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Spectral Fuzzy Classification System: A Supervised Approach

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Abstract

The goal of this paper is to present an algorithm for pattern recognition, leveraging on an existing fuzzy clustering algorithm developed by Del Amo *et al.* [3, 5], and modifying it to its supervised version, in order to apply the algorithm to different pattern recognition applications in Remote Sensing. The main goal is to recognize the object and stop the search depending on the precision of the application. The referred algorithm was the core of a classification system based on Fuzzy Sets Theory (see [14]), approaching remotely sensed classification problems as multicriteria decision making problems, solved by means of an outranking methodology (see [12] and also [11]). The referred algorithm was a unsupervised classification algorithm, but now in this paper will present a modification of the original algorithm into a supervised version.

1 Introduction

In a recognition system, three pillars have to be taken into account: hardware, software and a collection of concepts, methods and techniques that underlie automation of reasoning. These concepts methods and techniques can be based on soft computing, which is a set of methodologies whose role model is the capability of the human mind to exploit the tolerance for imprecision, uncertainty and partial truth to achieve tractability, robustness and minimality of cost or effort.

Remotely Sensed Image Classification is an extremely complex problem. Therefore, the usual approach is to divide it into more accessible mathematical problems, introducing some assumptions into each part of the recognition system. Since the Earth's surface is amazingly complex and not easily recorded, relatively complex remote sensing devices are required, introducing constraints such as spatial, spectral and geometric resolution. Errors typically slip into the data acquisition process.

Therefore, it is frequently necessary to preprocess the remotely sensed data prior to analyzing it. This part of the problem is called *image restoration*. Image restoration involves the correction of distortion, degradation and noise introduced during the rendering process. Image restoration produces a corrected image that is as close as possible, both geometrically and radiometrically, to the radiant energy characteristics of the original scene. This can be considered as a different problem which we will assume is already solved (see [8] for additional information on this issue). Such a previous mathematical treatment assures a right meaning of data. In this paper we shall be assuming that geo-referenced images have been stored in memory, once the images have been already corrected.

The simplest form of digital image classification is to consider each pixel individually, assigning it to a class based upon several values measured in separate spectral bands. This type of classifier is usually referred to as a *spectral* or *point* classifier. Although point classifiers offer benefits of simplicity and economy, they do not give the analyst the opportunity to exploit the information contained in relationships between each pixel and its neighbors (a nice overview on the state of the art on fuzzy techniques in digital image processing can be obtained from [10] and [9]).

By definition, a remotely sensed image illustrates a portion of the Earth's surface as recorded by reflection of solar energy to an instrument at high altitude. The following questions illustrate the information necessary to interpret these images:

- What forms of radiation have been used to record this pattern?
- How was this radiation gathered and recorded as an image?
- What is the scale? (How large is the ground area represented in the image?)
- What ground patterns are represented by variations in brightness? (Are they related to others not visible?)
- When was the image acquired? (What time of day? What season?)

The field of Automatic Target Recognition (ATR) has been widely studied for at least two decades with no fielded success. The lack of success can be attributed to many reasons including lack of adequate sensors and the appropriate mechanism to incorporate expert knowledge. In this sense, fuzzy sets [14] provide a interesting mechanism for incorporating expert knowledge into recognition systems, since one main difficulty with object recognition in images is the inability to define the objects in question. Definition is a prerequisite to recognition. Definability is concerned with whether and how a concept X may be defined in a way that lends itself to mathematical analysis and computation.

A *pattern* refers to the arrangement of individual objects into distinctive, recurring forms that permit recognition on aerial imagery. Patterns on an image naturally follow a functional relationship between the individual features that compose the pattern.

The definition of the patterns has to be done using the same mathematical model that the classification will be performed in. For this reason the system will have two phases. The first phase is a definition phase, in which the patterns are spectrally defined. The second phase is the classification and recognition.

