

Local Boolean Manifolds from Knowledge Representation Systems

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Abstract

We introduce a structure to represent “*observations*” on entities in order to obtain “*knowledge*” about some of their characteristic properties or “*attributes*”.

This structure is based on the Pawlak’s definition of “*information systems*” (also “*knowledge representation systems*”) and lead us to obtain algebraic structures of lattice depending from the choice of an observational “*context*”.

The semantical algebraic structure so obtained is of local Boolean manifold whose global structure is an orthoposet which shows a nontransitivity of the implication; this behaviour could be useful for a formal algebraic approach to “non-monotonic” features of reasoning.

1 Knowledge Representation Systems

Before introducing the notion of knowledge representation system, we quote from Vakarelov [Va, 91]: “The main theoretical concept in this [...] approach is the notion of *knowledge representation system* (KR-system) introduced by Pawlak in [Pa, 81] under the name of *information systems*. A KR-system is a formalism for representing knowledge about some objects in terms of *attributes* (e.g., colour) and values of attributes (e.g., green).”

Let us start now with the formal definition.

Definition 1.1 *A knowledge representation system is a structure*

$$\mathcal{KR} := \langle X, \text{Att}(X), \text{val}(X), F \rangle$$

where X is a non empty set of objects (situations, entities, states); $\text{Att}(X)$ is a non empty set of attributes valuable on objects of the set X ; $\text{val}(X)$ is the set of

possible values which can be assumed in any observation on objects from X ; F is a mapping

$$F : X \times \text{Att}(X) \rightarrow \text{val}(X)$$

called the information mapping and associating to any pair consisting of an object and an attribute, the value assumed by the attribute on this object.

The knowledge representation system just introduced can also be considered a *state–input–output* system, whose state set is X , the input set is $\text{Att}(X)$, the output set is $\text{val}(X)$ and F is the transition mapping.

1.1 Sentential language of Knowledge Representation Systems

In order to obtain a sentential language from the system \mathcal{KR} , we define, for any $\alpha \in \text{Att}(X)$, the following set:

$$\text{val}(\alpha) := \{\lambda \in \text{val}(X) : \exists x \in X : F(x, \alpha) = \lambda\}$$

For any $\Delta \in \mathcal{P}(\text{val}(\alpha))$ [the power set of $\text{val}(\alpha)$], the pair $\langle \alpha, \Delta \rangle$ describes the *elementary sentence (question)*: “a test (observation, measure) of the attribute α yields a value belonging to Δ ”; in particular for any singleton $\{\lambda\} \in \mathcal{P}(\text{val}(\alpha))$ the pair $\langle \alpha, \{\lambda\} \rangle$ describes the *atomic question*: “an observation of the attribute α yields the value λ ”.

Now we give the following

Definition 1.2 Let $\mathcal{KR} = \langle X, \text{Att}(X), \text{val}(X), F \rangle$ be a knowledge representation system. For any fixed attribute $\alpha \in \text{Att}(X)$, the α –sentential language of questions from \mathcal{KR} , denoted by \mathcal{Q}_α , is the set of all well formed formulas (wffs) obtained according to the following formation rules:

- (i) Symbols \top and \perp are constant questions.
- (ii) If $\Delta \in \mathcal{P}(\text{val}(\alpha))$, then $\langle \alpha, \Delta \rangle$ is an elementary question [in particular, if $\Delta = \{\lambda\}$, with $\lambda \in \text{val}(\alpha)$, then $\langle \alpha, \{\lambda\} \rangle$ is an atomic question].
- (iii) By induction, we define the composed questions: if Q is an α –question, then $\neg_\alpha Q$ (“not” Q) is an α –question; if Q_1 and Q_2 are α –questions, then $Q_1 \wedge_\alpha Q_2$ (Q_1 “and” Q_2), $Q_1 \vee_\alpha Q_2$ (Q_1 “or” Q_2), and $Q_1 \rightarrow_\alpha Q_2$ (“if Q_1 then Q_2 ”) are questions.

Only strings generated by rules (i)–(iii) above are wffs of \mathcal{Q}_α .

The union of the α –sentential languages:

$$\mathcal{Q} := \bigcup_{\alpha \in \text{Att}(X)} \mathcal{Q}_\alpha$$

can be considered as the “global” sentential language of \mathcal{KR} -questions, whose “local” sentential components are the (pairwise disjoint) α -languages \mathcal{Q}_α . (In the sequel, if there is no possibility of confusion, we omit the subscript α in the logical connectives).

