

The Process of Induction as a Non-classical Logic's Double Negation: Evidence from Classical Scientific Theories

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1 About the strategy for analysing induction processes

Although an induction process is always focused upon a local item, its general features may depend on the global context. In this hypothesis, the best context one may hope for an analysis on induction processes is a scientific context, such as it is given by a whole, systematic theory. Then, rather than to focus the attention on a single process of induction resulting in a statement, I will take in account the actual ways the most representative scientific theories, ranging they from calculus to chemistry or psycho-analysis, represent induction in their genetic processes.

The advantages are apparent. The set of past scientific theories represents one of the highest levels of intellectual activity, it constitutes a well-defined body of knowledge, it receives the most general agreement as possible on the statements, the symbols, the meanings, the arguments, the developments each theory involves. Moreover, we may refer no more to the low-level induction process for inducing a synthetic law from a collection of experimental data, but to the lot of the most relevant efforts performed by the greatest scientists for inducing from previous or experimental data some new decisive achievements in the history of the intellectual work. Last but not least, we may refer no more to an informal laboratory context, but we may refer to a formal context such as it is formalised by an author in his book presenting the theory. As a consequence, history of science becomes a relevant field of research for the discovery of the general features of the induction processes.

However, the disadvantages too are apparent. They amount to achieve a clear-cut appraisal of the foundations of scientific theories, since the induction processes

surely pertain to the most deep level of a theory, both in its genesis and in its logical development.

The present analysis is a consequence of my claim to have offered an appraisal of the foundations of science, by which I was able to characterise in a new way several theories. In few words, I found out that it is a misleading image the traditional image of science as a monolithic block, whose method is an unique one, whose logic is the classical one only, whose mathematics is at best the “rigorous” calculus, whose reference theory in theoretical physics is Newtonian mechanics. Instead, I discovered a basic bipolarisms in all the previous, basic aspects of science.¹⁾

In particular, let us consider logic. It is well-known that quantum mechanics suggests a non-classical logic. My studies showed that non-classical logic actually represents a more rooted phenomenon in history of science, being included by several instances of past scientific theories: Galilei’s exposition of his results, Boyle’s main theses, L. Carnot’s calculus, geometry and mechanics, S. Carnot’s thermodynamics, classical chemistry, Lobachevskii’s non-Euclidean geometry, Galois’ theory, etc. In these theories, non-classical logic constitutes a specific feature of a particular kind of a theory organisation, that is the organisation which is based on the search for a new method which is capable to solve an universal problem as well as correlated problems. Since this kind of organisation essentially depends from the principle of sufficient reason, it can be traced back to Leibniz’ philosophy. (This principle is one out of the two great principles of the human mind - the other one being the principle of non-contradiction.²⁾). After Leibniz, D’Alembert³⁾ distrusted the traditional deductive organisation (the “rational” one) for suggesting a new, “empirical” organisation. At last, L. Carnot founded his three scientific theories upon this new kind of organisation, which he was able to illustrate, although in intuitive terms only.⁴⁾

In the following, my claim is that a theory displaying a problem-based organisation preserves the genetic process by which it was built; thus, a qualified version of induction is represented by such a scientific theory through the statements belonging to non-classical logic. I mean “non-classical logic” in the literal meaning of a logic which differs from classical logic at least as the intuitionistic logic does.

2 Double negated statements in classical scientific theories

My work in History of Science started by focusing the attention upon those past scientific theories which made a deliberate use of an elementary mathematics. I saw in them an attitude favourable to a mathematics consistently using of potential infinity only, i.e. a mathematics that since 1967 is known as constructive mathematics⁵⁾. The first theories of this kind - L. Carnot’s theories, classical chemistry, S. Carnot’s thermodynamics and Lobachevskii’s non-Euclidean geometry - all belong to the period around the French revolution. Moreover, each of them

presents a common remarkable feature; it does not start from principles-axioms; rather, it presents as its crucial problem an universal problem, the solution of which requires the invention of a *new scientific method*; indeed, the development of the theory consists in a successful inquiry for this new method by starting from some *methodological* principles. I called such a kind of organization a *problem-based, or problematic organization*. L. Carnot emphasised both at the end of the first edition of his book on mechanics and in the preface of the second edition of the same book⁶⁾ that the ideal of his theory was at variance with respect to the “rational method”.