2 Georegistration

Georegistration is the process of adjusting one drawing or image to the geographic location of a "known good" reference drawing, image, surface or map. For brevity, this topic and other georegistration topics use images as examples. However, the same procedures apply when georegistering drawings or surfaces.

The drawing, image, surface or map being used as a reference is called the reference component. The drawing, image or surface being adjusted is called the target component.

Georegistration involves precise transformation of the image from the sensor based projection to an earth surface based projection. This process includes calculating a satellite mode, matching ground and image based control points and transformation and resampling the data to a map projection coordinates system. The most important factor to consider in geometric registration is the positional accuracy of each pixel as it is moved from the sensor based projection to the surface base projection.

Lets see some assumptions we will make for the reference component and for the target component.

- Geo-referenced images with identified areas of interest for "correlation" will be already stored in memory for use with on aircraft imagery.
- All images have been taken with the same resolution and have been stored in memory landscape.
- The set of images acquired for matching will be taken with the same resolution as the ones already in memory.
- All the images involved in the problem have been taken from the same angle.

This way problems like alignment of the images, resolution transformations, reference systems and so on will be part of a preprocessing problem so the problem we will be dealing here with will be bounded.

3 Object classification

Let

$$X = \{X_{ij} \,/\, i = 1, \cdots r;\ j = 1, \cdots , s\}$$

be the set of objects to be classified. Each object is considered to be an earth surface unit or *pixel*. Let

$$(x_{ij}^1, x_{ij}^2, \cdots , x_{ij}^n) \in I\!R^n$$

be the vectorial representation of each object X_{ij}, and x_{ij}^f is the value of feature f for object X_{ij}. These values could be a subproduct of a series of direct observations.

Most of the classification algorithms don't have an *a priori* fixed number of classes. The number of classes is usually set up after a set of comparisons between different classifications with a different number of classes. Anyway, in the case of a fuzzy classification the number of classes is considerably lower than in crisp classification due to the flexibility that a fuzzy class provides. We are working with an unknown number of classes k. For each one of the classes, in which the classification will be performed, a range of valid values has to be defined.

In the original algorithm (see [3]), for each class k and each feature f, the lower and upper extremes of the interval I_{fk}, inside which the membership function has a value of 1, where properly defined ($\underline{\alpha}_{fk}$ and $\overline{\alpha}_{fk}$). Analogously, two values $\underline{\omega}_{fk}$ and $\overline{\omega}_{fk}$ were defined as the lower and upper extremes of the interval outside of which the membership function has a value of 0. This means element X_{ij} crisply does not satisfy the f property for class C_k in a crisp way whenever $x_{ij}^f < \underline{\omega}_{fk}$ or $x_{ij}^f > \overline{\omega}_{fk}$ occurs; it satisfies the f properties of class C_k in a crisp way whenever $\underline{\alpha}_{fk} \leq x_{ij}^f \leq \overline{\alpha}_{fk}$ and satisfies properties of class k in a fuzzy way whenever $\underline{\omega}_{fk} \leq x_{ij}^f < \underline{\alpha}_{fk}$ or $\overline{\alpha}_{fk} < x_{ij}^f \leq \overline{\omega}_{fk}$ ($\underline{\omega}_{fk} < \underline{\alpha}_{fk} < \overline{\alpha}_{fk} < \overline{\omega}_{fk}$ should be verified). The membership function for each class C_k with respect to each f property can be then defined in the following way:

$$m_{fk}(x_{ij}^f) = \begin{cases} 0 & if \quad x_{ij}^f \in \quad]-\infty, \underline{\omega}_{fk}[\quad \cup \quad]\overline{\omega}_{fk}, \infty[\\ (\frac{x_{ij}^f - \underline{\omega}_{fk}}{\underline{\alpha}_{fk} - \underline{\omega}_{fk}})^2 & if \quad x_{ij}^f \in \quad [\underline{\omega}_{fk}, \underline{\alpha}_{fk}[\\ 1 - (\frac{x_{ij}^f - \overline{\alpha}_{fk}}{\overline{\omega}_{fk} - \overline{\alpha}_{fk}})^2 & if \quad x_{ij}^f \in \quad]\overline{\alpha}_{fk}, \overline{\omega}_{fk}] \\ 1 & if \quad x_{ij}^f \in \quad [\underline{\alpha}_{fk}, \overline{\alpha}_{fk}] \end{cases} \tag{1}$$

