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Formal Validation of Fuzzy Control Techniques. Perspectives

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Abstract

In this paper, a survey of the state of the art and perspectives of two main lines of research in fuzzy control systems is presented: on one hand, the *interpolative-functional line* representing fuzzy systems as parameterized universal function approximators, thus applying nonlinear control and neural network paradigms; on the other hand, a *logic-formal* approach where fuzzy systems are analysed in terms of logic interpretations, exploring validation, consistency and completeness, uncertainty management and knowledge supervision. The former approach is not widely used in the control field, and asserting its possibilities is one of the main motivations of this paper. References in both lines and some abridged results in the second case are cited, in particular some of the author's contributions, to which the interested reader is referred.

Keywords: Fuzzy Control, fuzzy logic, validation, expert control.

1 Introduction.

Quality improvement in industrial processes implies, to a great extent, efficient control strategies. The neural (connectionist) and fuzzy (knowledge-based) approaches, so called *intelligent control*, sprang in the last 25 years as candidate paradigms to achieve efficient control of complex, high-dimensional processes.

However, in industrial practice, application of connectionism with learning capabilities [12] is at an initial stage. Notwithstanding, practicioners accept with interest fuzzy logic applications, mainly due to the parallelism with their reasoning schemes and their explanation capabilities: Those capabilities are a great advantage with respect to neural networks towards satisfactory user interaction.

But, even with the increasing popularity of fuzzy systems in practice, many current applications of fuzzy control theory are in low-dimensional or nearly decoupled processes. Furthermore, fuzzy controllers, originated from a logic of "vague" and "imprecise" concepts, are used in most cases to approximate precise, deterministic controller functions, thus contradicting some of the principles that ground the usefulness of fuzzy logic.

A problem posed by current structures is the difficulty of learning and validation when applied to systems requiring a complex strategy. Validation is carried out experimentaly in most cases, with hundreds of simulations and, in industrial practice, modifications of rulebases in expert control systems are carried out in a manual, tedious way, by trial and error. An example of that is the authors' experience in application to cement kilns [11].

Integration of fuzzy and neural techiques gives rise to neurofuzzy structures that usually combine linguistic interpretation (amenable to logic analysis) with a parametric approximator structure, in many cases linear in them so that certain learning results [22, 8, 2] can be applied. *Granulation* and *locality* are the key concepts in these approaches. Notwhithstanding, most learning techniques in the fuzzy control arena use only the parametric interpretation, so that gradient and/or random search methods are used for parameter updates.

By means of the parametric approach,stability of adaptive control can be guaranteed for systems in a certain class of matemathical models [14, 22, 20]. In systems with unknown models apart from basic qualitative characteristics, those techniques cannot be applied with total confidence in practical situations, although paradoxically fuzzy control was born to control precisely those ones.

So, these reflections are the main motivation of the research line in this paper: the lack of use of tools from fuzzy logic in the control arena, outweighted by a "parameterised function approximator" approach in which differences between neural nets, fuzzy systems, splines, etc. vanish: only gradients and approximation errors are considered.

This introduction summarizes the authors' experience and opinion, the conclusion being that additional problems arise in order to create efficient knowledge-based controllers in complex systems. A possible solution might be a knowledge-acquisition supervision layer in fuzzy and neurofuzzy systems. This layer would evaluate the quality and quantity of available knowledge to decide between adaptation and restructuration, and to ensure the absence of internal contradictions.

A more detailed critical discussion of the current fuzzy control practice and the outline of the authors' contributions [15]–[18] towards creating that knowledge-acquisition supervision layer (in principle, focused towards function approximation for learning control purposes) is the main objective of the paper.

The structure of the paper is as follows: Next section details some of the important issues in current fuzzy control practice outlined in this introduction. Section 3 presents the possibilities of control performance enhancement that motivate the research line in formal techniques for fuzzy control. Definitions and results in that line are briefly outlined in section 4 (references for further details are presented). Some conclusions are presented in the last section.

2 Some issues about current fuzzy controllers.

Let us think of a configuration in which the objective is generating a set of fuzzy rules that control a system. The rule generator entity can be a human expert or a learning algorithm (numeric or symbolic).

