On the Viability of an Algebraic Approach to Non-monotonic Reasoning

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1 Introduction

Under the general heading of non-monotonic reasoning many different, although connected, phenomena (and the theories purporting to explain them) that are related to an empirical fact of commonsense reasoning are usually grouped together. The empirical fact is the following: when adding new information to a body of knowledge one can be very naturally induced (and in some cases is forced) to withdraw some of the conclusions previously obtained. Most research done in A.I. –indirectly or (especially starting from the late seventies) directly– has tried to clarify, classify and connect some of the issues pertaining to these *empirical* phenomena. More recently it has attracted interest and criticism too, on the part of the more properly logical community.

In the present paper we shall briefly argue in favour of the following points. 1) An algebraic analysis can contribute to understanding the mechanism underlying the empirical phenomenon of the withdrawal of conclusions and to addressing some of the difficulties arising on the way of an adequate formal treatment of this phenomenon. 2) Additional insights can be obtained if any new information which is added to a certain body of knowledge is not considered in isolation, but as part of a (limited but structured) body of knowledge. From a conceptual point of view, in fact, it is clear that it is just the *implicit* knowledge associated to the new single piece of information which requires the withdrawal of conclusions previously drawn: finding out a way of making explicit and formally representing this implicit knowledge cannot but help in understanding better the problem; our tenet is that an algebraic setting can at least help focusing on some of the problems which arise. 3) The separate extensions problem can be usefully looked at from the point of view of logical structures in which some statements are (declared to be) "incompatible". Algebraic analyses of points 2 and 3 can be carried out in standard algebraic struc-

tures and a possible way of doing this connects the two points. The idea is to consider families of partially overlapping boolean algebras (alternatively, of other suitable algebraic structures) satisfying some coherence conditions in the overlapping parts. Let us emphasize that this paper is only a preliminary investigation into the possibility of outlining a framework in which these questions and problems can be properly discussed: this, of course, is **not** the same as claiming that in this framework these questions and problems can actually be, or have already been, solved. The present paper, then, should be seen as nothing more than an attempt at chartering the territory.

The paper is structured in the following way. In section 2 the conceptual background of the approach is discussed. Section 3 presents a series of remarks showing both the conceptual relevance of non-distributive structures for our theme and some difficulties impeding an immediate migration of results from one field to the other. In particular, a series of very elementary examples suggests a possible attack on the problems arising in systems containing *Unless* operators by means of non-distributive structures. These remarks indicate the possibility of a useful interchange between Quantum Logic and problems in Artificial Intelligence. We refer to [?, ?] for background information to the problem of non-monotonic reasoning; some recent developments can be found in [?]. The present paper is a further step in the elaboration of ideas summarized in [?] and whose ancestors can be found in [?] and [?]; a more detailed and expanded discussion will be found in [?].

2   The conceptual background of the approach

The final aim of research programmes possibly emerging from the outline of this algebraic approach would be that of obtaining a general structure showing how the surface phenomenon of non-monotonicity can be explained at deeper levels, where the elementary logical steps behave in a more traditional way. Non-monotonicity would then appear as some sort of global effect manifesting itself at the surface of logico-linguistic behaviour, and pointing to the fact that complex phenomena are taking place at hidden levels, which crucially involve some sort of interaction between the premises already accepted and the new assumption. This interaction brings about the collapse of the traditional monotonicity property.

Non-monotonic reasoning, that is, the empirical fact that inferred conclusions can be subsequently withdrawn in the light of further evidence and information, has been always recognized in reflections upon commonsense reasoning and scientific inquiry alike (and, as *empirical fact* considered uncontroversial). It became a **hot** subject only when the general conceptual constraints of Artificial Intelligence forced one to see whether it was possible to afford this problem by means of *mechanical* tools which could be easily handled by the same machinery which was able to manage, successfully, other aspects of A.I. such as the representation of knowledge. More generally, one aims at developing formal tools of the "same type" as those that helped clarifying inference in mathematical reasoning, and applicable in every

field in which both the drawing of inferences from the available knowledge and the modifying of our concepts due to a real enrichment of knowledge are needed.

