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Hierarchical Decomposition of Fuzzy Controllers Based on Meta-knowledge

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Abstract

This paper focus on the problem of decomposing multivariable fuzzy controllers using a hierarchical approach based on the application of meta-knowledge. Usually, hierarchical fuzzy systems are based on a cascade structure of fuzzy logic controllers where the output of each level is considered as one of the inputs to the following level. The paper introduces a different approach, to the idea of hierarchy, where the output of a level is considered not as input to the following level controller but as a semantics modifier.

1 Introduction

Fuzzy Logic Controllers (FLCs) have being widely and successfully applied to different areas. However the application of FLCs to large scale systems presents different problems. The definition of Fuzzy Rules is a key task when designing FLCs, and its difficulty may range from the simplicity of designing a regulation system using a PD-like fuzzy controller, for which multiple Rule Bases are available in literature, to the complexity of most industrial applications with a large number of variables and a highly non-linear behavior. A fundamental problem that the application of FLCs to large-scale complex systems encounters is the high dimensionality of the rule base. As the number of variables increases, the number of rules increases exponentially ([7]).

The application of fuzzy techniques to large scale systems ([3]) is a challenging problem. Different approaches to reduce the complexity of the rule base have been proposed. These approaches try to cope with the complexity of the system in a variety of ways, that following [3] are:

1. Fusing sensory variables before feeding them to the inference engine, thereby reducing the size of the inference engine.
2. Grouping the rules into prioritized levels to design a hierarchical fuzzy controller.
3. Reducing the size of the inference engine directly using notions of passive decomposition of fuzzy relations.

4. Decomposing a large system into a finite number of reduced-order subsystems, thereby eliminating the need for a large-sized inference engine.

Most of these approaches are based in decomposing the system using different techniques. In this paper, hierarchical decomposition will be analyzed. In [8], game theory is applied, considering the different subsystems as players that correct each other actions.

2 Hierarchical Fuzzy Controllers

Hierarchical fuzzy controllers were proposed in [7] as a way of coping with the dimensionality problems of rule bases in FLCs.

A fuzzy controller with n input variables where each variable is represented by l linguistic labels, will have a rule base with l^n rules. This fuzzy controller will apply rules of the following form:

$$R_j : \textbf{if } (x_1 \text{ is } A_{1j}) \textit{ and } \ldots \textit{ and } (x_n \text{ is } A_{nj}) \textbf{ then } y \text{ is } B_j$$

where x_1 to x_n are the input variables, y is the output variable and A_{ij} and B_j are fuzzy sets related to the corresponding input or output variable.

In a conventional FLC, all the rules are used, in a single step, when calculating the output of the controller. In a hierarchical fuzzy controller, variables, and consequently rules, are divided into different levels in such a way that the most influential variables are chosen as input variables at the first level, the next most important variables are chosen as input variables at the second level, and so on. In addition, the output variable of each level is introduced as input variable at the following level.

The rules in the first level of such a controller are as follows:

$$\begin{array}{rll} R_{1i}: & \textbf{if} & (x_1 \text{ is } A_{11i}) \textit{ and } \ldots \textit{ and } (x_{n_1} \text{ is } A_{1n_1i}) \\ & \textbf{then} & u_1 \text{ is } B_{1i} \end{array}$$

The rules in the k-th level ($k > 1$) are as follows:

$$\begin{array}{rll} R_{kj}: & \textbf{if} & (x_{N_k+1} \text{ is } A_{k1j}) \textit{ and } \ldots \textit{ and } (x_{N_k+n_k} \text{ is } A_{kn_kj}) \\ & & \textit{and } u_{k-1} \text{ is } B_{k-1j} \textbf{ then } u_k \text{ is } B_{kj} \end{array}$$

where

$$N_k = \sum_{t=1}^{k-1} n_t$$

and n_t is the number of system variables used as input at level t.