In most applications of fuzzy logic in control, some questions arise that many times are neglected or solved in an ad-hoc manner. The following items discuss some of them.

Multiplicity of operators. Generalisation of key binary operators presents a multitude of variants (implication [9], defuzzification [22], rule chaining), based on formal or intuitive grounds. Some of them give rise to severe problems in case of nonconvex consequents or "fat shape" subsets. In certain cases, they can be solved by further refining via additional entropy-like coefficients or other heuristics [24]. The choice of logic operators, implication and defuzzification in industrial practice [1] is based in ease of implementation and end-user understanding.

Simple interpolation interface. In most applications and theoretic control developements fuzzy logic is considered only as a convenient, intuitive user-oriented interface to multidimensional interpolation routines, most with linear in parameter universal approximators in the form:

$$u = \frac{\sum \phi_i(\mathbf{x})\theta^i}{\sum \phi_i(\mathbf{x})} \tag{1}$$

where $\mathbf{x}$ is the system state, θ^i are the adjustable parameters for the learning phase and ϕ_i are named "membership functions", "basis functions´´, etc. Expression (1) may be formally equivalent to other interpolation techinques, such as splines, radial basis neuronal nets, etc. If functions ϕ_i also have adjustable parameters, equation (1) is analogous to some multilayer neural network configurations. The obtained results do not rely in any fuzzy-related concept.

Numeric vs. conceptual processing. In most references on fuzzy control, emphasis is biased towards the numeric aspect of fuzzy regulators, i.e. they stand as parameterised nonlinear approximators (1) in order to obtain results on stability, Lyapunov functions, linear parameterisations, etc. [22]. Thus, in that context, part of the initial spirit of fuzzy logic (concept processing: "computing with words") has been lost.

Where is vagueness?. The conclusion of the previous items is that fuzzy logic, born as a logic for vague and imprecise linguisitic models, is used in most control applications in a completely *deterministic* framework (such as the previously presented interpolators).

These ideas are presented here in an intentionally absolute, maybe exaggerated way. Of course, confronting these argumentations, other points of view based on different core interpretations of "fuzziness" [6], and the need of the final controller to be deterministic can be pointed out.

Notwithstanding, the present "parametric" and "interpolative" bias of fuzzy control has reached a series of achievements justifying it. Some of the most significant are referred here:

- Simple identification algorithms, in certain context, derived from standard Least Squares [22, 10].
- Linear matrix inequalities over convex combinations of linear systems [20].
- Direct applicability of nonlinear control results: small gain, descriptive function, identification and adaptive control of control affine systems [14, 3].
- Genetic algorithms and derivative-based neuronal-like learning [5]: backpropagation, Newton, Levenberg-Marquardt.

Those sucesses have to be confronted to these three drawbacks related to linguistic, conceptual processing:

- the multiplicity of fuzzy operators for connectives and implication with little practical relevance,
- the fact that in some references the capabilities of fuzzy controllers are exaggerated, without a proper comparison to other conventional techniques (such as PID's),
- many good-performance, expert-based regulators are adjusted in practice by trial and error

So, the result is a credible explanation of the referred emphasis in just the numeric processing (universal approximation property) regarding fuzzy systems in the control field.

3 The role of formal fuzzy methods in control.

In the previous section, the present situation of the fuzzy control theory field has been outlined. But, on the other hand, there has been a significant success in application of simple fuzzy controllers. The reasons of that success and the possibilities are the motivation of this section.

The success in practical control applications of fuzzy logic is due to the capability of using models with "ambiguous" concepts in order to *reduce the intuitive complexity* of a process, so that control, planification and supervision can be carried out, even perhaps in a crude, approximated way, over nonlinear and time-varying plants.

