

The Study of the L-Fuzzy Concept Lattice

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Abstract

The $L - Fuzzy$ concept theory that we have developed in [2] sets up classifications from the objects and attributes of a context through $L - Fuzzy$ relations. This theory generalizes the formal concept theory of R.Wille ([6]).

In this paper we begin with the $L - Fuzzy$ concept definition that generalizes the definitions of the formal concept theory, and we study the lattice structure of the $L - Fuzzy$ concept set, giving a constructive method for calculating this lattice. At the end, we apply this constructive method to an example that has been studied by other methods.

Keywords: $L - Fuzzy$ concepts, hierarchies of concepts, $L - Fuzzy$ sets, lattices.

1 Introduction

Let $(L, \leq, ', \top)$ be the algebraic system determined by:

- A complete lattice (L, \leq)
- A complementation $'$ in $L([1])$
- A t-conorm \top in $L([1])$.

We gave in [2] the basic definitions that support the $L - Fuzzy$ concept theory. These definitions, together with other results that appear in the following paragraph, will allow us to study and represent the lattice structure of the $L - Fuzzy$ concept set.

2 Derivation operators in structures $(L, \leq, ', \top)$.

Let L^X and L^Y be the classes of $L - Fuzzy$ sets associated with X and Y respectively, and let $\underset{\sim}{R} \in L^{X \times Y}$ be an $L - Fuzzy$ relation.

Given $\underline{A} \in L^X$, we can associate to it the L -Fuzzy set \underline{A}_{\sim_1} of L^Y such that

$$\underline{A}_{\sim_1}(y) = \inf_{x \in X} (\underline{A}'(x) \mathbf{T} \underline{R}(x, y)) \quad (1)$$

In the same way, given $\underline{B} \in L^Y$ we can associate to it $\underline{B}_{\sim_2} \in L^X$ such that

$$\underline{B}_{\sim_2}(x) = \inf_{y \in Y} (\underline{B}'(y) \mathbf{T} \underline{R}(x, y)) \quad (2)$$

Definition 1 *The operators denoted by the subscripts 1 and 2 are called derivation operators weighted by a complementation.*

Definitions (1) and (2) recover, in the case of $L = \{0, 1\} = \mathbf{2}$, the derivation operator concepts used in the formal concept analysis of R. Wille ([6]).

Proposition 1 *The derivation operators defined in (1) and (2) verify:*

$$i) \underline{A} \leq \underline{B} \implies \underline{A}_{\sim_1} \geq \underline{B}_{\sim_1}, \forall \underline{A}, \underline{B} \in L^X.$$

$$i') \underline{C} \leq \underline{D} \implies \underline{C}_{\sim_2} \geq \underline{D}_{\sim_2}, \forall \underline{C}, \underline{D} \in L^Y.$$

$$ii) \bigvee_{i \in I} (\underline{A}_{\sim_i})_1 \leq \left(\bigwedge_{i \in I} \underline{A}_{\sim_i} \right)_1, \forall \underline{A} \in L^X, i \in I.$$

$$ii') \bigvee_{i \in I} (\underline{B}_{\sim_i})_2 \leq \left(\bigwedge_{i \in I} \underline{B}_{\sim_i} \right)_2, \forall \underline{B} \in L^Y, i \in I.$$

If \mathbf{T} is an upper semicontinuous t -conorm in L , then

$$\alpha \mathbf{T} \bigwedge_{i \in I} (\beta_i) = \bigwedge_{i \in I} (\alpha \mathbf{T} \beta_i), \forall \alpha, \beta_i \in L$$

and it is true that

$$iii) \left(\bigvee_{i \in I} \underline{A}_{\sim_i} \right)_1 = \bigwedge_{i \in I} (\underline{A}_{\sim_i})_1, \forall \underline{A} \in L^X, i \in I.$$

$$iii') \left(\bigvee_{i \in I} \underline{B}_{\sim_i} \right)_2 = \bigwedge_{i \in I} (\underline{B}_{\sim_i})_2, \forall \underline{B} \in L^Y, i \in I.$$