Some more theories can be recognised - as we will see in the table of sect. 5 - as instances of a problem-based theory: for ex., Galois’ theory, Klein’s Erlangen program, Einstein’s special relativity, quantum mechanics, computability theory. I concluded that around French revolution a new way of organising a scientific theory began and then competed with the traditional, Aristotelian way of organising a theory in a full deductive way.

It is not apparent at first glance how to decide when a given theory is a problematic theory and then to extract its main problem. However, a crucial test for answering that question, is suggested by a further analysis on the previous theories of the period around the French revolution. In the original texts of such theories the authors themselves surprisingly presented their methodological principles under a same logical feature - i.e. each of these principles is a double negated statement (**DNS**), which is not equivalent to the corresponding affirmative statement⁷⁾.

In physics the most known statement of this kind is “the impossibility of a perpetual motion” (perpetual = which has no end, as Stevin wrote it⁸⁾). As Gillispie remarked,⁹⁾ this methodological principle played the role of a “demonstrative and tutelary principle” in L. Carnot’s mechanics, then it was currently used as the most suggestive principle of thermodynamics up to the time of Nernst. E. Mach maintained that it is more general and more fundamental than causality principle.¹⁰⁾

Actually, in no way such a statement can be changed in an equivalent positive statement enjoying scientific evidence. When following the classical logical law, $\neg\neg A = A$, we drop out the two negations: “Every motion has an end”; that yet lacks of scientific evidence. Indeed, we are unable to state the exact moment of the end of a specific motion - e.g. the Earth, a motor-car - since we need the previous knowledge of the friction on the whole trajectory which actually is a priori unknown, at least in its extension if not in its shape. We have to conclude that in such a case $\neg\neg A \neq A$, a typical law of intuitionistic logic¹¹⁾ and more in general non-classical logic.

For brevity’ sake, in the following I will list without comments some more DNSs - fitting the $\neg\neg A \neq A$ law - which I was able to recognize in the original texts of scientific theories: “Infinitesimals are not chimerical beings” (chimerical = not real; L. Carnot); “A body cannot change its state of motion by itself (= if not by the action of other bodies)” (L. Carnot); “It is impossible that matter can be divided at infinity” (19th Century chemists); “We call element what we cannot

still decompose” (A. L. Lavoisier); “Nothing is created” (A. L. Lavoisier); “It is not contradictory the hypothesis ...” (N.I. Lobachevskii); “It is patently absurd to guess that forces God gave to matter may be destroyed by man’s action” (J.P. Joule); “It is not true that heat is not equal to work” (thermodynamicists); “Under geometrical transformations there are un-changed quantities” (F. Klein).

3 Double negated statements in recent scientific theories

The recent scientific theories disvealed this problematic organisation of a theory, because in them the problem to meet the DNSs was unavoidable, since it was no more possible the traditional solution of inventing some *ad hoc*, unscientific words; e.g. to call an infinitesimal as “a different kind of zero” (Euler) or as a number that “tends” to zero; to say that a body “perseveres” in its state of motion (Newton); or that heat is “equivalent” to work (Clausius, Kelvin, etc.).

As a matter of fact the *birth of intuitionism* blurred the traditional picture of a mathematical theory as a full deductive organisation. Brouwer distrusted all axioms and moreover he conceived the deductions in a so different way from those of classical logic, that he rejected some well-established logical laws. One may suggest that what Brouwer actually changed was just the organization of the logical theory, since intuitionistic logic may be considered as the theory of logic yet organized in a problematic way.¹²⁾ In addition, his intuition of a number surely differs from any conceivable axiom.

