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On Four Intuitionistic Fuzzy Topological Operators

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Abstract

Four new operators, which are analogous of the topological operators "interior" and "closure", are defined. Some of their basic properties are studied. Their geometrical interpretations are given.

Keywords: Intuitionistic fuzzy set, Topological operator

Initially, we shall introduce some basic definitions, related to the Intuitionistic Fuzzy Sets (IFSs), following [1].

Let a set E be fixed. An IFS A in E is an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in E\},$$

where the functions $\mu_A : E \to [0,1]$ and $\nu_A : E \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in E$, respectively, and for every $x \in E$:

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

Let an universe E be given and let the figure F in the Euclidean plane with Cartesian coordinates be given (see Fig. 1).

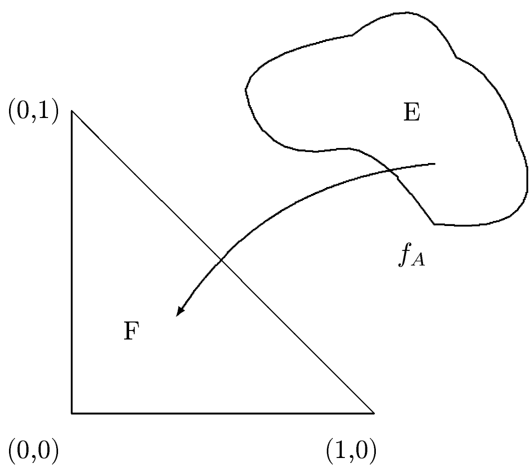

Fig. 1.

Let the IFS A be fixed. Then we can construct a function f_A from E to F, such that if $x \in E$, then

$$p = f_A(x) \in F,$$

the point p has coordinates $\langle a, b \rangle$ for which:

$$0 \leq a + b \leq 1,$$

and these coordinates are such that:

$$\begin{array}{l} a = \mu_A(x), \\ b = \nu_A(x). \end{array}$$

Therefore the function f_A is a surjection (see [2]).

For every two IFSs A and B a lot of operations, relations and operators are defined (see, e.g. [1,3]), the most important of which, related to the present research, are:

$$\begin{array}{lcl}
A \subset B & iff & (\forall x \in E)(\mu_A(x) \leq \mu_B(x) \& \nu_A(x) \geq \nu_B(x)); \\
A \supset B & iff & B \subset A; \\
A = B & iff & (\forall x \in E)(\mu_A(x) = \mu_B(x) \& \nu_A(x) = \nu_B(x)); \\
\overline{A} & = & \{\langle x, \nu_A(x), \mu_A(x) \rangle | x \in E\}; \\
A \cap B & = & \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x)) \rangle | x \in E\}; \\
A \cup B & = & \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x)) \rangle | x \in E\}; \\
\Box A & = & \{\langle x, \mu_A(x), 1 - \mu_A(x) \rangle | x \in E\}; \\
\Diamond A & = & \{\langle x, 1 - \nu_A(x), \nu_A(x) \rangle | x \in E\}; \\
C(A) & = & \{\langle x, K, L \rangle | x \in E\}; \\
I(A) & = & \{\langle x, k, l \rangle | x \in E\}.
\end{array}$$

where

$$K = \max_{y \in E} \mu_A(y), \quad L = \min_{y \in E} \nu_A(y)$$

and

$$k = \min_{y \in E} \mu_A(y), \quad l = \max_{y \in E} \nu_A(y).$$

The geometrical interpretations of the above operations and operators (and of the other ones) are discussed in [2]. Here we shall show only the geometrical interpretations of the last two operations about the IFS A from Fig. 2. - see Fig. 3 and 4.

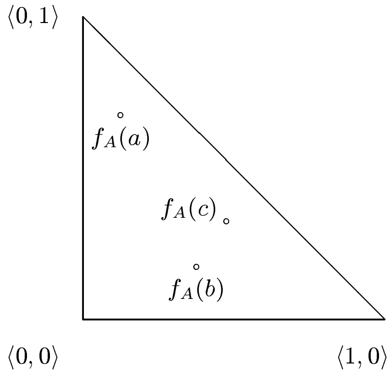

Fig. 2.

Up to now, these two operators, which are in some sense analogous of the topological operatos "closure" (C) and "interior" (I), transform a given IFS to a new one, all elements of which have equal degrees of membership and non-membership. This situation really correspond to the sense of the two topological operators, but this correspondences are in an extremally powerful forms.

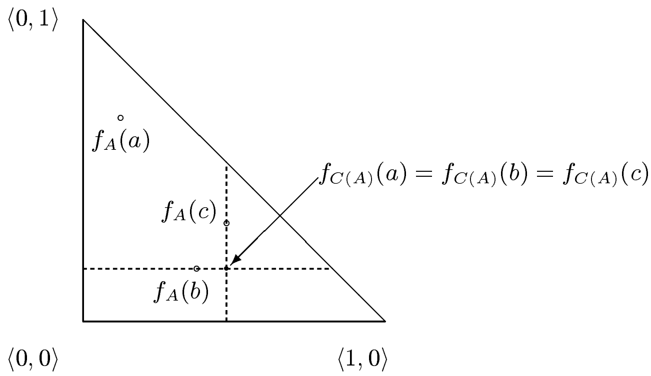

Fig. 3.