1.2 Propositional semantics of local languages from Knowledge Representation Systems

In a \mathcal{KR} system, the mapping F furnishes informations on objects from X depending from the attribute tested on them: precisely, for a chosen attribute $\alpha \in \text{Att}(X)$, an “*observation*” on object $x \in X$ produces the value $F(x, \alpha) \in \text{val}(\alpha)$. Thus, we can introduce a family of functions, each of which depends from the observed attribute $\alpha \in \text{Att}(X)$:

$$f_\alpha : X \rightarrow \text{val}(X), f_\alpha(x) := F(x, \alpha)$$

We make every f_α surjective restricting its range to $\text{val}(\alpha)$. For any $x \in X$, $f_\alpha(x) \in \text{val}(\alpha)$ represents the value assumed by the “*observable*” attribute α in this state (when the attribute α is mathematically realized by the function $f_\alpha : X \rightarrow \text{val}(\alpha)$). We emphasize an analogy with the *observable quantities* on the *phase* (or *state*) *space* X of Statistical Mechanics: in fact, any f_α can be considered as a *random variable*, once associated to X and $\text{val}(X)$ the σ -algebras of their power sets. Elements $E \in \mathcal{P}(X)$ describe *events* of the phase-space X , elements $\Delta \in \mathcal{P}(\text{val}(\alpha))$ subsets of possible values for the random variable f_α associated to the attribute α . In this context, the subset of X

$$f_\alpha^{-1}(\Delta) := \{x \in X : f_\alpha(x) \in \Delta\}$$

consists of all states in which a test of α (i.e., f_α) yields a value in Δ . So we can introduce the mapping

$$E_\alpha : \mathcal{P}(\text{val}(\alpha)) \rightarrow \mathcal{P}(X), \Delta \mapsto E_\alpha(\Delta) := f_\alpha^{-1}(\Delta)$$

which is a boolean algebra morphism and is called the “*observable*” related to the attribute α . Given the attribute α , to any subset of possible α -values $\Delta \in \mathcal{P}(\text{val}(\alpha))$, the corresponding observable E_α associates the event $E_\alpha(\Delta) \in \mathcal{P}(X)$.

For all fixed attribute $\alpha \in \text{Att}(X)$ in the \mathcal{KR} -system, we define the following set:

$$(1.1) \quad \mathcal{B}_\alpha(X) := \{f_\alpha^{-1}(\Delta) : \Delta \in \mathcal{P}(\text{val}(\alpha))\}$$

and so the set

$$(1.2) \quad \mathcal{L}(X) := \bigcup_{\alpha \in \text{Att}(X)} \mathcal{B}_\alpha(X).$$

We have the following result:

Proposition 1.1 *Let \mathcal{KR} be a knowledge representation system; let $\alpha \in \text{Att}(X)$ be any fixed attribute. Then $\mathcal{B}_\alpha(X)$ is a non empty set (since it contains \emptyset, X), closed with respect to set theoretic operations \cap, \cup and (\cdot) . Moreover, the structure $\mathcal{B}_\alpha(X) := \langle \mathcal{B}_\alpha(X), \cap, \cup, (\cdot), \emptyset, X \rangle$ is a boolean algebra of subsets of X .*

From the (1.2) we have that $\mathcal{L}(X)$, as union of the family of boolean algebras $\{\mathcal{B}_\alpha(X) : \alpha \in \text{Att}(X)\}$, is a *boolean manifold (atlas)*; each boolean algebra $\mathcal{B}_\alpha(X)$ being a *local chart* of the manifold [DPS, 95].