The very novelty carried on by *special relativity* was to destroy the basic notions of the deductive organisation of mechanics, i.e. absolute space and absolute time, together with the cause-force. It evidenciated rather the problematic organisation of a theory, inasmuch as it was based on a crucial problem, i.e. to find out the general covariance of the physical laws, or in other terms the Lorentz group as the relativity group. Its main problem is manifested by Einstein when, in addition to the postulate of relativity, he introduced “another postulate which is only apparently irreconcilable with the former.” (Einstein 1905)

Then, the disvealing process became as irreversible by the birth of *quantum mechanics*. Its novelty consisted in the fact that the typical problem of this theory - and hence its DNS - concerned no more some foundational notions only, but the definition of the state itself of the system. It is well-known that quantum mechanics includes an universal problem, i.e. the problem of the measurement of two physical magnitudes. This problem may be well-represented by a DNS: “It is not true that two magnitudes cannot be measured at the same time”. Indeed, owing to the indeterminacy principle the corresponding statement without negations is false; on the other hand, by dropping out one negation one obtains again a false statement, since quantum mechanics teaches a way for performing such measures; all depend on the method we are following.¹³⁾

I furthermore recall that *computability theory* cannot appeal to principle-axioms. Rather, it is aimed to solve the universal problem of what is a calculation; its methodological principle was stated by Church under an odd denomination (“thesis”), just since it was embarrassing either to call it as “axiom”, or to claim an entirely different organisation of the theory. Its DNS is apparently: “it is not true that the intuitive notion of computability is not equal to the abstract, formal one”; it is correctly a problem, not a thesis.

It is remarkable that the recourse to DNSs constitutes the subject of a little - yet, crucial - paper which was written by S. Freud in order to give a foundation to his *psycho-analytic* theory. “On Negation” (1925) suggests that when the patient states: “In my dream I did not want kill my mother”, in its turn the analyst has to deny this negated statement: “It is not true that he did not want to kill his mother”. “This way to mean the negation well agrees with the fact that in the analysis no instance of “not” coming from the unconscious is ever discovered as well as the fact that never the recognition of the unconscious by the Ego is formalised by a negative sentence. No more apparent evidence is possible for our success in our aim to discover the unconscious than the moment when the analysed person reacts to our discover by means of the sentence: “I did not think that” or even “To that I never thought”.¹⁴⁾ The whole Freud’s paper may be analysed and interpreted under the light drawn by DNSs. The result I obtained was a consistent theory of the psycho-analytic world.¹⁵⁾

I conclude that no more adequate illustration of an induction process may be the representation of the process by which an analyst by relying upon some little symptoms only, truly some negated statements (stated by the patient about his dreams) guesses the illness of a patient.

In addition, the DNSs have their philosophical source from the principle of sufficient reason, i.e. the second principle among - according to Leibniz - the two great principles of human mind. In fact, Leibniz stated the principle in the following way: “Nothing is without cause”¹⁶⁾, which itself is a DNS. All that I suggested may constitute an improvement of Leibniz’ program for an *ars inveniendi*, unsuccessfully searched by Leibniz himself and by several his followers for at least a Century and half.

Actually, the use of non-classical logic in scientific theories was effective since some centuries ago, but unfortunately this historical origin remained ever unrecognised. Only at present time, when mathematical logic formalised the very borderlines between classical logic and the several kinds of non-classical logic it is possible to sharply evidentiare this characteristic feature of the scientific activity.

All that lead to investigate furtherly this kind of arguing in scientific theories. Truly, when a DNS is put at the very beginning of a theory, each possible chain of subsequent deductions lacks of that deductive evidence Aristotle suggested for the organisation of a scientific theory according to the apodictic ideal. Without doubt, a DNS is an open statement, it cannot state anything with certainty if not a bound of our thinking, i.e. the impossibility that $\neg\neg A$ is the same that either A , or $\neg A$. Indeed, the introduction of statements of non-classical logic in the foundations

of a theory makes impossible the general validity of the law of excluded middle - either A or $\neg A$ - and hence the a priori choice - as the apodictic science does - for the deductive method too, which relies upon the use of that law. In physical terms most DNSs take the version of a physical impossibility - say, of a perpetuum motion.

Under this light, we may see each previous DNS as an instance of the core of an induction process which obtained as a result a new method for solving the general, universal problem of the corresponding scientific theory.¹⁷⁾

4 Induction as L. Carnot's new synthetic method

It is possible to improve this characterisation of an induction process?

Let us follow Brouwer's suggestion, according to which logic does not grew up by itself; rather, it developed as an epiphenomenon of the scientific knowledge. If my line of illustration of the problem is correct, now we dispose of a lot of scientific experiences -represented by the all above-mentioned theories- upon which we can study our problem to recognise in a general and adequate way the main features of this process of induction.