Therefore, each object X_{ij} has an associated vector

$$M_k(X_{ij}) = (m_{1k}(x_{ij}^1), m_{2k}(x_{ij}^2), \cdots, m_{nk}(x_{ij}^n))$$

for each class C_k, which shows the different degrees of verification each property has with respect to each class.

Let $P = \{P_1, P_2, \cdots, P_p\}$ be the family of patterns. Each pattern P_k will be a subset of objects characterized through the different observed features,

$$P_k = \{P_{i,j}^k \ / \ i = 1, \cdots, r_k \ j = 1, \cdots, s_k\}$$

verifying $1 \leq r_p \leq r$ and $1 \leq s_p \leq s$, and being

$$((p_k)_{ij}^1, (p_k)_{ij}^2, \cdots, (p_k)_{ij}^n) \in I\!R^n$$

the vectorial representation of each one of the pixels in the pattern, where $(p_k)_{ij}^f$ is the value of feature f for pixel $(p_k)_{ij}$ in object P_k .

Each pixel in the pattern P_u has been already classified or spectrally defined using the unsupervised classification system presented in [3] and [5]. So each one of the pixels in the pattern image will have a representation as follows:

$$\mu_{ij}^{u}(C_k) = \mu_{ij}^{u^{ND}}(C_k) \cdot b_{ij}(C_k)$$

is the membership degree to class C_k, where

- $C = \{C_1, C_2, \cdots, C_b\}$ is the family of classes. Each class C_k will become characterized through the different observed features.
- $\mu_{ij}^{u^{ND}}(C_k)$ is the fuzzy set of non-dominated classes representing the membership degree of pixel X_{ij} to class C_k.
- $b_{ij}(C_k)$ is basic pixel information (each object $(P_u)_{ij}$ has an associated vector

  $$M_t((P_u)_{ij}) = (m_{1t}((p_u)_{ij}^{1}), m_{2t}((p_u)_{ij}^{2}), \cdots, m_{nt}((p_u)_{ij}^{n}))$$

  for each class C_t, which shows the different degrees of verification each property has with respect to each class).

Pixel X_{ij} will verify the properties of class C_k with an intensity of

$$b_{ij}(C_k) = \min_{f} \left\{ m_{fs}(x_{ij}^{f}) \right\} \tag{2}$$

where $m_{fs}(x_{ij}^{f})$ is the membership function for each class (as defined above) C_u with respect to each f property.

4 Fuzzy Pattern Recognition System

The fuzzy pattern recognition system that we are presenting can be hence divided into two phases. In the first phase, the program reads through the dataset and sequentially builds a fuzzy classification of each pixel in the image or target component. Also, as an assumption we mentioned that a set of images or referenced components has been previously individually classified by the same method. Therefore, all the components in the system are defined by the same mathematical model, the unsupervised fuzzy classification algorithm. In the second phase of the algorithm, a comparison between the target component (acquired image) and the referenced images (stored images) will be performed.

The definition of a target component (in our case a georegistered image) after its classification using the fuzzy classification system outlined above (see [3] for a whole explanation of the classification algorithm) will be as follows: let P_u be a target component that has been classified using Del Amo *et al.* algorithm. The definition of the target component will be

$$P_u^{def} = \left\{ \phi_{ij}^{u}(C_k) \ / C_k \in \{C_1, \cdots, C_b\} \right\} \tag{3}$$

$\phi_{ij}^{u}(C_k)$ being the degree of membership of pixel ij to class C_k for pattern u.