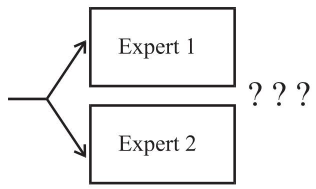
Hence, its success is an issue of simple representations, in a similar way to linear time invariant systems, which are, too, complexity reduction entities that allow the design of approximated controllers. Depending on the system, a few hand-tuned fuzzy rules may perform satisfactorily without the need of classical modelling and control approaches (which, in the case of complex systems may be costly and hard to validate). Although this issue of representation may not be a very clear case for perfectly-known academic example processes (differential equations), it is a fact in industry.

After analising the achievements of fuzzy control, the possibilities and limits for its future have to be considered. With respect to the possibilities, as fuzzy systems are universal approximators of *smooth* functions, further achievements in the arena of nonlinear adaptive and geometric control research can be expected.

In the following, a reflection will be made about the existence of possibilities for formal fuzzy logic tools to be applied in improving fuzzy control by solving

If T is $High$, u is $Open$
if T is $High$, u is $donotOpen$
if T is $Medium$, u is $donotchange$
if T is Low, y is $Close$
if T is Low, y is $CloseSlightly$
...
if h is $High$, (T is Low or Q is $High$)
if h isn't $VeryLow$ or T is $High$, Q is $Normal$

(a) Complex rule base.



(b) Knowledge fusion.

Figure 1: Need of logic-based methods.

the questions that arised in previous sections, and some additional ones, not easily dealt with by means of just the numeric-functional approach.

Vagueness of knowledge. In control engineering, two fundamental sources of ambiguousness have to be tackled by intelligent controllers:

- *Plant model ambiguousness*: the noise, time variability, nonlinearity and lack of knowledge about details of the process gives as a consequence that in most practical cases only an intuitive model of the plant to be controlled is available, approximately describing low order dynamics, in long time scales.
- *Control specification ambiguousness*: the "cost index" (control specifications) to be optimised, directly related to final quality and production cost, is often expressed in a qualitative, vague form. In practice, experience and "intelligence" of human operators are used to compute *setpoints* for the actual, hierarchically subordinate, regulators. Hence, there might be no noticeable final quality improvement by enhancing the performance capabilities of those low-level regulators, if setpoints are suboptimal.

Complex rulebases. Fusion. If a complex rule base is present (either created by an expert or by a learning algorithm), such as the one in figure 1(a), it would be useful to detect abnormalities such as contradiction between rules, redundancies, missing rules to complete a rulebase, etc. In a similar context, if linguistic information is coming in from different sources (human or learning algorithms, fig. 1(b)), mechanisms of verification of their agreement and fusion strategies should be devised.

Interpretability of results (readability). In the functional view of fuzzy systems, a model (1) with a sufficiently high number of basis functions ψ_i can uniformly approximate any smooth continuous function [12]. A problem arises in many cases: due to the presence of a great number of parameters, the (neuro-)fuzzy models lose their original intention (i.e., grasping the fundamental characteristics of the process to reduce complexity). So, there seems to exist a need to weight linguistic interpretability versus approximation accuracy. These questions may be discused in further detail: introduction of validation (regularisation) indexes in learning algorithms may open the possibility of a directed learning with better generalisation and end-user readability [13].

Learning and supervision. In the case of control with a learning algorithm, thus including an *evaluation* of performance in terms of control specifications, some decisions must be made: parameter modification, the decision to undergo structural modifications, the efficient coding of the available knowledge, the verification of readability of results, etc. Those could be carried out by a *knowledge acquisition supervisor*, hierarchically higher.

3.1 Perspectives of fuzzy control.

The conclusions of the previous considerations are, as a matter of fact, that there is a role to be played in fuzzy control for the formal, logic based, knowledge management:

- *Fuzzy methods must deal with uncertainty management in intelligent control systems, providing information about modelling error, confidence, etc.* A truly "intelligent" system should combine a model and a rough idea of its validity range and accuracy.
- *The research line is different, complementary to the "interpolative" approach*: a list of parameters θ^i is a useful model but, in fact, it's a poor representation of what "knowledge of the process" means for a human operator.
- *Knowledge management must aim to improve learning capabilities by introducing validation notions such as*:
  - Consistency (knowledge *quality*).
  - Completeness (knowledge *quantity*).
  - Eficiency (used resources, redundancy detection, simplicity of representation, preservation of user interpretability).