The problems of non-monotonic reasoning became *problems* as a consequence of the clash between the (1) mechanical constraints posed by the very nature of an Artificial Intelligent System and (2) the qualitative behaviour of human agents manifesting the usual features of commonsense reasoning.

Among the main causes of the withdrawal of conclusions previuosly obtained in the light of new information (i.e. non-monotonicity in reasoning) there are, certainly, the two following facts:

a) the (possible) incompatibility of some facts and statements and

b) the merging of knowledge from different sources which can produce clashes and (again!) incompatibilities.

We now briefly present our working hypothesis. Let us assume that each small piece of local knowledge which is available to a reasoner can be considered complete (until explicit information to the contrary is provided). We can then make the hypothesis that each such piece of knowledge can be represented through some elementary relational structure, possibly a boolean algebra. What usually happens in everyday life (but the same holds also in the realm of scientific discovery) is that different elementary pieces of knowledge can interact in a non-trivial way. During this process, new relevant information for a certain piece of local knowledge can be borrowed from another piece of local knowledge. This mechanism in many cases produces an enrichment of local knowledge without any negative effect. In some cases, however, some undesirable consequences can spring out. We propose to describe this phenomenon algebraically through a family of relational structures (possibly, boolean algebras) each of which is a model of a small piece of local, elementary knowledge. These structures can overlap. Our tenet is that some familiar examples recurring in the literature on non-monotonic reasoning can be accounted for, at least qualitatively, by some procedure which constructs, step by step, the composite structure (and indicates possible wrong identifications of the elements in the overlapping parts of the basic structures). Two worth-mentioning difficult problems are the following ones. 1) Is it possible to find out purely mechanical rules ensuring a right development of this process? 2) Is it possible to provide a nice algebraic characterization of the global structure obtained by composing the basic structures (paralleling the achievements of Finch [?] for the case of Quantum Logic)? Let us explicitly observe that underlying the previous remarks there is the (trivial but nonetheless crucial) idea that the correct logical relationships emerge from both the actual context and the past history of a given situation. A suitable algebraic setting should guarantee the correct management of the data[1].

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[1]In a preliminary investigation, following the attempt outlined in [?], the algebraic structures one can try to test are: i) single boolean algebras, ii) families of boolean algebras and iii) nondistributive structures (lattices and posets). Finch's result [?], just quoted, provides a nontrivial connection between ii) and iii). See the comments in [?], stressing both the role that this connection can play for the problem we are dealing with and the fact that families of boolean algebras connected *a' la* Finch, could also give rise to different structures, depending on the

Some further points which could contribute to the present conceptual analysis, but will not be discussed here, are the following ones. 1) An analysis of the connections between approaches starting from an analysis of implication connectives, and from the logical consequence relation, respectively. 2) An analysis of the possibility of obtaining, at a global level, Gabbay's non-monotonic consequence relation [?] starting from conditions imposed on the composition of elementary algebraic structures. 3) An analysis of the presence of both non-monotonicity and vagueness of the involved predicates, following also a path suggested in [?]. For some recents aspects of Fuzzy Quantum Logics see [?]. Some of these issues will be touched on in [?].

Let us finally observe that, since the general philosophy underlying the approach outlined here is consonant with the remarks made, in different settings, by Peter Gardenfors [?, ?, ?] and by Fausto Giunchiglia (see for instance [?, ?] and the bibliography therein), an analysis of convergences and differences should be made.

3 On "commonsense reasoning" and non-distributive structures

As was indirectly suggested in the previous section, Quantum Logic seems capable of playing a role in an unconventional analysis of some aspects of non-monotonic reasoning. In this section we shall present additional information which should both reinforce our point and show some of the difficulties impeding a straightforward "application" of the formalism of Quantum Logic to problems of commonsense reasoning. In particular, we shall argue for the following thesis. The situations which are informally presented through *Unless* operators are good candidates to be modelled by means of non-distributive algebraic structures. To corroborate this thesis we shall limit ourselves here to present a series of very elementary examples indicating some of the constraints to be imposed on a non-distributive structure in order to fulfil the qualitative requirements of systems containing *Unless* operators.