A fundamental issue to take into account when a spectral classification is being performed is that a difference exists in the spectral response of a pixel on two dates if the biophysical materials within the Instantaneous Field of view (IFOV) have changed between dates. Ideally, the spectral resolution of the remote sensor system is sufficient to record reflected radiant flux in spectral regions that best capture the most descriptive spectral attributes of the object.

One of the possible objectives of the pattern recognition system is to detect an object in a particular location. In this case, it will make sense to try to either emphasize the characteristics or features of the object being detect or to emphasize the edges.

Once the set of referenced components and the target component have been spectrally classified or defined the recognition phase starts. One of the target components itself, or a subset of a target component will be the object in the comparison or recognition part of the algorithm. The target component will be compared with the referenced components until a possible match has been found. Each possible match has different levels of verification depending on the application and the degree of accuracy we are pursuing. Therefore, depending on these hypothesis we will continue with the whole set of analysis levels or we will stop at the first level once our requirements have been met. The system is being envisioned as a two modes system, where the expert knowledge is included in the decision making process (using a possibilistic network) or as a semiautomatic or interactive system where the user can interactively make decisions depending on the level of precision that wants to reach.

The system will also have the capability to let the user interactively select a particular area from the target component when the user considers this selected area is highly probable to meet the requirements for the present goal.

Let's say that a subset of size $r_u \times s_u$ has been selected in the target component. Let's say that we are comparing subsets of smaller size than the selected one. Once a possible match has been found, a superset of the subset has to be compared also in order to make sure that the hypothesis of similarity between the two is true to a certain level of accuracy. This can be done by comparing the first line of pixels r_u in the pattern image with the image sliding the pattern over the image, starting on the left top corner and then spanning the area of comparison. Another possibility could be to create a filter and use the filter in the whole set of referenced components. The filter will be used to determine possible sets of pixels in a target component and in the referenced components that can be considered as similar for a particular similarity relation.

A similarity measure has been used in the first place to decide whether or not two objects are members of the same cluster. Now a similarity measure has to be defined to determine if a referenced component or a subset of it, it is similar to a certain level to a target component or to a subset of it in order to conclude that the selected objective meets the requirements.

There exists several ways to study similarity. A pixel by pixel comparison without a rational argument for this approach will be very time consuming and will require extensive computational resources. The goal is to find a single representative of a group of pixels that will contain the aggregated information of all the pixels in the group in order to save computational resources.

Moreover, it is very important to consider the neighbors of each pixel in order to perform a classification. Some of the relations between pixels, and also erroneous or non classifiable data can only be handle by consideration of the neighbors. Very often some of the errors due to acquisition difficulties can be solved or not be considered only if the neighbors are being taken into account in the analysis.

As has been established above, a first (unsupervised) classification of the pixels in the pattern images has already been performed according to the algorithm developed in [3, 5]. Therefore, we have a classification, pixel by pixel, to each one of the classes in the class set (each matrix Γ_{C_k} represents each one of the individual pixel as a vector representing the degrees of membership of the pixel to each one of the classes in the classification). Let Γ be the target component already classified using Del Amo *et al.* algorithm [5] as follows (when we consider Γ for a referenced component we will use the notation Γ_p and the elements of the matrix will be noted $\rho_p \mu_{ij}(C_k)$):

$$\Gamma = \left\{ \Gamma_{C_k} \;/\; k = \{1, \cdots, b\} \right\} \tag{4}$$

where Γ_{C_k} is defined as follows:

$$\Gamma_{C_k} = \left\{ \phi_{ij}(C_k) \quad i = 1, \cdots, r; \quad j = 1, \cdots, s \right\} \tag{5}$$

Therefore, Γ represents the original target component (an image) but its colormap now represents the degree of membership to each one of the classes that each one of the pixels in the original image has been classified in. The second part of the algorithm will consider the neighbors, it won't be a pixel by pixel classification, so in the particular case of an indexed image, the classification algorithm can not be performed over the colormap matrix like it was done in the original one. In this case we need to perform the classification in both matrices. We need to consider not only the particular characteristics of each pixel but also their location in the image.