One may start by choosing the most suitable theory for this kind of analysis. Surely, it is calculus the most astonishing achievement of Western science. About it we will refer to the interpretation by Lazare Carnot, i.e. the same author that offered some suggestions about the two ways for organising science.

In the book on calculus, **Reflexions...**, Lazare Carnot made a so abundant use of DNSs to cumulate six negations in the statement of the one principle of his theory.¹⁸⁾ An analysis of his merely linguistic re-formulation of calculus shows that he well perceived the great relevance of negations, although he was unable to recognise the basic laws of non-classical logic - actually, in his time classical logic too lacked of a formalization.

It is possible to select a set of such statements whose sequence is sufficient to synthesise his new foundation of calculus¹⁹⁾ - that evidentiates the consistent use by L. Carnot of non-classical logic in the most advanced mathematics of his time, although he was unaware of modern mathematical logic.

After two introductory remarks - "Does not exist no one discovery that produced in mathematics a revolution so successful and abrupt..."²⁰⁾, "Nothing is more plain than the exact notion of infinity." (p.174) - his starting point was stated in a sharp way: "Infinitesimals are not chimerical beings" (chimerical = not real; p. 182). This statement constitutes the main problem of the theory at issue.

This statement is followed by a new definition of an infinitesimal and then he presents the interpretation based upon the theory of the double fault, previously advanced by both Leibniz and Berkeley. An infinitesimal "...is no other thing than the difference of two quantities, both having for limit a third quantity" (p. 174). Then according to him, calculus adds to the mathematical system some auxiliary variables (they are commonly called infinitesimals). That leads to consider an

auxiliary system (imperfect equations), where it is easier than before to find out the solution of the problem.

“...it is apparent that the theory of infinity is no other thing than a calculation of compensated faults, and that the advantage of this calculus consists in the fact that, while the conditions of a question are often too much difficult to be exactly expressed by means of rigorous equations they are instead easily expressed by means of imperfect equations; the calculus gives the way for extracting from these imperfect equations the same results and relations each of which is at all sure, as if the primitive equations truly were of the most certain exactitude, and that procedure is developed by means of the mere elimination of the quantities whose presence caused these faults.” (p. 217) That is, by just managing to suppress the auxiliary variables (as traditional calculus is able to do by suppressing higher order infinitesimals in comparison to lower order infinitesimals) the desired solution of the original system results.

“What matters in fact whether these quantities are or not chimerical beings, when relations are not so; and when these relations are the one thing we are interested in?” (p. 225)

His result was a so deep insight in the foundation of science to be recognised as an anticipation of non-standard analysis.²¹⁾

In a final “Note” of his celebrated book²²⁾ he presented for the last time his general method as constituting an improvement of the old synthetic method. In the calculus, “infinitesimal quantities are no other thing than auxiliary quantities, which are necessarily to be eliminated in order to obtain the wanted results.” This kind of interpretation follows from the synthetic method, which “never employs other thing than the very quantities which it wants to mutually compare, and it never compares them if not directly or by the intermediation of some other quantities which are effective as the previous ones. Instead, the analytical method often takes as comparison terms among the very quantities some imaginary beings, some purely algebraic forms; yet, these algebraic forms carry on by themselves the index which is useful for their elimination, by means of several transformations

which want progressively clean the calculation by reducing it to the explicit forms; otherwise, this calculation will be useless, just as an unaccomplished calculation is.”

In other words, this method is apparent only when the auxiliary variables “are effective”. When instead they are ideal, “imaginary beings”, then the process may give evenly rise to the desired solution, yet it changes of nature; it becomes an idealised process of a genius’ mind. L. Carnot’s method was then paralleled by modern, Hilbert’s method - based upon highly idealised notions - for the elimination of the quantifiers.²³⁾ However, L. Carnot’s method, by “never losing sight of the content of the auxiliary variables”, that is without never allowing “beings of reason”, always made use of effectively computable beings for investigating the foundations of mathematics. In other terms, induction processes are manifest when the mathematics is bounded to be the constructive one.

I conclude that L. Carnot’s genius was to follow Leibniz’ investigations on the method of the most advanced achievement of past scientific thinking, calculus. Being successful, L. Carnot applied the resulting method for giving new foundations to the whole body of the formalised science of his time.