The conjunction of both approximations to fuzzy control could give rise to control systems with features such as:

- Hierarchic systems, combining several plant models with different precision.
- Consistent combination of several knowledge sources: functional (first-principle models), heuristic (linguistic information from humans) and experimental (plant data).
- Fault detection, when the plant contradicts the knowledge the controller has about it.
- Knowledge acquisition supervisors.
- Advanced learning: combining structural and parametric modifications while preserving readability. Automatic discovery of dimensionality-reducing approximations (for example, if the long term behaviour can be considered first-order, if two sets of measurements are interdependent, etc.).

4 Formal approach for fuzzy control. An outline.

The discussion of the various unsolved issues in fuzzy control applications presented in previous sections motivated the work of the authors in formal fuzzy methods for function approximation in control [15]–[18]. Some other works [7, 13, 19] also address related problems from a point of view not specific of the control area.

4.1 Objectives of knowledge analysis (validation).

The knowledge a controller has about the process it regulates should be an "intelligent" compilation of past experience, thus requiring a suitable analysis of the knowledge gathered.

Fuzzy systems, to be used in the control context, should be previously validated to avoid wrong outputs. Under the term "validation" appears the set of operations to determine the fitness and usefulness of the knowledge to fulfill a certain set of control specifications. The validation applies for either algorithm implementation, fuzzy process models or the actual fuzzy logic controllers. Some general properties and definitions will outline the available tools in the following subsections.

Fuzzy controller validation can be carried out under two complementary points of view: *internal* (logic structure verification) and *external* (experimental verification of fitness, reduced error).

The most common learning paradigms aim at the second one. From the point of view of improving learning methods, experimental invalidity can have different consequences, depending on its grade: slight discrepancies would pinpoint the need of parametric adaptation, while greater discrepancies would perhaps either imply the need of structural adaptation (rule addition or deletion) or just to be used for fault detection purposes.

Internal validation can be used to improve quality and speed of the learning process [13, 15]. At the current situation, internal validation can be approached in two ways:

- Logic verification of complex rulebases (oriented to expert control applications).
- Verification of the validity of numeric interpolative-like inference algorithms over a particular rulebase (assumed consistent): for example, centroid defuzzification methods are not advised when some of the consequents are wider than the rest, or there are nonconvex sets. If a learning algorithm can produce any of those set shapes, this validation should be carried out in an *automatic* way, so that the system can always ensure that the algorithm used to obtain control actions efficiently recovers the conceptual information encoded in the rulebase [17].

Rule validation originated in the binary logic field [19, 4], being recently extended to the fuzzy case [23, 7, 17] taking as a starting point, in some cases, first definitions of consistency from the compatibility measure (peak of the intersection) in [25]. In [15] a sound examination of the meaning of validation of functional approximation for control purposes is carried out. A selection of some significant results is outlined in the rest of this article. Proofs and examples may be consulted in the referenced works.

4.2 Basic definitions.

The starting point is the equivalence between a rule and an inequality inspired by some implications of the Lukasievicz class [17, 9].

Definition 1 *Let u and y be numeric variables belonging to two universes of discourse (input universe U and output Y, respectively), and let $A \subset U$ and $B \subset Y$ be two fuzzy sets defined by membership functions $\mu_A : U \to [0,1]$ and $\mu_B : Y \to [0,1]$.*

Then, the rule "If u is A, then y is B" is defined equivalent to the inequality [17]:

$$\mu_A(u) \leq \mu_B(y) \tag{2}$$

and in the same way the rule "If and only if u is A, then y is B" is defined equivalent to the equation $\mu_A(u) = \mu_B(y)$.