Therefore, we will compute a transformation matrix.

In order to compute a transformation matrix we are going to use the following definitions:

Definition 4.1. *The (-1)-maximum value of size q of a set of neighbors of size $(r_u)_q \times (s_u)_q$ of element $(p_u)_{ij}$ will be defined as follows:*

$$[\phi_{ij}(C_k)]_{max} = \max \left\{ \phi_{\alpha,\beta}(C_k) \;/\; \begin{array}{ll} \alpha = i - q, \cdots, i + q & i \neq j \\ \beta = j - q, \cdots, j + q & \alpha \geq 0, \quad \beta \geq 0 \end{array} \right\} \tag{6}$$

where $[\phi_{ij}(C_k)]$ represent the membership degree of pixel ij to class C_k in pattern p.

Analogously, we can define the *(-1)-minimum* of size q as follows:

Definition 4.2. *The (-1)-minimum value of size q of a set of neighbors of size $(r_u)_q \times (s_u)_q$ of element $(p_u)_{ij}$ will be defined as follows:*

$$[\phi_{ij}(C_k)]_{min} = \min\left\{\phi_{\alpha,\beta}(C_k)/ \quad \begin{array}{lll} \alpha = i-q,\cdots,i+q & i \neq j & \\ \beta = j-q,\cdots,j+q & \alpha \geq 0, & \beta \geq 0 \end{array}\right\} \qquad (7)$$

Notice in the above definitions that for the pixels located in the borders or at a distance smaller than the selected size of the filter the following definitions vary. For each subset of pixels of size q we are going to define a filter for the subset image that is going to depend on the distance of the pixel that has not been considered to the maximum and the minimum intensity values of the other ones. The objective of this filter is to exaggerate differences. A matrix that we will call Ξ_q-*transformation* will be constructed. The construction process for this matrix will be explained in the next section. But let's see some definitions previous to that:

Definition 4.3. *Let X be a component (target image, for example),*

$$X = \{X_{ij} \,/\, i = 1,\cdots,r \; j = 1,\cdots,s\}$$

and let X_{r_u,s_u} be a subcomponent of X which size is the size of pattern P_u. Therefore, we will define Ξ and call it the Ξ-transformation to the matrix which elements will be defined as follows:

If

$$\|[\phi_{ij}(C_k)]_{max} - [\phi_{ij}(C_k)]\| \leq \|[\phi_{ij}(C_k)]_{min} - [\phi_{ij}(C_k)]\|$$

we will modify each pixel in subcomponent X_{r_u,s_u} except for the center as follows:

$$[\phi_{ij}(C_k)] = \begin{cases} [\phi_{ij}(C_k)] + [\phi_{ij}(C_k)]_{min} & if \quad [\phi_{ij}(C_k)] + [\phi_{ij}(C_k)]_{min} \leq 1 \\ 1.0 & otherwise \end{cases}$$

and if $\|[\phi_{ij}(C_k)]_{max} - [\phi_{ij}(C_k)]\| > \|[\phi_{ij}(C_k)]_{min} - [\phi_{ij}(C_k)]\|$ then

$$[\phi_{ij}(C_k)] = \begin{cases} [\phi_{ij}(C_k)] - [\phi_{ij}(C_k)]_{min} & if \quad [\phi_{ij}(C_k)] - [\phi_{ij}(C_k)]_{min} \geq 0 \\ 0.0 & otherwise \end{cases}$$

Once we have calculated this for every pixel except the center one we will proceed to define the *geo-number* for that pixel

Definition 4.4. *An Individual Geo-number of size q for class k and pixel (i,j) or a Geo_{-q} number will be the determinant of the $q \times q$ sub-matrix obtained from the Ξ-transformation matrix.*

$$IndGeo_{-q}(i,j,k) = |\; |[\phi_{ij}(C_k)]|_q| \qquad (8)$$

(for pixels located in the border or at a distance smaller than the size selected for the matrix the $q \times q$ sub-matrix will be defined as the matrix which center is the selected pixel and the non existing elements will be defined as 1.0). For each group of $q \times q$ pixels, we will have to calculate the Geo_{-q} number. A fuzzy similarity relation will be defined based on the Geo_{-q} numbers.