With this structure it is shown ([7]) that:

- For a hierarchical fuzzy controller with L levels of rules, n system variables, m fuzzy sets per variable, and n_k variables (including the output variable of

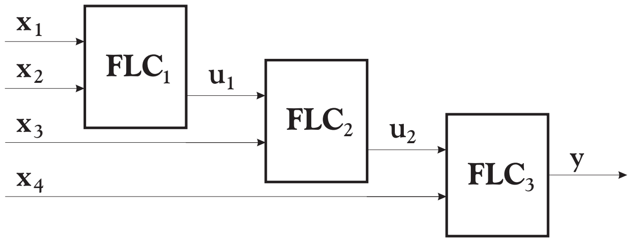

Figure 1: Minimal hierarchical structure.

the previous level) in the k^{th} level of the hierarchy, the total number of rules is given by

$$T = \sum_{i=1}^{L} m^{n_k}$$

with

$$n_1 + \sum_{i=2}^{L} (n_k - 1) = n$$

- In addition, if there is a constant number of input variables per level, i.e. $n_1 = n_2 = \ldots = n_L$, the number of rules is

  $$T = (\frac{n - N}{N - 1} + 1) * m^N$$

  with $N = n_1 = n_2 = \ldots = n_L$.

- Finally, if m and n_k satisfy conditions $m \geq 2$ and $n_k \geq 2$, the total number of rules will take on its minimum value when $n_k = 2$, and this minimum value is

  $$T = (n - 1) * m^2$$

This third approach producing the lowest dimension of the rule base is illustrated in Figure 1 using a four input one output system.

The result is that in such a hierarchical FLC, the number of rules in a complete rule base is a linear function of the number of variables, while in a conventional one it was an exponential function of the number of variables.

In general, this hierarchical approach is based on a cascade structure where the output of each level is considered as one of the inputs to the following level. This approach represents only a functional decomposition by which an n-dimensional FLC that can be represented, from a functional point of view, as

$$y = F(x_1, \ldots, x_n)$$

is replaced by (in the case illustrated in fig. 1, with four input variables) the functional composition of $n-1$, in this case three, two-dimensional controllers, that is,

$$y = f_3(f_2(f_1(x_1, x_2), x_3), x_4)$$

This approach defines the decomposition problem from a merely functional point of view, but it is possible too to apply a knowledge based point of view to the decomposition problem. The idea is to use meta-knowledge about the problem to be solved, this meta-knowledge defines the semantics of the hierarchy, while the syntax is defined by the structure of the hierarchy. This semantics try to describe how the variables of the system are coupled, in such a way that using the semantics information it is possible to *decouple* the variables. The meta-knowledge (the semantics) is then incorporated into the hierarchical FLC as contextual information.

3 Designing a hierarchical fuzzy controller

A fuzzy system can be viewed from two different points: as a knowledge based system or as a nonlinear map with some properties, as being an universal approximator. That applies to hierarchical fuzzy systems (HFSs) too. It is possible to analyze a HFS as an universal approximator ([10]), but as a knowledge based system too. In the following, the study will be centered on this second approach.

To apply a knowledge based design to a HFS, special attention has to be paid to variables interaction and coupling. The hierarchy has to be designed considering how and why the different variables of the system are coupled, trying to translate that information to the hierarchical structure.

To illustrate the design process, a well known control problem will be used, the cart pole balancing system.

3.1 The Cart-pole Balancing System

The cart-pole balancing system is a classical control problem that the literature has established as a benchmark on learning control system evaluation. The objective of such a system is the control of the translational forces applied to a cart, to position it at the center of a finite track, while balancing a pole hinged on the top of the cart. In our experiments we use the model applied in [1] maintaining the parameters and dimensions of the system.

The problem is represented through a set of nonlinear differential equations simulating the dynamics of the cart-pole system. The nonlinear differential equations are:

$$\ddot{x} = H_1(\dot{x}, \theta, \dot{\theta}, F) = \frac{F - \mu_c sign(\dot{x}) + \tilde{F}}{M + \tilde{m}} \tag{1}$$

and

$$\ddot{\theta} = H_2(\dot{x}, \theta, \dot{\theta}, F) = -\frac{3}{4l}(\ddot{x}\cos\theta - g\sin\theta + \frac{\mu_p\dot{\theta}}{ml}), \tag{2}$$