3.1 On "commonsense reasoning" and quantum implication connectives

Marisa Dalla Chiara on page 442 of [?] makes reference to the fact that the quantum-logical material conditional shows a somewhat anomalous behaviour, since it violates most of the laws that are valid for a positive conditional. In a sense, this *anomalous behaviour* is not negative in the present setting of looking at Quantum Logic as a possible instrument for capturing "unusual" aspects of commonsense reasoning, if really there is –as we maintain– a "consonance", at a qualitative level,

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conditions imposed (on the relationship among the different boolean algebras) by the qualitative properties of the specific problem under consideration. Some preliminary results regarding single boolean algebras have been obtained by Trillas [?, ?].

between some of the basic features of Quantum Logic and aspects of commonsense reasoning. Of course, only detailed technical work will show how many aspects of the latter can be captured in the formalism of the former as well as how the formalism of the former should be trasformed to take into account meaningful aspects of the the latter. Granted these quite trivial considerations, let us note that it is not at all obvious that the following law (which is violated by the quantum-logical material conditional)

$$(\alpha \to \beta) \to ((\beta \to \delta) \to (\alpha \to \delta)),$$

should be maintained in commonsense reasoning, for instance, in the case in which the following interpretation is given:

$\alpha = $ the flight is in the timetable

$\beta = $ the airplane will leave

$\delta = $ there is no strike

It seems, in fact, that, *in commonsense reasoning*, one could accept $(\alpha \to \beta)$ –in the previous interpretation– at least as a working hypothesis, while no one would accept as reasonable, *in commonsense reasoning*, that $(\alpha \to \beta)$ has $(\beta \to \delta) \to (\alpha \to \delta)$ as one of its consequences.

3.2 A remark on the *modification* of concepts

Giovanni Sambin –in his essay [?] on an innovative point of view on the foundations of mathematics– proposes a very radical solution to the problem of non-monotonic reasoning: according to his suggestion, in the case in which the necessity of the withdrawal of a conclusion arises, we must change the concepts involved, not the logic: it is the application of the *usual* logic that, focusing or making explicit the contradiction, pushes towards a modification of the concepts involved. Sambin (page 206) makes a step further along this direction when he claims that we should concentrate on the search for formalisms allowing a dynamical interpretation of predicates, which can be modified as the available information increases, and suggests that topoi theory could be a good candidate to play this role. Close to this position (which could be summarized in the slogan: *Don't change the logic, change the concepts*), is the interesting view heralded by Peter Gardenfors (see, among his contributions, also the one in this same volume [?]).

   The analysis proposed here, in our opinion, takes seriously into account this idea, but at the same time tries to reconstruct and *describe* –at the level of the algebraic structures– what is taking place in a way that could be "understood" and accepted by the supporters of non-monotonic *logics*.

3.3 Families of (partially overlapping) algebraic structures

In this subsection we resume to the conceptual analysis presented in section 2 above. We have available some algebraic structures which, at least in principle, are good candidates for being the natural formal *explicatum* of the informal idea that when we add a new information to a body of knowledge, we, in fact, let two structured complete pieces of knowledge interact. As already stressed, this is compatible with the observation that Quantum Logic is a potential candidate for clarifying some problems of A.I.