For any attribute α , we can associate to the “local” α -sentential language \mathcal{Q}_α the semantical model $\langle \mathcal{B}_\alpha(X), v_\alpha \rangle$ based on the local chart $\mathcal{B}_\alpha(X)$, where the *valuation mapping* $v_\alpha : \mathcal{Q}_\alpha \rightarrow \mathcal{B}_\alpha(X)$ associates to any elementary question $\langle \alpha, \Delta \rangle \in \mathcal{Q}_\alpha$ the proposition [i.e., subset of states (objects)] $v_\alpha(\langle \alpha, \Delta \rangle) := E_\alpha(\Delta)$. Intuitively, the α -valuation of the α -elementary question $\langle \alpha, \Delta \rangle$ consists of all states $x \in X$ in which this question is “*verified*” (or “*true*”) [i.e., a test of the attribute α on the state x yields a value $f_\alpha(x) \in \Delta$]. Moreover, the following must be satisfied for arbitrary $Q, Q_1, Q_2 \in \mathcal{Q}_\alpha$: $v_\alpha(\neg Q) = v_\alpha(Q)$, $v_\alpha(Q_1 Q_2) = v_\alpha(Q_1) \cap v_\alpha(Q_2)$, $v_\alpha(Q_1 \cup Q_2) = v_\alpha(Q_1) \cup v_\alpha(Q_2)$, and $v_\alpha(Q_1 \rightarrow Q_2) = v_\alpha(Q_1) \cup v_\alpha(Q_2)$. Moreover, $v_\alpha(\perp) = \emptyset$ and $v_\alpha(\top) = X$.

Two α -questions $Q_1, Q_2 \in \mathcal{Q}_\alpha$ are α -semantically equivalent, written $Q_1 \equiv_\alpha Q_2$, iff $v_\alpha(Q_1) = v_\alpha(Q_2)$; as expected we have that

$\neg \langle \alpha, \Delta \rangle \equiv_\alpha \langle \alpha, \Delta \rangle$, $\langle \alpha, \Delta_1 \rangle \langle \alpha, \Delta_2 \rangle \equiv_\alpha \langle \alpha, \Delta_1 \cap \Delta_2 \rangle$, $\langle \alpha, \Delta_1 \rangle \cup \langle \alpha, \Delta_2 \rangle \equiv_\alpha \langle \alpha, \Delta_1 \cup \Delta_2 \rangle$, and $\langle \alpha, \Delta_1 \rangle \rightarrow \langle \alpha, \Delta_2 \rangle \equiv_\alpha \langle \alpha, (\Delta_1) \cup \Delta_2 \rangle$.

In particular, from the trivial property

$$(1.3) \quad E_\alpha(\Delta) = \bigcup_{\lambda \in \Delta} E_\alpha(\{\lambda\})$$

we get that $\langle \alpha, \Delta \rangle \equiv_\alpha \bigcup_{\lambda \in \Delta} \langle \alpha, \lambda \rangle$.

An α -wff $Q \in \mathcal{Q}_\alpha$ is an α -tautology, written $\vdash_\alpha Q$, iff $v_\alpha(Q) = X$; in particular we have that for any pair of α -questions $\vdash_\alpha (\langle \alpha, \Delta_1 \rangle \rightarrow \langle \alpha, \Delta_2 \rangle)$ iff $\Delta_1 \subseteq \Delta_2$.

1.3 Partitions, indiscernibility relation and rough approximation in a Knowledge Representation System

Given any attribute α , we introduce the following subset of the boolean algebra $\mathcal{B}_\alpha(X)$:

$$(1.1a) \quad \pi_\alpha(X) := \{f_\alpha^{-1}(\{\lambda\}) : \lambda \in \text{val}(\alpha)\} \subseteq \mathcal{B}_\alpha(X)$$

and so the set:

$$(1.2a) \quad \mathcal{L}_\alpha(X) := \bigcup_{\alpha \in \text{Att}(X)} \pi_\alpha(X) \subseteq \mathcal{L}(X).$$

Proposition 1.2 *Let \mathcal{KR} be a knowledge representation system; let $\alpha \in \text{Att}(X)$. Then $\pi_\alpha(X)$ is a partition of X called the α -partition.*

Thus, starting from a \mathcal{KR} -system, we can choose an attribute α and then induce the α -partition $\pi_\alpha(X)$ of X making use of the “observable” related to α : $E_\alpha = f_\alpha^{-1}$. To be precise, the α -partition is obtained by the valuation of f_α^{-1} on the singletons $\{\lambda\}$, with $\lambda \in \text{val}(\alpha)$. The α -equivalence class $E_\alpha(\{\lambda\})$ is then the collection of all states in which a test of attribute α produces the fixed value λ .