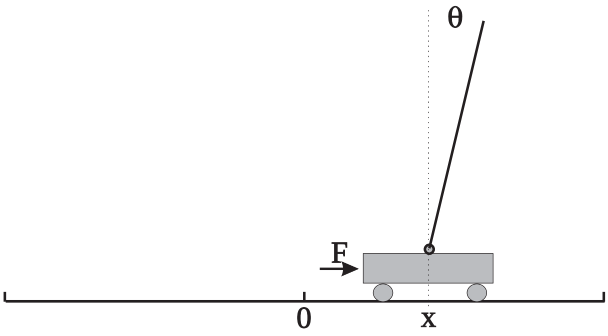

Figure 2: Cart-pole balancing system

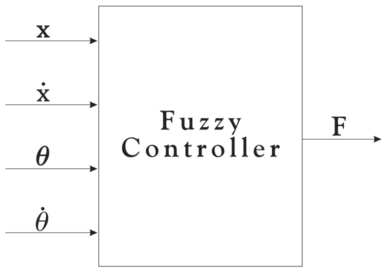

Figure 3: Non-hierarchical fuzzy controller.

where

$$\tilde{F} = ml\dot{\theta}^2 \sin\theta + \frac{3}{4} m \cos\theta (\frac{\mu_p \dot{\theta}}{ml} - g \sin\theta) \quad (3)$$

and

$$\tilde{m} = m(1 - \frac{3}{4}\cos^2\theta). \quad (4)$$

3.2 Knowledge based design

The first possibility when designing a fuzzy controller to balance the cart-pole system is to use a non-hierarchical fuzzy controller like that shown in fig. 3. Considering a controller with five linguistic terms per variable, the obtained rule base will contain $5^4 = 625$ rules.

Different approaches to solve that problem from a non-hierarchical point of view has been proposed, as the use of an *indirect adaptive fuzzy controller* ([9]) or the application of learning techniques as in [2, 5, 6]. It is possible to use techniques that reduce the dimensionality of the rule base, as the sensor fusion described in [4], integrating the four system variables into two input variables ($u_1 = ax + b\dot{x}$ and

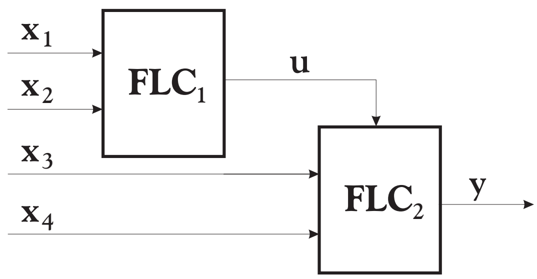

Figure 4: Hierarchical structure using contextual information.

$u_2 = c\theta + d\dot{\theta}$). The objective of this paper is not to outperform these controllers but to focus on a different approach, a knowledge based hierarchical design.

A possible approach is to use the structure defined in Figure 1, consisting of three fuzzy controllers, selecting which variables have to be assigned to each level and what are the intermediate variables. Considering again five linguistic terms per variable, the overall number of rules will be $3 \times 5^2 = 75$ rules. But our proposal is to follow a third approach based on the block diagram in Figure 4. With the same assumptions the overall number of rules will be $2 \times 5^2 = 50$ rules.

To select the variables applied at each level of the hierarchy and to obtain the corresponding rule bases, a knowledge extraction process will be applied to the problem. The main idea is that of generating a hierarchical structure and a rule base (in this case two rule bases) that reflects the expert knowledge (in this case we play the role of experts).

3.3 Knowledge extraction

In a first step the four input variables will be grouped according to its role in the system and with the different levels of coupling they have.

The cart-pole balancing system is a multi-objective system where a first objective is balancing the pole and a second one is centering the cart on the track. Each one of these two objectives is related to a subset of the input variables. The achievement of the first objective (balancing the pole) relates to the values of the variables θ and $\dot{\theta}$, while the variables x and $\dot{x}$ reflect the attainment of the second objective (centering the cart). That means that variables θ and $\dot{\theta}$ are tightly coupled and must be considered as a single subset of the input variables, consequently, they will probably constitute the input variables of one of the FLCs composing the hierarchical structure. The same applies to x and $\dot{x}$.