Proof. i) Let $\underline{A}, \underline{B} \in L^X, \underline{R} \in L^{X \times Y}$. If $\underline{A} \leq \underline{B}$, then $\underline{A}(x) \leq \underline{B}(x) \forall x \in X \implies \underline{A}'(x) \geq \underline{B}'(x) \forall x \in X \implies \underline{A}'(x) \mathbf{T} \underline{R}(x, y) \geq \underline{B}'(x) \mathbf{T} \underline{R}(x, y) \forall x \in X, \forall y \in Y$. Taking infimum $\inf_{x \in X} (\underline{A}'(x) \mathbf{T} \underline{R}(x, y)) \geq \inf_{x \in X} (\underline{B}'(x) \mathbf{T} \underline{R}(x, y)) \implies \underline{A}_{\sim_1}(y) \geq \underline{B}_{\sim_1}(y) \forall y \in Y \implies \underline{A}_{\sim_1} \geq \underline{B}_{\sim_1}$.

ii) $\forall A \in L^X, i \in I$ it is verified that $A \geq \bigwedge_{i \in I} A_{\sim i}$. Applying the derivation operator $(A_{\sim i})_1 \leq (\bigwedge_{i \in I} A_{\sim i})_1, \forall i \in I$; then $(\bigwedge_{i \in I} A_{\sim i})_1$ is an upper bound of $\{(A_{\sim i})_1, i \in I\}$.

As the supremum is the least upper bound:

$$\bigvee_{i \in I} (A_{\sim i})_1 \leq \left(\bigwedge_{i \in I} A_{\sim i} \right)_1.$$

iii) Since L is a complete lattice and \top is an upper semicontinuous t-conorm, hence

$$\begin{aligned} \left(\bigvee_{i \in I} A_{\sim i} \right)_1 (y) &= \inf_{x \in X} \left\{ \left(\bigvee_{i \in I} A_{\sim i} \right)' (x) \top R_{\sim i}(x, y) \right\} = \\ &= \inf_{x \in X} \left\{ \left(\bigwedge_{i \in I} A'_{\sim i} \right) (x) \top R_{\sim i}(x, y) \right\} = \\ &= \inf_{x \in X} \left\{ \left(\inf_{i \in I} A'_{\sim i} (x) \right) \top R_{\sim i}(x, y) \right\} = \\ &= \inf_{x \in X} \left\{ \inf_{i \in I} \left(A'_{\sim i} (x) \top R_{\sim i}(x, y) \right) \right\} = \\ &= \inf_{i \in I} \left\{ \inf_{x \in X} \left(A'_{\sim i} (x) \top R_{\sim i}(x, y) \right) \right\} = \\ &= \inf_{i \in I} \left\{ \left(A_{\sim i} \right)_1 (y) \right\} = \bigwedge_{i \in I} (A_{\sim i})_1 (y), \forall y \in Y. \end{aligned}$$

The proofs of i') , ii') and iii') are analogous to the previous ones. ■

If we write $A_{\sim 12}$ and $B_{\sim 21}$ to represent the L – Fuzzy sets $(A_{\sim 1})_2$ and $(B_{\sim 2})_1$ respectively, then we can define the operators φ and ψ :

$$\varphi : L^X \rightarrow L^X / \varphi(A) = A_{\sim 12} \tag{3}$$

$$\psi : L^Y \rightarrow L^Y / \psi(B) = B_{\sim 21} \tag{4}$$

which will help us to define the L – Fuzzy concepts.

Definition 2 The operators φ and ψ defined in (3) and (4) are called constructor operators.

These operators preserve the order as we proved in [2].

3 L-Fuzzy concept lattice.

At this point, we can introduce the basic definitions that support this work.

Definition 3 An L -Fuzzy context is a tuple (L, X, Y, R) where X and Y are the object and attribute set respectively, and $R \in L^{X \times Y}$ is a L -Fuzzy relation.