Definition 1.3 *Let \mathcal{KR} be a knowledge representation system and let $\alpha \in \text{Att}(X)$. Then we define the indiscernibility (equivalence) relation induced by α (denoted by \sim_α) in the following way. Let $x_1, x_2 \in X$ and let $\pi_\alpha(X)$ be the partition of X from α ; then we have that x_1, x_2 are indistinguishable*

$$\begin{aligned} x_1 \sim_\alpha x_2 &\Leftrightarrow \exists E \in \pi_\alpha(X) : x_1, x_2 \in E \\ &\Leftrightarrow f_\alpha(x_1) = f_\alpha(x_2) \end{aligned}$$

The discernibility relation induced from α (denoted by $\not\sim_\alpha$) is obviously the following (irreflexive and symmetric) binary relation on X :

$$\begin{aligned} x_1 \not\sim_\alpha x_2 &\Leftrightarrow \exists E_1, E_2 \in \pi_\alpha(X), \text{ with } E_1 \cap E_2 = \emptyset : \\ &\quad x_1 \in E_1 \text{ and } x_2 \in E_2 \\ &\Leftrightarrow f_\alpha(x_1) \neq f_\alpha(x_2) \end{aligned}$$

At this point, we recall the *indiscernibility* concept introduced by Pawlak; he says: “The basic idea underlying classification consists in the fact that objects being in the same equivalence class of equivalence relation cannot be discerned, therefore we will call these the *indiscernibility classes*.” ([Pa, 92]). In Pawlak’s terminology, equivalence classes $E_\alpha(\{\lambda\}) \in \pi_\alpha(X)$ are called *elementary sets*; moreover, any α -event as set theoretic union of elementary sets [see the (1.3)], is a *definable set*. For any subset $A \in \mathcal{P}(X)$ one can introduce the *lower* (also *inner*) α -approximation of A :

$$I_\alpha(A) := \cup \{E_i \in \mathcal{B}_\alpha(X) : E_i \subseteq A\}$$

and the *upper* (also *outer*) α -approximation of A :

$$C_\alpha(A) := \cap \{E_j \in \mathcal{B}_\alpha(X) : A \subseteq E_j\}$$

Following Pawlak, the α -rough approximation of A is the pair:

$$r_\alpha(A) := (I_\alpha(A), C_\alpha(A)).$$

An attribute α realizes a “*context*” in which we execute observations; such context is formally characterized by the corresponding α -partition.

Definition 1.4 Let \mathcal{KR} be a knowledge representation system; let $\alpha_1, \alpha_2 \in \text{Att}(X)$. Then α_1 is contextually equivalent to α_2 (denoted by $\alpha_1 \equiv \alpha_2$) if and only if $\pi_{\alpha_1}(X) = \pi_{\alpha_2}(X)$. The equivalence classes of attributes induced by \equiv are the contexts of \mathcal{KR} -system. Any context (equivalence class) $[\alpha]_{\equiv}$ is represented by the (unique) partition associated to any of its attributes.

2 An interesting example

Let us consider the following \mathcal{KR} -system $\langle X, \text{Att}(X), \text{val}(X), F \rangle$, where

$$X = \{1, 2, 3, 4\} \quad \text{Att}(X) = \{\alpha_0, \alpha_1, \alpha_2\} \quad \text{val}(X) = \{Y, R, G, M, L, S, A, T\}.$$

The random variables describing these attributes are given by the following table:

	$ c c c c - $
$x \in X$	$f_{\alpha_0}(x) \quad f_{\alpha_1}(x) \quad f_{\alpha_2}(x)$
1	A G S
2	A R S
3	T Y M
4	T Y L

We have three α -partitions, one for each attribute:

$$\pi_{\alpha_0}(X) = \{\{1, 2\}, \{3, 4\}\} \quad \pi_{\alpha_1}(X) = \{\{1\}, \{2\}, \{3, 4\}\}$$

$$\pi_{\alpha_2}(X) = \{\{1, 2\}, \{3\}, \{4\}\}$$

Thus the contexts of this system are three, each with a single attribute: $j = 0, 1, 2$, $[\alpha_j]_{\equiv} = \{\alpha_j\}$. The boolean algebras generated by these attributes are the following:

$$\begin{aligned} \mathcal{B}_{\alpha_0}(X) &= \{\emptyset, \{3, 4\}, \{1, 2\}, \{1, 2, 3, 4\}\} \\ \mathcal{B}_{\alpha_1}(X) &= \{\emptyset, \{1\}, \{2\}, \{1, 2\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\} \\ \mathcal{B}_{\alpha_2}(X) &= \{\emptyset, \{4\}, \{3\}, \{3, 4\}, \{1, 2\}, \{1, 2, 4\}, \{1, 2, 3\}, \{1, 2, 3, 4\}\} \end{aligned}$$

We depict these boolean algebras by using Hasse diagrams:

Figure 1a.