Considering the two objectives separately it is easy to solve the problem of centering the cart or balancing the pole, a simple regulator such as a PD could control the applied force and achieve the objective. But the question is not so simple because of the coupling between both tasks. This coupling creates two opposite situations:

1. Both objectives require an opposite action, i.e., its deviations have the same sign. This is the situation when cart and pole are simultaneously in a positive (negative) position, as the one illustrated in Figure 2.

2. The two objectives require a similar action, i.e., its deviations are opposite. This is the situation when cart is in a positive (negative) position while pole is in a negative (positive) one.

Considering now the way a human would solve this control problem, each one of the two previously described situations will have a partial solution that will be described in following paragraphs.

The first situation could be reduced to the second one by applying a force to move away the cart from the track center, where the intensity of the force has to be high enough to change (in sign) the orientation of pole.

The second situation requires a force towards the center of the track, i.e., with the opposite sign to the cart position, but the intensity of that force has to be bounded to stop the pole changing its position and consequently converting the system situation from the second to the first one.

These two actions could be expressed in the following single rule

> *anywhere the cart is, apply the force required to place and maintain the pole deviated towards the center of the track.*

3.4 Adapting the knowledge to the hierarchical structure

The control rule obtained in the previous section is now going to be adapted to a two level hierarchical fuzzy controller.

The knowledge base of a FLC has three main components:

1. the scaling factors or the scaling functions,

2. the membership functions, and

3. the control rules.

¿From a linguistic point of view, the scaling factors could by interpreted as context information. While the membership functions describe the relative semantics (context independent) of the linguistic variables contained in the rules, the union of the scaling functions and the membership functions generates the absolute semantics of the linguistic variables (context dependent through the scaling functions).

In this paper context information will be considered as intermediate variables applied to connect the levels of the hierarchical structure, as shown in fig. 4. In this case, the intermediate variable u will constitute not an input variable to the second level fuzzy controller but a different kind of information.

At the first level of the hierarchical fuzzy system a control meta-rule is obtained that constitutes context information (in this case the objective) for the second level controller. The meta-rule is:

maintain the pole deviated towards the center of the track.

At the second level a control rule ensures the achievement of the objective (that changes according to first level output). The control rule is:

achieve the desired pole position by applying a force with the same sign as that of the deviation (from the desired position) of the pole.

When Interpreting these two levels of knowledge in the context of the application, it is important to notice that the control problem is, in this case, a multi-objective problem. The control system tries simultaneously to center the cart and balance the pole. From this point of view, the meta-rule represents meta-knowledge describing how to *synchronize* the attainment of both objectives. The control rule contains the information to achieve one of these objectives while the meta-rule keeps both objectives synchronized.

3.5 Increasing the Contextual Information

It is possible to incorporate more contextual information into the interface between both levels of the hierarchical structure.

In the previous paragraph the output of the first level defines an objective value for the pole position. The main idea is to maintain different levels of angular deviation for the pole, according to the linear deviation of the cart. When comparing the objective of maintaining the pole in a vertical position or maintaining it with a certain deviation it is easy to notice (without needing to know the mathematical model) that the vertical position represents a point of unstable equilibrium while the other cases are not equilibrium points. If only considering the pole it intuitively seems possible to transform any pole position in an (unstable) equilibrium position only by applying the suitable *force* to the system. In other words, if the controller does not consider the cart position, it is possible to maintain a certain deviation of the pole with a first derivative equal to zero by only applying an appropriate constant force to the cart.

The idea is to generate as the output of the first level a pair (θ_0, F_0) representing the equilibrium condition. To do that it is possible to use a two input two output system generating (θ_0, F_0), or only a two input one output system assuming a linear relationship between θ_0 and F_0. The hierarchical controller described in previous subsection is the one considering $F_0 = 0$.

4 The hierarchical fuzzy controller

Considering again the block diagram in Figure 4, a first fuzzy controller (FLC_1) incorporates as inputs the variables x and $\dot{x}$, generating as output a desired *equilibrium point* (θ_0, F_0). The second fuzzy controller (FLC_2) has as inputs the variables θ and $\dot{\theta}$ and generates the force to be applied to the cart (F). The role of θ_0 and F_0 in the second controller will be described below. The dimensions and parameters of the system are represented in table 1.