Definition 4 Let $\varphi : L^X \rightarrow L^X / \varphi(\underline{A}) = \underline{A}_{\sim_{12}}$, and let $fix(\varphi)$ be the set $fix(\varphi) = \{\underline{A} \in L^X / \underline{A} = \varphi(\underline{A})\}$.

If $\underline{M} \in fix(\varphi)$ then the pair $(\underline{M}, \underline{M}_1)$ is said to be the L -fuzzy concept of the L -Fuzzy context (L, X, Y, R) .

We can see that this definition generalizes that given in the formal concept theory where $L = \{0, 1\}$.

In our case, L^X and L^Y are complete lattices and the constructor operators φ and ψ defined in (3) and (4) preserve the order; therefore, due to the fixed points theorem of Tarski ([5]), $\Omega = (fix(\varphi), \leq, 0_{\sim_{\Omega}}, 1_{\sim_{\Omega}}, \vee_{\Omega}, \wedge_{\Omega})$ and $\Sigma = (fix(\psi), \geq, 0_{\sim_{\Sigma}}, 1_{\sim_{\Sigma}}, \vee_{\Sigma}, \wedge_{\Sigma})$ are complete lattices.

To calculate the minimum and maximum elements of these lattices, and the supremum and infimum of a family, we will look at the work carried out by P. Cousot and R. Cousot ([3]), which provides a constructive version of the theorem of Tarski ([5]). In this work, given a function f that preserves the order, they define $luis(f)(\underline{A})$ as the limit of a stationary upper iteration sequence for f starting with \underline{A} , and $llis(f)(\underline{B})$ as the limit of a stationary lower iteration sequence for f starting with \underline{B} .

Therefore, we can calculate

$$0_{\sim_{\Omega}} = luis(\varphi)(0_{\sim_{\Omega}}) \quad 0_{\sim_{\Sigma}} = luis(\psi)(0_{\sim_{\Sigma}}) \quad (5)$$

$$1_{\sim_{\Omega}} = llis(\varphi)(1_{\sim_{\Omega}}) \quad 1_{\sim_{\Sigma}} = llis(\psi)(1_{\sim_{\Sigma}}) \quad (6)$$

$$\forall \{A_{\sim_i}, i \in I\} \in \Omega, \bigvee_{\Omega} A_{\sim_i} = luis(\varphi) \left(\bigvee_{\sim_i} A_{\sim_i} \right) \quad \bigwedge_{\Omega} A_{\sim_i} = llis(\varphi) \left(\bigwedge_{\sim_i} A_{\sim_i} \right) \quad (7)$$

$$\forall \{B_{\sim_i}, i \in I\} \in \Sigma, \bigvee_{\Sigma} B_{\sim_i} = luis(\psi) \left(\bigvee_{\sim_i} B_{\sim_i} \right) \quad \bigwedge_{\Sigma} B_{\sim_i} = llis(\psi) \left(\bigwedge_{\sim_i} B_{\sim_i} \right) \quad (8)$$

Let $\mathcal{L}_1 = \{(\underline{A}, \underline{A}_1) / \underline{A} \in fix(\varphi)\}$ and $\mathcal{L}_2 = \{(\underline{B}_2, \underline{B}) / \underline{B} \in fix(\psi)\}$.

Proposition 2 The sets \mathcal{L}_{\sim_1} and \mathcal{L}_{\sim_2} are equal.

Proof. We are going to see that $\mathcal{L}_{\sim_1} \subseteq \mathcal{L}_{\sim_2}$:

$\forall (A, A) \in \mathcal{L}_{\sim_1}, A \in \text{fix}(\varphi)$ and $A \in \text{fix}(\psi)$ since $\psi(A) = (A)_{\sim_1} = (A)_{\sim_{12}} = (\varphi(A))_{\sim_1} = A_{\sim_1}$. Moreover: $(A)_{\sim_1} = A_{\sim_{12}} = A_{\sim_1}$, therefore $(A, A) \in \mathcal{L}_{\sim_2}$.