Figure 1b.

Thus the boolean manifold generated by the above boolean charts $\{\mathcal{B}_{\alpha_0}(X), \mathcal{B}_{\alpha_1}(X), \mathcal{B}_{\alpha_2}(X)\}$ is the following:

$$\begin{aligned}\mathcal{L}(X) &= \{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{3, 4\}, \{1, 2\}, \{1, 3, 4\}, \\ &\quad \{2, 3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 2, 3, 4\}\} \\ \mathcal{L}_a(X) &= \{\{1\}, \{2\}, \{3\}, \{4\}, \{3, 4\}, \{1, 2\}\}\end{aligned}$$

whose associated Hasse diagram with respect to set theoretic inclusion is the following:

Figure 2.

Note that the resulting global structure is of poset which is not a lattice (e.g., the pair $\{1, 2, 3\}$ and $\{2, 3, 4\}$ does not possess the g.l.b., similarly the pair $\{2\}$ and $\{3\}$ does not possess the l.u.b.), with standard set theoretic complementation. This poset is obtained just by the “*pasting*” of the boolean algebras $\mathcal{B}_{\alpha_i}(X)$.

The subsets of $\mathcal{L}_a(X)$ are atoms of the single boolean charts (e.g., $\{1\}$, $\{2\}$, and $\{3, 4\}$ are atoms of chart $\mathcal{B}_{\alpha_1}(X)$), but in general they are not necessarily atoms of the global boolean manifold $\mathcal{L}(\mathcal{X})$ (e.g., $\{3, 4\}$ is not an atom).

In general X represents some objects with their own characteristics properties, specified by the attributes, and the information mapping F connects these attributes to objects. In the present example, the meaning of attributes can be, for example, the following: α_0 is the shape of the object (A as arched and T as thin), α_1 is the colour (Y as yellow, G as green, and R as red), α_2 is the dimension (S as small, M as medium, L as large).

With respect to the attribute “colour” α_1 , we have that the atomic question $\langle \alpha_1, G \rangle =$ “the colour is Green” is semantically valuated by the proposition $\{1\}$, the atomic question $\langle \alpha_1, R \rangle =$ “the colour is Red” is semantically valuated by the proposition $\{2\}$, and the atomic question $\langle \alpha_1, Y \rangle =$ “the colour is Yellow” is semantically valuated by the proposition $\{3, 4\}$; hence, for instance, $\{1, 3, 4\}$ is the semantical valuation of the question $\langle \alpha_1, \{G, Y\} \rangle \equiv_{\alpha_1} \langle \alpha_1, G \rangle \langle \alpha_1, Y \rangle =$ “the colour is Green or Yellow”, and so on.

Let us notice that the proposition $\{3, 4\}$ is the semantical valuation of both the atomic questions $\langle \alpha_1, Y \rangle =$ “the colour is Yellow” and $\langle \alpha_0, T \rangle =$ “the shape is Thin”, and of the complex question $\langle \alpha_2, \{M, L\} \rangle =$ “the size is Medium or Large”, all pertaining to different attributes.

Notwithstanding the transitivity of the set theoretic inclusion (e.g., from $\{1\} \subseteq \{1, 2\}$ and $\{1, 2\} \subseteq \{1, 2, 3\}$ it follows $\{1\} \subseteq \{1, 2, 3\}$), this property of transitivity cannot be applied to the \mathcal{KR} sentential language \mathcal{Q} ; roughly speaking, and without entering in technical details, we can say that in the context of colour “to be green” implies “to be green or red” ($\vdash_{\alpha_1} (G \rightarrow GR)$ since semantically $\{1\} \subseteq_{\alpha_1} \{1, 2\}$); similarly, in the context of size “to be small” implies “to be small or medium” ($\vdash_{\alpha_2} (S \rightarrow SM)$ since semantically $\{1, 2\} \subseteq_{\alpha_2} \{1, 2, 3\}$). But, there is no context α which allows to say that $\vdash_{\alpha} (G \rightarrow SM)$ (i.e., $\nexists \alpha : \{1\} \subseteq_{\alpha} \{1, 2, 3\}$), also if from the set theoretical point of view $\{1\} \subseteq \{1, 2, 3\}$.