Table 1: Model Dimensions and Parameters

Symbol	Description	Value
x	Cart position	[-1.0,1.0] m
θ	Pole angle from vertical	[-0.3,0.3] rad
F	Force applied to cart	[-10,10] N
g	Force of gravity	9.8 m/s^2
l	Half-length of pole	0.5 m
M	Mass of the cart	1.0 kg
m	Mass of the pole	0.1 kg
μ_c	Friction of cart on track	0.0005 N
μ_p	Friction of pole's hinge	0.000002 kg m^2

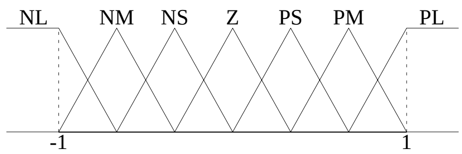

Figure 5: Membership functions.

The linguistic labels are similar for all the variables (including the output variables of both controllers) and are described in Figure 5. All the term sets contain seven terms {NL (Negative Large), NM (Medium), NS (Small), Z (Zero), PS (Positive Small), PM (Medium), PL (Large)}. The membership functions are defined in [-1,1] and consequently the four input variables have to be normalized from their ranges to [-1,1] interval. This normalization process is based on a linear function applying the following normalization ranges:

- ranges for cart position and its first derivative (x, $\dot{x}$) are [-1,1] in meters and m/s respectively,
- the ranges for the output of the first level representing the desired *equilibrium* point (θ_0, F_0) will be obtained below,
- the range for pole position (θ) and applied force (F) are defined as a function of the desired equilibrium point generated at first level, producing ranges contextualized by means of the defined condition. The applied ranges are [-0.3+θ_0,0.3+θ_0] rad. and [-10+F,10+F] Nw., and
- the range for pole position derivative ($\dot{\theta}$) is [-1,1] rad/s.

The hierarchical controller will use the structure shown in Figure 4 with the normalized fuzzy sets in Figure 5 and the previously defined normalization ranges.

Table 2: Control Rule Base (FLC_1 and FLC_2)

$\dot{x}$ \ x (θ_0, F_0)	NL	NM	NS	Z	PS	PM	PL
NL	PL	PL	PL	PL	PM	PS	Z
NM	PL	PL	PM	PM	PS	Z	NS
NS	PL	PM	PS	PS	Z	NS	NM
Z	PL	PM	PS	Z	NS	NM	NL
PS	PM	PS	Z	NS	NS	NM	NL
PM	PS	Z	NS	NM	NM	NL	NL
PL	Z	NS	NM	NL	NL	NL	NL

$\dot{\theta}$ \ θ (F)	NL	NM	NS	Z	PS	PM	PL
NL	NL	NL	NL	NL	NM	NS	Z
NM	NL	NL	NM	NM	NS	Z	PS
NS	NL	NM	NS	NS	Z	PS	PM
Z	NL	NM	NS	Z	PS	PM	PL
PS	NM	NS	Z	PS	PS	PM	PL
PM	NS	Z	PS	PM	PM	PL	PL
PL	Z	PS	PM	PL	PL	PL	PL

The definition of the rule bases will consider the meaning of the control meta-rule and rule defined in the previous section. Both of them reflect a regulation action that could be implemented by using a standard fuzzy PD, in this case the applied rule base will be similar to that described in [11] (2).

The next step is the determination of normalization ranges for θ_0 and F_0, and the analysis of the behavior of the controller. To determine the suitable range for both θ_0 and F_0 tests will be carried out by applying different values that will be evaluated using the following expression

$$\text{ITAE} = \sum_{10 init.} \sum_{n=1}^{6000} n\tau|x|\tau \qquad (5)$$

This expression is equivalent to the ITAE (Integral Time Absolute Error) index, with $n\tau$ representing the time, $|x|$ the error and τ the *integration step*.

The evaluation results of considering the ranges $[-\alpha, \alpha]$ and $[-\beta, \beta]$ for θ_0 and F_0 respectively, with the values of α from 0 to 0.35 (rad.) by 0.05 and β from 0 to 3.5 (Nw) by 0.5, are shown in Figure 6. The system presents a very similar performance for a wide range of values, finally the point (0.35,3.5) that generates the minimum is selected and consequently the variables θ_0 and F_0 are normalized in [-0.35,0.35] and [-3.5,3.5] respectively.