5 Interpretation of all problematic theories by means of L. Carnot’s method

Let us improve furtherly previous interpretation of the induction process. At present we have a wider basis for investigating this process than Carnot’s calculus only. We will investigate all theories which - after L. Carnot’s analysis - followed the problematic type of organisation, that is all theoretical experiences cumulated by mankind in two-Centuries of inductive work for building scientific theories in a different way than the Aristotelian one. All they may be interpreted by means of the new specific method which was suggested by L. Carnot.

For brevity’ sake, I will present a synthetic table listing the main characteristic features of these theories as they are emphasised by L. Carnot’s method. However, we cannot hope that the new interpretation suggested by L. Carnot’s method may be sharply recognised in the original writings presenting each theory, since we have to take in account that the past theories are born from the ingenuity of their founders, who only in a partial way went beyond their particular technicalities for perceiving the general scheme of the induction process. For brevity’s sake I will omit the discussion of dubious evidences - which do not at all mean contrary evidences - for the characterisations at issue.

— l p(.7in) — l p(1.in) — l p(1.in) — l p(.7in) — l p(.7in) — _ — **Theory** — **What has to be induced** — **Methodological principle** — **Auxiliary variables** — **Operations’ cycle** —
 _ — *L. Carnot’s Calculus* — The nature of the infinitesimals — It is not true that infinitesimals are chimerical beings — Infinitesimals — yes —

- — *L. Carnot's Geometry* — The nature of the negative quantities — Isolated negative quantities are inadmissible — Displacements by insensible degrees — —
- — *L. Carnot's Mechanics* — What is conserved in a shock — It is not true that there are not invariant quantities — Geometrical motions — Invertibility of a geometrical motion —
- — *Chemistry* — The ultimate components of matter — Matter is not divisible at infinity — Atomic mass increment — Periodicity —
- — *S. Carnot's Thermo-dynamics* — The relationship between work and heat — It is impossible a perpetual motion even in presence of heat — Infinitesimal adiabats — Carnot's cycle —
- — *Galois Theory* — A decision procedure for the solvability or not — It is not true that the eq. of degree > 4 is unsolvable — Radicals — Resolvent equation —
- — *Lobachevskii Non-Eucl. Geometry* — What is a parallel line — It is not absurd the hypothesis of two parallel lines — Triangles — Propositions 19-22 —
- — *Erlangen Program* — What is a geometry — There exist invariants — Groups of spatial transformations — —
- — *Special Relativity* — Invariance or not of the speed of light — c is independent of the state of motion of the emitter — Transformations of reference system — —
- — *Quantum Mechanics* — The measurement of physical quantities — It is not true that two conjugate variables are not measurable — Amplitude of probability — —
- — *Recursive Functions* — The notion of computation — Induction principle — Functions included in a recursive definition — Recursion —
- — *Psycho-Analysis* — The removed. The unconscious — It is not false what patient denies — Dreams — Transfer process —
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6 A comparative analysis of the induction process in problematic theories.

I add to the previous list one more theory, i.e. number theory, which is very relevant because it is the basic one for the entire mathematics and moreover it formalises the induction process by means of a specific, basic principle, that is the induction principle.

Elsewhere,²⁴⁾ I revisited Poincar's polemics with logicians about number theory. I showed that his idea of the "intuitive principle" in opposition to the "formal principle" may be consistently interpreted according to a DNS which actually was stated by Poincar as follows:

$$\forall n [T(0) \ \& \ T(n) \rightarrow T(n+1)] \rightarrow \neg \exists n \neg T(n)$$

where the predicate T represents Poincar's words "no contradiction" (truly, a DNS by itself).

One may add that Brouwer, when he stated the induction principle quoted Poincaré's comments and reiterated almost the same words.²⁵⁾ He added one specific remark. He considered the intuition of mathematics as "*invariance in change*" as well as *unity in similitude*".²⁶⁾ That is, he conceived the induction principle as a rule for adding a new element - as in L. Carnot's synthetic method - in order to achieve the invariant of this process, i.e. the very notion of an integer number.