While describing the process of the interaction of elementary (but structured) small pieces of knowledge we observed that an algebraic description of the phenomenon of interaction could be provided by considering a family of relational structures each of which could be considered as an algebraic model of a corresponding piece of local elementary knowledge. We should then construct a global algebraic structure and the rules used to construct the global structure should be such as to respect the interaction among these small pieces of knowledge. The fact that, in section 2, we added that these relational structures should possibly be boolean algebras and, at the same time, Finch's paper [?] was mentioned, may induce one to think that the hypothesis that each elementary piece of knowledge should always be represented by a boolean algebra was surreptitiously made. In fact, this can be considered as a "desideratum". It is not clear, however, that this can really be done, at least straightforwardly. Think, for instance, of the well known simple example of a database containing the information that $N \to S$ and $E \to \neg S$, where N = It is noon; S = It sunshines; E = There is an eclipse. If we try to model this situation according to the lines discussed above, we should embed each of the two previous pieces of information into structured small bulks of knowledge and, later on, let them interact. The final global algebraic structure should take into account both the initial information and the mode of their *interaction*. However, the second of the previous pieces of information contains already, in itself, a *message* of incompatibility between E and S. So, formally, we can certainly construct the elementary bulk of knowledge expressed by $E \to \neg S$ by means of the boolean algebra generated by E and S and containing the information that $E \to \neg S$. However, it is not clear whether in pursuing this path we are, in fact, losing some basic information. Perhaps, in cases of this type we should start from a non-distributive structure just at this level. One could, possibly, follow a strategy similar to the one suggested, in a different context, by Claudio Garola on page 208 of [?]. Also in very simple cases, then, incompatibility could arise at the level of just one (structured) elementary piece of knowledge. We are left with the following dilemma. If we want to pursue the strategy that each elementary piece of knowledge must be modelled by a boolean algebra (which is simpler both from a conceptual and a technical point of view) we remain with the problem of taking into account and representing this elementary incompatibility in an efficient way. If we admit that the incompatibility can appear also at the level of the representation of elementary pieces of knowledge than we must face the more difficult technical

and conceptual problem of starting from different basic algebraic structures[2].

3.4 *Unless* operators and non-distributive structures

In the previous subsection we have briefly touched upon mechanisms which can produce non-distributive structures starting from more elementary basic levels. We have also seen that this is not so straightforward. Let us indicate now a possible connection between *Unless* operators and non-distributive structures. Let us consider the case of Sandewall's interlocked defaults [?]

$$\left\{ \begin{array}{l} Unless A \rightsquigarrow B \\ Unless B \rightsquigarrow A \end{array} \right.$$

The conceptual feature behind the notion of interlocked defaults is *simply* that A and B are mutually incompatible. We could then, *simply*, look for some algebraic structure which takes into account *automatically* the fact that some elements are incompatible, and cannot be both present in the set of derived conclusions (or, better, in the set of declared facts plus the derived conclusions from the facts that have been assumed explicitly). Now, the first algebraic structure which comes to the mind is, in general, a non-distributive poset. The cases in which defaults of the *Unless* type are present can be different from the one in which they are *interlocked* as in the case considered above; however, the same general considerations can be made.

The proposal, then, is the following: describe these situations in a non-distributive structure declaring that A and B are not compatible.
So, if we have something like:

$$\left\{ \begin{array}{l} C(Unless A) \rightsquigarrow B \\ D(Unless B) \rightsquigarrow A \end{array} \right.$$

after the declaration that A and B are not compatible we should simply have, in the new structure:

$$\left\{ \begin{array}{l} C \Rightarrow_N B \\ D \Rightarrow_N A \end{array} \right.$$

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[2]A discussion of these kinds of examples and the associated problems was first given in [?]; subsequent discussions with Giampiero Cattaneo have helped both of us to clarify the questions involved but have not provided yet a solution, see [?]. The idea presented above of starting from a non-distributive structure (or, alternatively, the one of decomposing into smaller boolean components those elementary bulks of information which present already messages of incompatibility) has not yet been tested technically to see whether it can provide a viable solution to the problem under consideration. Possible connections with Gabbay's weaving of logics [?, ?] should be also investigated.

(where $\Rightarrow_N$ is the implication in the new structure) since the *same* structure should take care *automatically* of the information previously provided *locally* by the *Unless* operator.

If we have the knowledge that C holds, the system should *automatically* derive B if A has not been already derived, otherwise, again *automatically*, the structure should **not** allow one to derive it.

As an example, let us consider a database containing the following information:

$$\begin{cases} C \Rightarrow_N B \\ C \end{cases}$$

Starting from the previous information, we should be able to derive B, while from

$$\begin{cases} A \\ C \Rightarrow_N B \\ C \end{cases}$$

the same structure should forbid the derivation of B if A and B have been declared incompatible.