2.1 Non-transitive reasoning: an example with knowledge representation systems

In this paragraph we give an example of non-transitive reasoning in the context of a knowledge representation system obtained by an empirical *ordered structure of implication*. To be precise, we are concerned with the problem of extracting knowledge from experimental results; in particular we discuss the well known example of getting information about the physical events “*it is noon*” (n), “*there is sun*” (s), and “*there is an eclipse*” (e).

If we choose a *bottom-up* methodology, we start from experiments and detect the partial informations we have during the experimentation. With respect to the above experimental knowledges, we consider the two possible contexts:

$$\alpha_1 := (s, e) \text{ "sun light" and "eclipse" context:}$$

In this case, let us suppose that the first observation leads us to the conclusion that during an eclipse the sun cannot lights earth (obviously each experiment is not executed during the night). Thus we have the partial information: " $e \rightarrow \neg s$ " and, clearly, its contraposition " $s \rightarrow \neg e$ ".

$$\alpha_2 = (n, s) \text{ "sun light" and "noon" context:}$$

Then let us suppose to perform some new experiments which confirm that each time we perform the test at noon, then the sun lights the earth (eclipses are not frequent, so it is very unlikely that in the first step of experiments we can get the event of an eclipse at noon). Thus we obtain another implication: " $n \rightarrow s$ ", and obviously its contraposition " $\neg s \rightarrow \neg n$ ".

We depicts these implication relations by the diagrams of Figure 3 which describe the empirical conclusions from these two different contexts.

Figure 3:

We can argue that, with respect to our *knowledge*, the only "*situations*" (*states*) of the triple (n, s, e) to be taken into account are the ones in which there are no contradictions with respect to the above experimental situations. In the following table we denote by $n, s, e = 1$ the fact that "it is noon", "there is soon", and "there is an eclipse" respectively (otherwise we pose 0); moreover, we denote by $a = 1$ the noncontradictory situation with respect to our partial experimental knowledge (otherwise we pose $a = 0$):

	c	c	c	c	n	s	e	a
0	0	0	0	1				
0	0	1	1					
0	1	0	1					
0	1	1	0					
1	0	0	0					
1	0	1	0					
1	1	0	1					

	c	c	_	state	condition
1	$\neg n$	\wedge	$\neg s$	\wedge	$\neg e$
2	$\neg n$	\wedge	$\neg s$	\wedge	e
3	$\neg n$	\wedge	s	\wedge	$\neg e$
4	n	\wedge	s	\wedge	$\neg e$

The situations involved by contexts α_1 and α_2 are described by the following table [where we label different situations of the (s, e) context by the conventional arbitrary symbols (G, R, Y) , and different situations of the (n, s) context by the arbitrary symbols (S, M, L)]:

	c	c	_	state	condition
α_1	1	G	$:=$	$(\neg s \wedge \neg e)$	S
α_2	2	R	$:=$	$(\neg s \wedge e)$	S
α_1	3	Y	$:=$	$(s \wedge \neg e)$	M
α_2	4	Y	$:=$	$(s \wedge \neg e)$	L

By the comparison of this (labelled) table with the table of the knowledge representation system exemplified in section 2, we immediately realize that they show the same features; thus we are able to translate the involved diagrams in the present ones. In particular, the two boolean diagrams of Figure 3 are the “translations” of the two latter boolean diagrams of Figure 1; moreover, the global orthoposet diagram of Figure 2 is translated in the one pictured in Figure 4.

Figure 4.

We can give now the following interpretation of the structure: supposing that (as shown in Figure 3) one scientist gets the conclusion that in the \mathcal{KR} -system it is true that “ $e \rightarrow \neg s$ ”, and a second one reach the conclusion that it is true that “ $n \rightarrow s$ ”, then the diagram of Figure 4 shows the deductions we can obtain from joining the contexts in a unique “environment” (“pasting”).

In this case we observe that we cannot infer “ $e \rightarrow \neg n$ ”, since the implication relation is not transitive; from $\vdash_{\alpha_1} (e \rightarrow \neg s)$ and $\vdash_{\alpha_2} (\neg s \rightarrow \neg n)$, we cannot find

an “actual” context of experimental knowledge α such that $\vdash_{\alpha} (e \rightarrow \neg n)$. Also if we have improved our knowledge from the context $\mathcal{A}_1 := \{\alpha_1\}$ to the context $\mathcal{A}_{1,2} := \{\alpha_1, \alpha_2\}$, single “contextual” inferences cannot be extended by transitivity to obtain new implications.

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