This hint for a mathematical formalisation of an induction process was better developed by L. Carnot again. In mechanics, L. Carnot's method of adding auxiliary variables to the original system gave rise to a group of transformations - the group of geometrical motions - whose invariants are the conservations of the mechanical quantities which are sufficient to solve the main problem of his theory, i.e. the shock of bodies. In fact, L. Carnot was proud to be the founder by means of his theory of geometrical motions of an entirely new theory, qualified as "intermediate between geometry and mechanics".²⁷⁾ Today this theory is easily recognised as the group theory, or in other terms the arguing in theoretical physics by means of symmetry. Unfortunately, this discovery was not appreciated by his contemporaries and at the time of Einstein too the symmetry was not considered a very subject for theoretical physics. It was widely recognised as an essential tool for a theoretical physicist not before the years '50.

This postponed, abrupt change suggested some reflections. Some physicists and mathematicians suggested that symmetry is essentially linked to Leibniz' thinking, in particular the principle of sufficient reason and the "ambiguity" - really, a loose way to suggest the content of a DNS.²⁸⁾ A.O. Barut re-visited almost all past theories by starting from Kepler's version of gravitation theory²⁹⁾, which is recognised as a first hint for an arguing by means of symmetry. He is able to develop this hint up to a definite result by means of the modern techniques of theoretical physics. Then, he lists almost all physical theories and he decides for each of them whether it is founded or not upon the symmetry. He concludes that this mathematical technique of arguing played a role which was sometimes a cooperative role, sometimes a complementary role to the "dynamic" technique, that is the differential equations.

Unfortunately, Barut ignored the first introduction of mathematical symmetry in science by Lazare Carnot's mechanics. By adding to Barut's list this case, one obtains in mechanics the lacking, alternative formulation of the well-known formulations - which are all based upon the differential equations. Moreover, he ignored that according to a recent paper³⁰⁾ classical thermodynamics also is based upon symmetry. This further case completes Barut's list, which now covers all theories of classical physics.

By means of the above facts I successfully tested in all theories of classical physics³¹⁾ the following hypothesis which improves Barut's one: In a scientific theory, the technique of differential equations, i.e Barut's "dynamics", constitutes the typical way of mathematical arguing in a formulation founded upon classical mathematics and an Aristotelian, fully deductive organisation, whereas "symmetry" is the typical mathematical arguing in the case the formulation is based upon constructive mathematics and a problematic organisation.

Really, being no more qualified induction process possible than the mathematical discovery of the invariants under a group of transformations, I conclude that we obtained the mathematical formulation of induction processes. The last pre-conditions of the above hypothesis offer evidence for what I suggested at the beginning of the present paper, that is the induction depends upon the context.

Bibliography and notes

1) A. Drago: Le due opzioni. Per una storia popolare della scienza, La Meridiana, Molfetta BA, 1991; “Incommensurable scientific theories. The rejection of the double negation, logical law”, in D. Costantini, M.G. Galavotti (eds.): Nuovi problemi e temi della logica e della filosofia della scienza, CLUEB, Bologna, 1991, vol. 1, 195-202.

2) I suggested a new account of this philosophy of science in “The Modern Fulfilment of Leibniz’ Program for a <Scientia Generalis>”, in H. Breger (ed.): Leibniz und Europe. VI Int. Leibniz-Kongress, Hannover, 1994, 185-194.

3) J. Le Ronde D’Alembert: “Imens” in J. D’Alembert, D. Diderot (eds.): ncyclopedia Franaise, 1754, Paris, vol. 17, p.411.

4) A synthesis of all these theories is C.C. Gillispie: Lazare Carnot Savant, Princeton U.P., Princeton, 1971. However, Gillispie did not gave much relevance to the new kind of organisation.

5) E. Bishop: Foundations of Constructive Mathematics, Mc Graw-Hill, New York, 1967. A recent presentation is D. S. Bridges: Computability, Springer, Berlin, 1994.

6) L. Carnot: Essai sur les machines en gnral, Defay, Dijon, 1783 (It. transl. and critical edition by A. Drago and S.D. Manno, CUEN, 1994) 104-106. These remarks are reiterated in p. XIV-XVII of the second book on mechanics: Principes fondamentaux de l’equilibre et du mouvement, Deterville, Paris, 1803.

7) I presented the first instances of some DNSs belonging to the original texts of scientific theories in ASL Colloquium, Berlin, 1989 and then in A. Drago: “Incommensurable...”, op. cit. in n. 1.