Definition 4.5. *A Group Geo-number of size q for class k around pixel (i,j) be the minimum of the Individual Geo-numbers of the $(q \times q-1)$-neighbors for that class.*

$$GroupGeo_{-q}(i,j,k) = \{ \min_{(q \times q-1)neigh} \mid |[\phi_{ij}(C_k)]|_q| \} \tag{9}$$

Definition 4.6. *A Geo-number of size q for (i,j) would be the maximum of the GroupGeo-numbers of the $(q \times q-1)$-neighbors considering all the classes.*

$$Geo_{-q}(i,j) = \{ \max_{(q \times q-1)neigh} GroupGeo_q \} \tag{10}$$

Definition 4.7. *A Geo_q matrix is the matrix which elements are the Geo numbers of size q of the original matrix.*

We will denote the elements of matrix Geo_q as $geo_q(i,j)$.

The above numbers would give us the opportunity to mark an area of size q with a single representative for comparison. Homogeneity between q-neighborhoods can be defined and studied. This way the number of computations in the comparison will be considerably reduce.

Also, a comparability degree can be defined that will represent the degree of accuracy that we are looking for and it will depend on the neighborhood size. A sensibility analysis can be also performed in order to obtain a reasonable level of accuracy not compromising the number of false detections included.

¿From the Geo_q matrix we will be able to define another matrix we will call Ξ_q which elements will be selected elements from the Geo_q matrix. The process to compute the Ξ_q matrix is explained in the following section.

5 Ξ_q-transformation construction

Once we have established some definitions in the previous section, we can proceed to explain how to compute the Ξ_q matrix. The construction of the Ξ_q matrix for the target image $I_t(r,s)$ would start with pixel

$$x_{q+1,q+1} \in I_t(r,s) \qquad q \le r \quad q \le s$$

The $IndGeo_q$ numbers will be computed for each one of the pixels and each one of the features in which the target component or an area of the target component has been previously classify. Once the $IndGeo$ numbers have been computed the algorithm the system will proceed to calculate the $GroupGeo_q$ again for each one of the pixels in the target component or in an area of the target component. Finally the Geo_q number can be computed for each one of the pixels. This Geo_q already contains the information of each one of the features that has been used to perform the classification in the first place. The value of pixel $x_{q+1,q+1}$ and all its q-neighbors will be substituted by the Geo_q already calculated as explained in the above section.

The same process will be done with the rest of the pixels but following the pixel

$$x_{q,(2n+1)q+(n+1)}$$

starting with $n = 1$ and continue in the same row of pixels until

$$(2n+1)q + (n+1) \geq s$$

Then go to row $(2n+1)q + (n+1)$ and start with pixel

$$x_{(2n+1)q+(n+1),q}$$

and go on till

$$(2n+1)q + (n+1) \geq r$$

The same process has to be done with the referenced components. This process will reduce the number of comparison that would be done in order to find a possible target area. Depending on the accuracy required by the system would also reduce the number of comparisons that need to be done between the possible areas.

Once the Geo_q numbers have been calculated, the q-neighbors of each one of the pixels that have been used to compute each one of the Geo_q numbers have to be substituted by that Geo_q numbers. The elements of this new matrix that has been computed will be denoted as $\chi_q(x_{i,j})$.

The same matrix will be calculated for an equivalent area in each one of the referenced components that are going to be compared with the target component. So now we have the target component and the referenced components all defined by a single number for each one of the $q - areas$.