Figures 7 and 8 have been obtained by applying the hierarchical fuzzy controller to the cart-pole system with initial conditions $x = 0.5$ m. and $\theta = 0.15$ rad. The

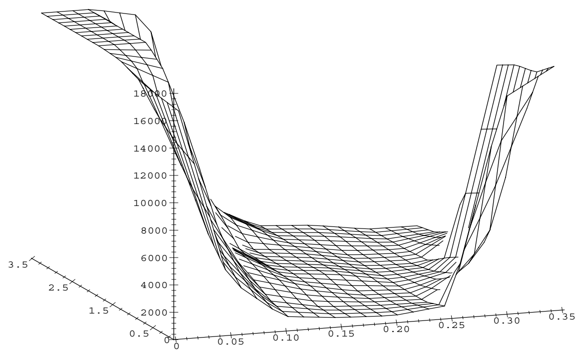

Figure 6: Evaluation for different ranges of θ_0 and F_0.

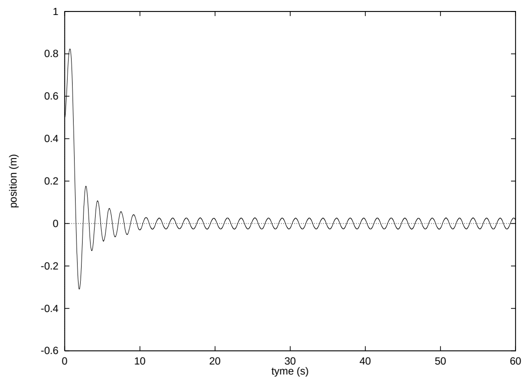

Figure 7: Cart position (using linear normalization).

controller achieves a stabilization of the system in a limit cycle with maximum deviation of 2.6 cm. and 0.018 rad.

4.1 Refining the fuzzy controller

The results shown in the previous subsection are clearly outperformed by other fuzzy controllers, but the main question was the reduction of complexity of the design process that has been obtained with the hierarchical structure.

Once the idea of an easy design process has been established, and the possibility of using a standard rule base has been shown, a second step will be added to improve the behavior of the hierarchical fuzzy controller. The idea is the use of the non-linear normalization functions previously described in [6], that replaces the linear normalization with a non-linear normalization of the input (output) variables from (to) its universe of discourse to (from) the normalized universe [-1,1].

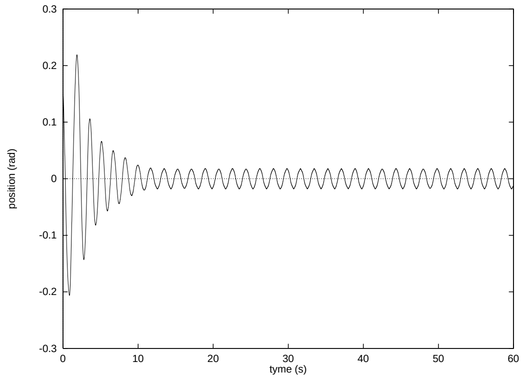

Figure 8: Pole position (using linear normalization).

A two step normalization (denormalization) process is applied. In a first step, the variable is linearly mapped from its range defined through the lower and upper limits ([V_{min}, V_{max}]) to [-1,1]. In the second step the non-linearity is introduced through the parameterized function ($f : [-1, 1] \rightarrow [-1, 1]$)

$$f(x) = \text{sign}(x) \times |x|^a, \text{ with } a > 0. \tag{6}$$

The final result is a value in [-1,1].

Working with a normalized fuzzy partition like the one shown in Figure 5 (where the fuzzy sets are uniformly distributed in [-1,1] interval), it is possible to obtain a denormalized fuzzy partition with uniform sensitivity ($a = 1$), higher sensitivity in medium values or higher sensitivity in extreme values ([6]).

Applying genetic algorithms to obtain the suitable values of V_{max} (as the system has a symmetric behavior the value of V_{min} is $-V_{max}$) and a for the different input and output variables of the hierarchical controller (and maintaining the rule base) it is possible to considerably improve the behavior of the system as shown in Figures 9 and 10 being equivalent to Figures 7 and 8 but using the non-linear normalization functions.