8) R. Dugas: Histoire de la Mcanique, Griffon, Neuchtel, 1950, p. 121. Of course, the emphasis in this quotation as well as in all the following quotations is added by myself.

9) C.C. Gillispie: op. cit. in n. 4, p. 99.

10) E. Mach: Principles of the Theory of Heat, (1887), Reidel, 1986, ch. XVIII.

11) D. Prawitz: “Meaning and Proof. The Conflict between Classical and Intuitionistic Logic”, Theoria, **43**, (1977), 6-39.

12) A. Drago: “Logic and the Aristotelian ideal of a scientific theory” Proc. Conference in honour of R. Magari, Siena, 1994 (in press).

13) See A. Drago: “Incommensurable..”, op. cit. in n. 1; “Alle origini della meccanica quantistica: le sue opzioni fondamentali” in G. Cattaneo A. Rossi (eds.): I fondamenti della meccanica quantistica, Editel, Cosenza, 1991, 59- 79; “Dualism

and incompleteness of quantum mechanics. Towards more consistent theories”, in R. Garola, A. Rossi (eds.): The Foundations of Quantum Mechanics, Kluwer Acad. P., 1995, 213-227.

14) S. Freud: “Die Verneinung”, *Imago*, **11**, (3) (1925) 217-221.

15) A. Drago: “<Sulla negazione> di S. Freud e i fondamenti della scienza”, in G. Sala, M. Cesa-Bianchi (eds.): La presenza di G. Iacono nella psicologia italiana, La Milanese, Rho, 1992, 137-150. C.E. Caorsi: “El sistema C_1 de da Costa y el premier modelo freudiano de la mente”, *Papeles de filosofo*, Montevideo, oct. 1989, 6-11, independently showed that Freud’s theory may be formalized by means of a para-consistent logic.

16) A. Drago: “The modern fulfilment...”, op. cit. in n. 2. Leibniz exactly says: “deux verits primitives, savoir en premier lieu le principe de contradictionet en deuxieme lieu, que rien n’est sans raison, ou que toute verit a sa preuve a priori, tire de la notion des termes, quoiqu’il ne soit pas toujours en notre pouvoir de parvenir cette analyse.” Here it is apparent that the affirmative version - “toute verit a sa preuve” - cannot be always stated and hence we have to state it as a DNS - “rien n’est sans raison”.

17) To my knowledge the hypothesis that an induction process may be represented by a non-classical logic’s argument was not taken in account by the several debates about this subject. See for instance, I. Lakatos (ed.): The problem of inductive logic, North-Holland, Amsterdam, 1968; J. Giedymin: “Confirmation, counterfactuals and projectibility”, in R. Klibansky (ed.): La Philosophie contemporaine, La Nuova Italia, Firenze, 1968, 71-87 and also M.L. Dalla Chiara, G. Toraldo di Francia: Le teorie fisiche, Boringhieri, Torino, 1980 (As an evidence of this difficulty to conceive induction by means of non-classical logic, let us remember that ironically, two pages before the last authors try to formalise induction by means of classical logic alone, they state that “...today we well know that this belief [in classical logic alone] represents a mere prejudice”, p. 20). Only N. Rescher: Hypothetical Reasoning, North-Holland, Amsterdam, 1964, introduces in Ch. 5 and in App. 3 a distinction which approaches the different versions of an induction result in respectively classical logic and non-classical logic. In fact, R. Carnap: “Inductive logic and inductive intuition”, in I. Lakatos (ed.): op. cit., 258-267 recalls that “When I talked [in early years of 1900] to philosophers about logical probability and inductive logic, I had to spend most of my time in an attempt to convince them that such things existed” (p.258). One may see in that late reception of the induction problem, together with the traditional disregard for non-classical logics (let us remember the title of the book S. Haak: Deviant Logics, Oxford, 1970), the source of the ignorance of the capability of non-classical logic for explicating the induction process. In particular the paradoxes of the confirmation seem have to played the Kuhnian role of the anomaly for suggesting a revolution in the way of thinking about this field of logic. Past philosophers unsuccessfully spent much time for solving them. Actually, these paradoxes merely disappear when one represents an induction process by means of a DNS, since the law $\neg\neg A \neq A$ denies the disconfirmation of the negated statement of the result of an induction.

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