Definition 2 *Given a rule set, the* ideal conclusion *is defined as the set of solutions of the equivalent equation set, assuming as known the value of certain variables (premises).*

Example. The rule base:

1- u is Low $\Rightarrow$ y is Not Low AND z is Low
2- u is Medium $\Leftrightarrow$ y is Low
3- u is High$\Rightarrow$ y is High OR z is High

is, by definition, equivalent to:

$$Low(u) \leq \min(1 - Low(y), Low(z))$$
$$Medium(u) = Low(y)$$
$$High(u) \leq \max(High(y), High(z))$$

Inference consists in, for a certain u, finding which values of y and z verify the equations and inequalities, given a certain shape for the membership functions. If it were the case, for example, in which $Low(u) = 0.5$ and $Medium(u) = 0.6$, no solution would exist: a contradiction between rules 1 and 2 arises.

Note that *defuzzification* does not exist in the presented framework as a fundamental theoretical entity. The presence of defuzzifiers in practical implementations is just a quick way of obtaining, under appropriate assumptions, a solution to the equations or a good approximation of it.

The expression of a fuzzy model as a set of equations opens up the possibility of combining those equations with other ones obtained by experimentation or first principles.

The logic properties of the rulebase are defined in parallel with those of existence and uniqueness of solutions in the equivalent equation set:

Definition 3 *If the equation set representing a rule base has no solution, the rulebase is said to be* contradictory *(for a given set of premises). It it exists at least one solution to them, the rulebase is said to be* coherent. *If the solution is unique, the rulebase is* complete, *and if it is nonunique, the rulebase is said to be* incomplete. *If the solutions are the same after ellimination of one of the equations, then the corresponding rule is said to be* redundant.

To allow inference over contradictory rulebases, a contradiction index associated to each rule will be defined, and named *inference error*. Defined $\epsilon : U \times Y \to [0,1]$ it has the expression:

$$\epsilon_{IF}(u,y) = \begin{cases} 0 & \mu_A(u) \leq \mu_B(y) \\ \mu_A(u) - \mu_B(y) & \mu_A(u) > \mu_B(y) \end{cases} \tag{3}$$

$$\epsilon_{IIF}(u,y) = |\mu_A(u) - \mu_B(y)| \tag{4}$$

The inference error is null if y is an ideal conclusion of the rule for premise u, and 1 if y totally contradicts the rule for that premise.

Once the inference error for each rule is given, a cumulative error function for a N-rule rulebase is formed:

$$\epsilon(u,y) = \left(\sum_{i=1}^{N} \phi_i(u,y)\epsilon_i(u,y)^p \right)^{\frac{1}{p}} \tag{5}$$

where p is a user-defined parameter, with unity reference value. The functions ϕ_i are weights that can be interpreted as confidence coefficients, defined by the rulebase designer.

In this way, ideal inference (def. 2) is generalised by defining the *ideal conclusion* as the set of y values minimising $\epsilon(u,y)$. Inference is thus converted to the minimisation of a *contradiction cost index*.

4.3 Validation formulae.

Suppose that either an expert or a learning algorithm provide a controller $u = f(e)$ or process model $y = f(u)$, represented by a rulebase to be validated.

Knowledge structure. Coherence and redundance **structure** can be evaluated, for example, by coherence and redundance coefficients, as presented in [15]. When particularised to a pair of rules, they have the expression:

$$c_{ij}(u) = \min_{y|\epsilon_j(u,y)=0} \epsilon_i(u,y) \quad \rho_{ij}(u) = \max_{y|\epsilon_j(u,y)=0} \epsilon_i(u,y) \tag{6}$$

These definitions allow for the possibility of deriving formal tools to modify, in an automatic way, the shape of membership functions to improve internal consistency. Moreover, given two rules with similar antecedents and consequents, upper bounds on the contradiction (inference error) incurred if one of them is deleted can be calculated by means of the redundancy coefficient.

Inference algorithm validation. With respect to validation of conventional fuzzy inference-defuzzification algorithms, if $y = f(u)$ is the output of a certain algorithm (for example, some of the many ones similar to (1)), then the rule inference error $\epsilon(u, f(u))$ provides an invalidity measure of that algorithm. That measure depends both on the particular algorithm used and on the shape of the fuzzy sets. Obviously, in order to interpret it correctly, the rulebase over which it is applied is assumed to be coherent, in the sense previously defined.