5 Concluding remarks

The paper focus on the problem of decomposing multivariable fuzzy controllers using a hierarchical approach based on the application of meta-knowledge, introducing a new approach to the application of FLCs to large-scale systems. For this kind of systems, where the high dimensionality of the rule base represents a significant difficulty, the use of a hierarchical structure produces an interesting reduction of the rule base. The proposed hierarchy introduces the idea of contextual information as a way of obtaining a larger reduction in the size of the rule base if comparing it with previously defined hierarchical structures. To allow an easier description of the method, the cart-pole balancing system is used as an application example,

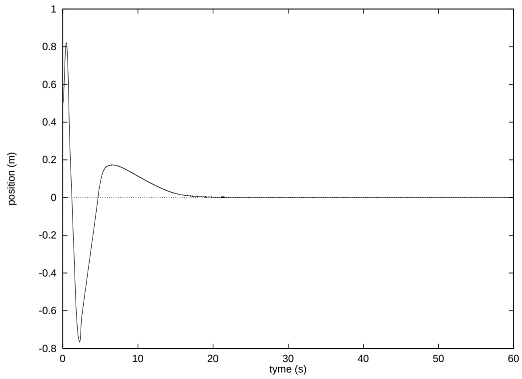

Figure 9: Cart position (using non-linear normalization).

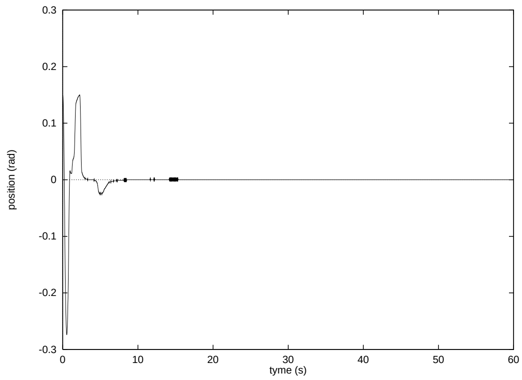

Figure 10: Pole position (using non-linear normalization).

demonstrating the reduction of the number of rules and the good performance of the generated FLC when controlling the system.

References

[1] A.G. Barto, R.S. Sutton, and C.W. Anderson. Neuronlike adaptive elements that can solve difficult learning control problems. *IEEE Transactions on Systems, Man and Cybernetics*, 13(5):834–846, Sep/Oct 1983.

[2] M.G. Cooper and J.J. Vidal. Genetic design of fuzzy controllers. In *Proceedings 2nd International Conference on Fuzzy Theory and Technology*, October 1993.

[3] M. Jamshidi. Fuzzy control of complex systems. *Soft Computing*, 1(1):42–56, April 1997.

[4] M. Jamshidi. *Large-Scale Systems: Modeling Control and Fuzzy Logic*. Prentice Hall PTR, 1997.

[5] M.A. Lee and H. Takagi. Integrating design stages of fuzzy systems using genetic algorithms. In *Proceedings 2nd IEEE International Conference on Fuzzy Systems, FUZZ-IEEE'93*, volume 1, pages 612–617, March 1993.

[6] L. Magdalena. Adapting gain of FLCs with genetic algorithms. *International Journal of Approximate Reasoning*, 17(4):327–349, Nov 1997.

[7] G.V.S Raju, J. Zhou, and R.A. Kisner. Hierarchical fuzzy control. *International journal of control*, 54(5):1201–1216, 1991.

[8] C. Rivero, P. Lasso, and F. Monasterio-Huelin. Cooperative-corrector multivariable fuzzy controller. *Mathware & Soft Computing*. In this issue.

[9] L.X. Wang. *Adaptive Fuzzy Systems and Control: Design and Stability Analysis*. PTR Prentice-Hall, Englewood Cliffs, New Jersey, 1994.

[10] L.X. Wang. Universal approximation by hierarchical fuzzy systems. *Fuzzy sets and systems*, 93(2):223–230, 1998.

[11] R. R. Yager and D. P. Filev. *Essentials of Fuzzy Modeling and Control*. John Wiley & Sons, Inc., 1994.