6 Detecting Similarities

Most of the time, similarity measures are used as mathematical tools to express how close the characteristics or features of two objects are from each other. Similarity can be considered as a conceptual way to study possible relationships between objects. In our case we are looking at how close are the *geo* numbers between two images or part of two images. We need to choose a multidimensional similarity measure since we are looking at the *geo*-matrices for each one of the classes that have been used in the unsupervised classification algorithm.

The system will have the option of real time selection of possible target areas. If the user suspects that a specific area seems to be a possible target area (target detection case) or the user wants to know a specific area location (georegistration case) he/she will be able to select a possible candidate area to be inspected. The selected area will be compared with the referenced components for possible matching. The process would be as follows: The first step after a target component has been acquired is to perform a fuzzy classification using the unsupervised classification algorithm (see Del Amo *et al.* [3]). After the classification has been performed the Ξ-transformation and the Geo_q matrix will be computed for a specific q that could be chosen depending on the size of the selected area for matching. Two aspects of this selection will need further investigation. The variation of the Geo_q numbers depending on the size of the selected component, and also how the specific q will increase or decrease accuracy in the detection of similar areas. These two topics will be the subject of a future research.

The algorithm will scan through the elements of the Geo_q matrix looking for similar $Geo{-}q$ numbers between the target component or the subimage of the target component and the referenced components. We will say that two q-neighborhoods can be further compared only if the Geo_q numbers of those two neighborhoods are similar. The similarity relation [1] will be as follows:

Definition 6.1. *Let R be a similarity relation:*

$$\begin{aligned} R: \quad & Geo_q(I1) \times Geo_q(I2) \rightarrow & [0,1] \\ & g(a,b)\, Rg(a',b') \rightarrow & 1 - \sqrt{|geo_q^2(g(a,b)) - geo_q^2(g(a',b'))|} \end{aligned} \tag{11}$$

$geo_q(g(a,b)) \in Geo_q(I1)$ and $geo_q(g(a',b')) \in Geo_q(I2)$ elements of the Geo_q matrix of the target component and referenced component respectively. We will say that two Geo_q elements are similar if $g(a,b)Rg(a',b') \simeq 1$

Once a similarity relation has been defined, we can explore partition trees and similarity classes of the similarity relation. Level sets can be defined and Geo_q groups of pixels from a particular level will be considered for further comparison. This can also be used to defined similarity classes or different level sets between the areas involved. We can defined the level sets as follows:

$$\begin{aligned} R_\alpha = \{ & [Geo_q(g(a,b)), Geo_q(g(a',b'))] \\ & R(Geo_q(g(a,b)), Geo_q(g(a',b'))) \leq 1-\alpha, \quad \alpha \in [0,1]\} \end{aligned} \tag{12}$$

Now, depending of the value of α we will decide that an area in a target component is allegeable to be label as matching pattern with a referenced component. The system will be able to give a solution to the user at this moment or if a more accurate solution need to be obtained a study of the homogeneity of both groups of pixels can be performed before an answer is output to the user.

Definition 6.2. *We will say that two Geo_q numbers $Geo_q(g(a,b))$ and $Geo_q(g(a',b'))$ are*

- *a very strong match if and only if they belong to the same class $[R_0, R_{0.1}]$*
- *a strong match if and only if they belong to the same class $(R_{0.1}, R_{0.3}]$*
- *a fairly strong match if and only if they belong to the same class $(R_{0.3}, R_{0.5}]$*
- *a fairly weak match if and only if they belong to the same class $(R_{0.5}, R_{0.7}]$*
- *a weak match if and only if they belong to the same class $(R_{0.7}, R_{0.9}]$*
- *a very weak match if and only if they belong to the same class $(R_{0.9}, R_{1.0}]$.*

Once a characterization of the degrees of matching between Geo_q numbers has been defined a study of the homogeneity of the individual pixels to the neighborhood can be performed. When the degree of homogeneity between the pixels of two

[1] It can be easily proof that R is a similarity relation

matching neighborhoods is fairly similar then we can concluded that we have found a matching pattern, and we can proceed to the identification of the geolocation or description of the target component based on the characteristics of the referenced components that is been match to.

