightarrow[F]{\theta_3} A_5\}$

be set a FFDs over scheme R, where is noted byA_1 -*Name*, A_2 - *Intelligence*, A_3 - *Capability*, A_4 - *Job*, A_5 -*Success*.

Prove that holds

$$\Gamma \Rightarrow A_1A_2 \xrightarrow[F]{\theta} A_5,$$

where is $\theta = \min(\min(\theta_1, \theta_2), \theta_3)$.

Lets prove in two ways that this examples holds , using following

a) Calculus of fuzzy functional dependences

b) The resolution principle in fuzzy logic.

a)

1) $A_2 \underset{F}{\overset{\theta_2}{\rightarrow}} A_4$ (hypothesis)

2) $A_1A_2 \underset{F}{\overset{\theta_2}{\rightarrow}} A_1A_4$ (**IR3** , 1))

3) $A_4 \subseteq A_1A_4$

4) $A_1A_4 \underset{F}{\overset{\theta_2}{\rightarrow}} A_4$ (**IR2** , 3))

5)$A_1A_2 \underset{F}{\overset{\theta_2}{\rightarrow}} A_4$(IR4, 2), 4))

6)$A_1A_2 \underset{F}{\overset{\theta_1}{\rightarrow}} A_3$(hypothesis)

7)$A_1A_2 \underset{F}{\overset{\min(\theta_1,\theta_2)}{\rightarrow}} A_3A_4$(IR5, 5),6))

8)$A_3A_4 \underset{F}{\overset{\theta_3}{\rightarrow}} A_5$(hypothesis)

9) $A_1A_2 \underset{F}{\overset{\theta}{\rightarrow}} A_5$ (IR4, 7),8))

where is $\theta = \min(\min(\theta_1, \theta_2), \theta_3))$.

b)

According to the previous theorems it is enough to prove that hold $\Gamma \Rightarrow A_1 \wedge A_2 \rightarrow A_5$. Let' s assert , as first , to FFDs the corresponding formulas :

$$A_1A_2 \underset{F}{\overset{\theta_1}{\rightarrow}} A_3 \quad F1 : (A_1 \wedge A_2) \rightarrow A_3$$
$$A_2 \underset{F}{\overset{\theta_2}{\rightarrow}} A_4 \quad F2 : A_2 \rightarrow A_4$$
$$A_3A_4 \underset{F}{\overset{\theta_3}{\rightarrow}} A_5 \quad F3 : (A3 \wedge A_4) \rightarrow A_5$$

According to the definition logical consequence and already said mentioned , it is enought to show that

F: $F1 \wedge F2 \wedge F3 \wedge \neg G$

unsatisfiable , where is $G : (A_1 \wedge A_2) \rightarrow A_5$.

To be able to apply a rule of resolution, it is needed , at first transform F in conjuctive normal form so to get a set F*, as a represent of F.

F*= $\{\neg A_1 \vee \neg A_2 \vee A_3, \neg A_2 \vee A_4, \neg A_3 \vee \neg A_4 \vee A_5, A_1, A_2, \neg A_5\}$

The following set of disjunct show resolvent inference.

1) $\neg A_1 \vee \neg A_2 \vee A_3$ (element from F*)
2) $\neg A_3 \vee \neg A_4 \vee A_5$ (element from F*)
3) $\neg A_1 \vee \neg A_2 \vee \neg A_4 \vee A_5$ (Resolvent 1) and 2))
4) $\neg A_5$ (element from F*)
5) $\neg A_1 \vee \neg A_2 \vee \neg A_4$ (Resolvent 3) and 4))
6) $\neg A_2 \vee A_4$ (element from F*)

7) $\neg A_1 \vee \neg A_2$ (Resolvent 5) and 6))
8) A_2 (element from F*)
9) $\neg A_1$ (Resolvent 7) and 8))
10) A_1 (element from F*)
11) $\min(A_1, \neg A_1) \leqslant 0.5$ (Resolvent 9) and 10)

6 Conclusion

In this paper we proved the equivalence between theory of fuzzy functional dependencies for fuzzy database and the part theory of fuzzy logic.

To achive such an aim, we introduced the definition of truth assignment of attributes in relation r over the relation scheme R. Based on this definition of FFD was attached to the fuzzy formula and was proved that if relation r satisfies FFD then this fuzzy formula is satisfied in the given interpretation and vice verse. The equivalence between set of the FFDS and fuzzy formulas was proved as well. This equalence makes possible an application of the resolution principle. With this equivalence, we may substitute calculation of fuzzy functional dependencies by calculation of fuzzy formulas ,applying the resolution principle as inference rules. The resolution principle in fuzzy logic enables a complete automatic proving, what is significant advantage over to the classic approach.

It is a progress a further study that will prove an equivalence of implication of fuzzy multivalued dependencies and of fuzzy logic.

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