3.5   Non-distributive structures and non-monotonicity

One could rightly object that even though the term *'non-monotonic'* has been used repeatedly, it has not been specified how it is –or could be– connected with non-distributive structures. More, until now the phenomenon of non-monotonicity has never *explicitly* appeared. The structure envisaged appears to be robust enough to take into account the fact that some statements can be mutually incompatible. Thus, it *does take care* of impeding the derivation of incompatible conclusions but, *as such*, it does not provide a model of the mechanism of a real withdrawal of a previously obtained conclusion. Can it contribute to explicating the phenomenon of the revisability of conclusions already obtained? Is it possible to enrich a non-distributive structure in order to obtain a real withdrawing? It is reasonable to attempt to do this by introducing a dynamical aspect. This is natural in the case in which B has been already derived and we add from the outside A, successively. What does it happen in a scheme of the following type?

$$\begin{cases} C \Rightarrow_N B \\ C \\ A \end{cases}$$

The first two lines should allow one to derive B. In a standard non-distributive

setting we are not allowed to add A if B has been already derived; however in an expert system it may happen that we want to add A from the outside as new information which has been in the meanwhile obtained.

A possible way out from this difficulty could be that of imposing the distinction between FACTS (obtained from outside) and CONSEQUENCES which are formally derived (by the expert system). In a mechanical artificial reasoning system we should impose that the facts known from an external source, which we regard as reliable, MUST have a preheminence upon derived conclusions. In these systems we should then introduce a metarule imposing to withdraw those conclusions which have been previously derived and which are incompatible with the facts that progressively one comes to know. We see that a non-distributive logical structure, if properly used, can show an interesting aspect of revisability of the obtained conclusions by using only classical tools. In the case in which both A and B are facts, the structure itself should warn us that either we are making a mistake or something should have changed in the global information, since A and B are incompatible (facts). Thus, the non-distributive structure should guarantee the incompatibility between derivations and the incompatibility between facts. In the case of an incompatibility between fact and derivation the preheminence rule of facts over derivations is activated. Let us summarize the situation in the following way: We propose TO MODEL those aspects of nonmonotonicity arising from *Unless* situations through a non distributive structure in such a way that:

(a) all incompatibilities are declared at the beginning, and

(b) some sort of metarule imposes to withdraw the conclusions previously obtained by means of *derivations*, if −at a later moment− some *facts* incompatible with those conclusions are asserted by an external source.

Incompatibility, in the context of systems studied in A.I., seems necessarily to require a distinction between the facts which are declared and those which are obtained through derivations[3]. The former ones, of course, have a preheminence upon the latter ones. However, the features of non-distributive structures should manage automatically the incompatibities declared at the beginning.

4   Conclusions

In the previous pages, the *algebraic* point of view briefly outlined has been defended as a useful approach to the clarification of *some aspects* of non-monotonic reasoning. In these last lines we want to make some general considerations regarding the conceptual analysis of the problem. Very informally, one family, at least, of the attempts at formalizing commonsense reasoning (the default logics) can be seen as *"approaches that introduce non-standard inference rules, sanctioning deductions based not only on what has been proved but on what has not been proved. Such inferences may have to be withdrawn as new data becomes available."* [?], p. 269.

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[3] Differences between the ways in which incompatibility arises in these systems and Heisenberg's uncertainty principle, should be carefully analyzed.

Now, it seems evident that the new data that become available can force one to withdraw something previously deduced only if the new data are *incompatible* with something already deduced or with what allowed to deduce them. So, from this point of view the notion of incompatibility should be one of the cornerstones of any general approach to the problem of the revisability of conclusions previously drawn; moreover, it should be in a sense *embedded* in the basic structure. From this point of view, as a simple corollary of a purely conceptual analysis, it follows that non-distributive structures should be one element of the basis on which a general theory of non-monotonic reasoning must be constructed. The fact that non-monotonicity does not appear –in this setting– just from the start is an interesting feature at least for two reasons: on the technical side since the structures we propose to use have –in themselves– no special connection with either monotonicity or non-monotonicity; on the conceptual side, since non-monotonicity is in essence a dynamical notion. Non-monotonicity, however, very naturally arises if we consider –dynamically– the process of *using* non-distributive structures in commonsense reasoning. We think that this aspect –minor as it can be– could help in a non-negligible way to the conceptual clarification of the problem.

Acknowledgments

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