In the same way, we prove that $\mathcal{L}_{\sim_2} \subseteq \mathcal{L}_{\sim_1}$ and therefore $\mathcal{L}_{\sim_1} = \mathcal{L}_{\sim_2}$. ■

We will call the previous set \mathcal{L} . We can see that $\mathcal{L} \subseteq \text{fix}(\varphi) \times \text{fix}(\psi)$.

Definition 5 We define the following order relation \preceq in \mathcal{L} :

$$(A, B) \preceq (C, D) \text{ if } A \leq C, \forall (A, B), (C, D) \in \mathcal{L}. \quad (9)$$

Proposition 3 If $(A, B), (C, D) \in \mathcal{L}$, then $A \leq C$ if and only if $B \geq D$.

Proof. It is evident from proposition 1 i), taking into account that $A = B$ and $C = D$. ■

The order relation \preceq is induced by the order relation \leq in Ω and its opposite \geq in Σ .

Theorem 1 (\mathcal{L}, \preceq) is a complete lattice.

Proof. Let

$$\mathcal{F} = \{(A, (A)_1), A \in \Omega, i \in I\} \subseteq \mathcal{L}$$

be a concept family.

We are going to see that we can calculate its supremum in \mathcal{L} :

As $\bigvee_{\Omega} A$ exists and Ω is a complete lattice, then $\left(\bigvee_{\Omega} A, \left(\bigvee_{\Omega} A\right)_1\right)$ is an upper bound since $\bigvee_{\Omega} A \geq A$ (and $\left(\bigvee_{\Omega} A\right)_1 \leq (A)_1$), $\forall i \in I$.

Furthermore, we check that $\left(\bigvee_{\Omega} A, \left(\bigvee_{\Omega} A\right)_1\right)$ is the least upper bound:

If $(A, A) \in \mathcal{L}$ is other upper bound, then $A \geq A, \forall i \in I$, and $A \geq \bigvee_{\Omega} A$ (and respectively $A \leq \left(\bigvee_{\Omega} A\right)_1$).

The minimum element is $\left(0_{\sim_{\Omega}}, \left(0_{\sim_{\Omega}}\right)_1\right)$ where $0_{\sim_{\Omega}}$ is the minimum element of the lattice Ω . ■

Definition 6 The pair (\mathcal{L}, \preceq) is said to be the L – Fuzzy concept lattice of the L – Fuzzy context (L, X, Y, R) .

Proposition 4 The maximum and minimum elements of (\mathcal{L}, \preceq) are respectively:

$$0_{\mathcal{L}} = (0_{\Omega}, 1_{\Sigma}) \quad \text{and} \quad 1_{\mathcal{L}} = (1_{\Omega}, 0_{\Sigma})$$

Proof. Since $\forall A \in \Omega$, it is verified that $0_{\Omega} \leq A \leq 1_{\Omega}$; then $(0_{\Omega})_1 \geq A \geq (1_{\Omega})_1$, $\forall A \in \Sigma$.

Furthermore, $\Sigma = \{A_{\Sigma} \text{ t.q. } A \in \Omega\}$; and therefore, $(0_{\Omega})_1 = 1_{\Sigma}$ and $(1_{\Omega})_1 = 0_{\Sigma}$. Then, the least element of \mathcal{L} with the order \preceq defined in (9) is $0_{\mathcal{L}} = (0_{\Omega}, (0_{\Omega})_1) = (0_{\Omega}, 1_{\Sigma})$, and the greatest is $1_{\mathcal{L}} = (1_{\Omega}, (1_{\Omega})_1) = (1_{\Omega}, 0_{\Sigma})$. ■

Theorem 2 If we use the upper semicontinuous t -conorm Υ for the definitions of the functions φ and ψ , then for every family

$$\begin{aligned} \mathcal{F} &= \{(A_{\Omega}, (A)_{\Sigma}), A \in \Omega\} = \\ &= \{((B)_{\Sigma}, B_{\Omega}), B \in \Sigma\} \subseteq \mathcal{L} \end{aligned}$$

we can write the supremum and infimum of \mathcal{F} in the following way:

$$\begin{aligned} \bigvee_{\mathcal{L}} (A_{\Omega}, (A)_{\Sigma}) &= \left(\bigvee_{\Omega} A, \bigwedge_{\Sigma} (A)_{\Sigma} \right) \\ \bigwedge_{\mathcal{L}} ((B)_{\Sigma}, B_{\Omega}) &= \left(\bigwedge_{\Sigma} (B)_{\Sigma}, \bigvee_{\Omega} B_{\Omega} \right) \end{aligned}$$

where the supremum and infimum in Σ and Ω are calculated with the expressions (7) and (8).