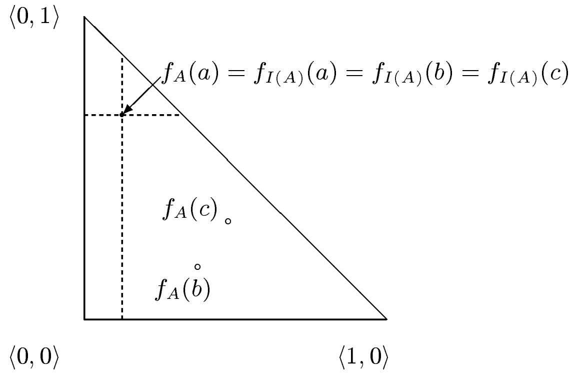

Fig. 4.

Now, we shall modify these two operators. They will be again analogous of the two topological operatos "closure" and "interior" (I), but now their forms will be not so powerful. Their definitions are the following:

$$\begin{aligned} C_\mu(A) &= \{\langle x, K, min(1-K, \nu_A(x))\rangle | x \in E\}; \\ C_\nu(A) &= \{\langle x, \mu_A(x), L\rangle | x \in E\}; \\ I_\mu(A) &= \{\langle x, k, \nu_A(x)\rangle | x \in E\}; \\ I_\nu(A) &= \{\langle x, min(1-l, \mu_A(x)), l\rangle | x \in E\}, \end{aligned}$$

where K, L, k, l have the above forms.

The geometrical interpretations of the new operators about the IFS A from Fig. 2 are shown on Fig. 5 - 8.

Obviously, for every IFS A:

$$I(A) \subset I_\mu(A) \subset I_\nu(A) \subset A \subset C_\nu(A) \subset C_\mu(A) \subset C(A).$$

Theorem 1. *For every IFS A:*
(a) $C_\mu(C_\nu(A)) = C_\nu(C_\mu(A)) = C(A)$,
(b) $I_\mu(I_\nu(A)) = I_\nu(I_\mu(A)) = I(A)$,
(c) $C_\mu(I_\mu(A)) = I_\mu(C_\mu(A))$,
(d) $C_\nu(I_\nu(A)) = I_\nu(C_\nu(A))$,
(e) $\Box C_\mu(A) = C_\mu(\Box A)$,
(f) $\Diamond C_\mu(A) \subset C_\mu(\Diamond A)$,
(g) $\Box C_\nu(A) \subset C_\nu(\Box A)$,
(h) $\Diamond C_\nu(A) \supset C_\nu(\Diamond A)$,
(i) $\Box I_\mu(A) \subset I_\mu(\Box A)$,
(j) $\Diamond I_\mu(A) \supset I_\mu(\Diamond A)$,
(k) $\Box I_\nu(A) \supset I_\nu(\Box A)$,

(l) $\diamondsuit \underline{I_\nu(A)} = I_\nu(\diamondsuit A)$,
(m) $\overline{C_\mu(\overline{A})} = I_\nu(A)$,
(n) $\overline{I_\mu(\overline{A})} = C_\nu(A)$.

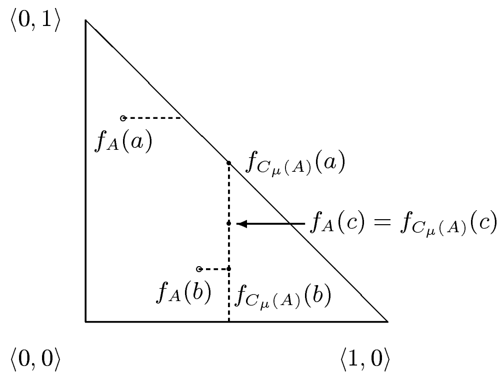

Fig. 5.

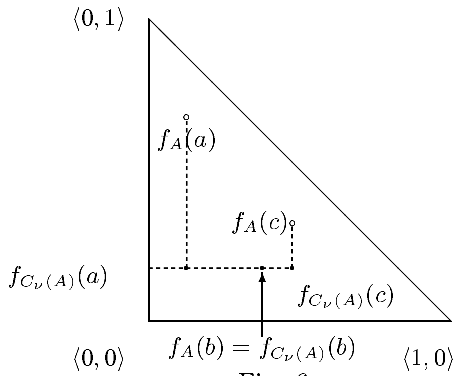

Fig. 6.

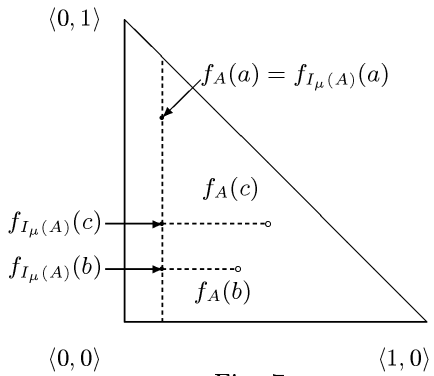

Fig. 7.

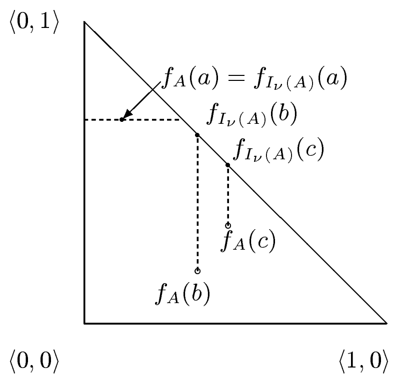

Fig. 8.

Theorem 2. *For every two IFSs A and B:*
(a) $C_\mu(A \cup B) = C_\mu(A) \cup C_\mu(B)$,
(b) $C_\nu(A \cup B) = C_\nu(A) \cup C_\nu(B)$,
(c) $C_\mu(A \cap B) = C_\mu(A) \cap C_\mu(B)$,
(d) $C_\nu(A \cap B) = C_\nu(A) \cap C_\nu(B)$.

The vallidities of the above assertions follow directly from the definitions.

References

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