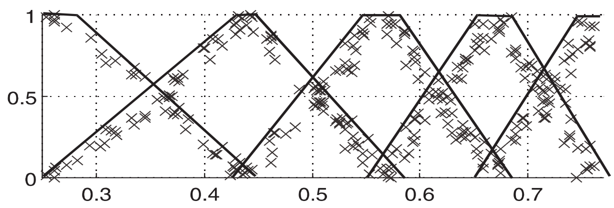


Figure 2: Consequents for coherent modeling of noisy data.

If $f(u)$ is a generic function to be modelled by the rulebase (used as a functional approximator), the rulebase is said to consistently model the function f if $\epsilon(u, f(u)) = 0$ for all u.

In that case, it can be proved that the rule antecedents (A_i) and consequents (B_i) verify $A_i \subseteq f^{-1}(B_i)$ where f^{-1} is defined according to the *extension principle* $\mu_{f^{-1}(B_i)}(u) = \mu_{B_i}(f(u))$. For IIF rules, the equality $A_i = f^{-1}(B_i)$ holds.

This functional interpretation can explain some intuitive procedures for constructing rulebases ensuring coherence. In the same way, the inequalities and functional interpretation enable to devise minimum contradiction learning algorithms [16].

4.4 Fuzzy Modelling.

Intelligent control systems must deal with uncertain, ambiguous fuzzy models. These fuzzy models can be obtained from various sources: either from expert rule assertions, by fuzzifying an approximate algebraic model, or by identification over a set of (maybe noisy or partial) experimental data. In the third case, if the algebraic interpretation (2) is used, given a point (u, y) from a training set and an antecedent A, if the rule equations are fulfilled, at point y, the consequent membership has to be greater than $\mu_A(u)$. So after setting a mark in output space at $(y, \mu_A(u))$ for each point in the training set, any fuzzy set that encompasses all marks can be a coherent consequent for the antecedent A. Hence, this one-pass algorithm can provide a coherent fuzzy set representation of the underlying function in the form of a multivalued mapping [21]. It will output wider and more overlapping consequents for fixed antecedents according to the amount of uncertainty presented on a data set. For example, noisy data from a nonlinear valve, given an add-1 triangular partition of antecedents, produce the consequents depicted in figure 2. The reader is referred to [16, 15] for details.

4.5 Control.

With respect to control, the formal methods make easier the task of designing coherent rulebases and faithfully extracting the information from them in an expert control environment.

In addition to that, if a fuzzy plant model is available, an easy way of dealing with model-based controller design problems is the plant inversion. Based on the equivalent equations (if the process is stable, minimum-phase), the equations of the fuzzy regulator are the same than those of the model, but in this case the regulator infers u assuming the desired ouput $y_{\rm ref}$ as a premise. With ambiguous models (incomplete rulebases), the solution is the set of control actions that *possibly* produce $y_{\rm ref}$. This is called the *lower* plant inverse [21]. To deal with the ambiguousness, the control actions that *necessarily* produce an output inside $[y_{\rm ref} - \delta, y_{\rm ref} + \delta]$ must be calculated. Hence, instead of the inference just mentioned, the equivalent equations are used to find the complementary of the actions that produce an output with nonzero possibility of laying outside the interval. This inverse is named *upper* plant inverse. If model uncertainty is big, then the upper inverse might be empty: the chosen output interval must be made wider.

In [15, 18] academic examples on feedback linearisation and sliding control are presented, not described here due to space limitations. They apply the plant inversion strategy just mentioned.

5 Conclusions

In this work, presentation and discussion of a number of current issues about fuzzy logic applications in control have been made, as well as about their future possibilities.

The combination of a "logic" approach with a "functional" one is necessary to obtain results with greater "intelligence" with respect to, for example, multimodel integration and knowledge acquisition supervision.

The conjunction of the two approximations to fuzzy control could give rise to control systems with innovative features (several plant models with different precision, consistent combination of several knowledge sources, fault detection, advanced learning combining structural and parametric modifications while preserving readability, etc.).

After the motivation and discussion of the need of validation issues, a summary of results in contradiction and redundancy and its use in control have been outlined, with bibliography references for further details and examples.

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