The system that is been envisioned will have the option of interactive selection. For example, let's say that a very quick decision has to be made, in this case the system can decided that if a *very strong match* has been found between two q-areas no further analysis has to be performed and it can be concluded that the two areas match. But if we are more interested in the accuracy of the decision to be made then further analysis can be performed. For example, the homogeneity of the pixels in a neighborhood can also be studied. But homogeneity can be studied at two different levels. We can define homogeneity on respect to the elements of Geo_q matrix or we can also define homogeneity on respect of the $GroupGeo_q$ numbers.

¿From the level sets a matching degree between the Geo_q numbers can be defined.

Definition 6.3. *We will say that β_q is the matching degree between $Geo_q(g(a,b))$ and $Geo_q(g(a',b'))$. The definition of β as follows:*

$$\beta_q(Geo_q(g(a,b)), Geo_q(g(a',b'))) = 1 - \alpha \tag{13}$$

where α is the similarity class both pixels belong to.

Let's define what we will call homogeneity.

Definition 6.4. *Let φ_Ξ be the degree of homogeneity of pixel $x_{r,s}$ on respect to the Geo_q matrix*

$$\varphi_\Xi(x_{r,s}) = |\chi_q(x_{i,j}) - geo_q(x_{r,s})| \tag{14}$$

where $\chi_q(x_{i,j})$ is the GroupGeo number for pixel $x_{i,j}$ and $Geo_q(x_{r,s})$ is one of the elements of the Geo_q matrix in the q-neighborhood of element $x_{r,s}$.

Definition 6.5. *If $\varphi_\Xi < \epsilon \quad \forall x_{r,s}$ in the target component and $\varphi_\Xi < \epsilon \quad \forall x_{r',s'}$ in the reference component,we will say that a match has been found and the degree*

The variation on the degree of homogeneity of a particular neighborhood will show the possibility of edges and therefore the possibility of different objets in the same region.

7 An improved algorithm

The original unsupervised classification algorithm proposed by the authors was applied to a remotely sensed image obtained from Sevilla surroundings, south Spain (see [3] for a description of the unsupervised algorithm and [5] for an application to a real digital image). Results were considered extremely good, as the representations included a natural description of vegetation cover (forest, scrubland, swampland, etc.) The advantage of a fuzzy approach against a classical crisp approach was obvious: that particular picture was quite well explained assuming the existence of a few fuzzy classes, allowing of course natural mixtures between those *in between* classes, showing in addition a natural structure of our classification system. The results not only showed a list of classifications, but also identified some relationships between the classification of different areas. Moreover, results obtained by means of a few fuzzy classes needed a significantly larger number of crisp classes in order to get equivalent results; and a naive look at the considered image showed the existence of three main fuzzy classes, with no sharp boundaries among them. When talking about Earth vegetation cover, fuzzy classification may be much more natural and accurate than a corresponding crisp classification. The key issue is the conceptual accuracy given by fuzzy classes. If reality shows natural fuzzy classes, with no crisp borders but gradation between classes, we should expect that fuzzy approaches will be more accurate.

8 Final comments

The goal of this research is to build up a model for classification whenever a comparative analysis is an essential part of the available information. In particular, we pursue an automatic detection system based on a previous algorithm developed by Del Amo *et al.* [3, 5], which was the core of a classification system based on fuzzy set theory (see [14]). In this way we can gain the advantage of non probabilistic imprecision (entities with no sharp borders), in the automated process of assigning image areas to pre-defined surface types. Such an algorithm was presented by the authors as a multicriteria decision making problem, by means of an outranking methodology [12], as considered also in [11].

The original algorithm [3, 5] was a unsupervised classification algorithm based on modified classic outranking methods in order to fit the objective of the model. This paper offers key modifications on the original algorithm in order to allow its supervised version plus its application to georegistration and target detection.

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