Proof. For the supremum expression we can take into account theorem 1 and proposition 1; and for the infimum we can proceed in the same way. ■

In general, we can not construct the supremum of a concept family as Wille does in his theory ([6]). It is only verified:

Proposition 5 Let $\mathcal{F} = \{(A_{\sim_i}, (A_{\sim_i})_1), A \in \Omega, i \in I\} = \{((B_{\sim_i})_2, B_{\sim_i}), B \in \Sigma, i \in I\} \subseteq \mathcal{L}$ be a concept family, it is verified that

$$\bigvee_{\mathcal{L}} (A_{\sim_i}, (A_{\sim_i})_1) \geq (\varphi(\bigvee A_i), (\varphi(\bigvee A_i))_1)$$

Proof. As $\Omega \subseteq L^X$, we can say that $\bigvee_{\Omega} A_i \geq \bigvee A_i, \forall A_i \in \Omega$. Taking φ on both sides and as $\bigvee_{\Omega} A_i$ is an element of $\Omega = \text{fix}(\varphi)$, then we have:

$$\bigvee_{\Omega} A_i \geq \varphi(\bigvee A_i), \quad \forall A_i \in \Omega, \text{ and we can conclude that}$$

$$\bigvee_{\mathcal{L}} (A_{\sim_i}, (A_{\sim_i})_1) \geq (\varphi(\bigvee A_i), (\varphi(\bigvee A_i))_1), \quad \forall A_i \in \Omega, i \in I. \quad \blacksquare$$

4 Constructive method of the L-Fuzzy concept lattice.

If L, X and Y are finite with cardinality k, m and n respectively, we can always take the k^m L -Fuzzy subsets of L^X and see if they are fixed points of φ .

We denote

$$\mathcal{M} = \{A \in L^X / A = \varphi(A)\}$$

Now, for every $A \in \mathcal{M}$ we construct the concept $(A_{\sim_i}, (A_{\sim_i})_1)$ and calculate the whole lattice. This method is only valid when L, X and Y are finite sets.

However, for a large amount of data, the method is quite slow.

If we calculate the derivation concepts of the sets $\underline{0}$ and $\underline{1}$, then we can reduce the number of operations needed. In this case, we only have to verify whether the sets that are between the first components of these concepts are fixed points of φ . We apply the method to the following:

Example 1. We take the relation used by B. Ganter, J. Stahl and R. Wille in [4]:

	1	2	3	4	5	6	7	8	9	10	11	12	13
logic	2	1	3	2	4	2	4	5	2	5	4	2	2
algebra	4	4	2	1	1	3	1	1	1	1	1	1	1
func. anal.	5	2	1	5	2	5	2	2	3	3	3	3	4
diff. ec.	1	5	4	3	3	4	3	3	4	2	2	5	1
num. anal.	3	3	5	4	5	1	5	4	5	4	5	4	5

This table contains the opinion (valued between 1 and 5) given by a group of 13 students to a question relating to their preferences in 5 areas of mathematics. In [4] the authors indicate that this is an example of multivalued context and they make a study of this table using scales.

In order to apply our *L - Fuzzy* concept model, we first define the context that we will use: We take the lattice $L = \{0, 0.25, 0.5, 0.75, 1\}$ and the t-conorm $\top = \vee$. We choose this lattice because it allows us to use Zadeh's complementation and, furthermore, its cardinality is equal to 5. This result allows us to translate the valuations of the students to this scale taking into account that 1 represents a greater preference and 5 a lesser one. The set X is defined as follows: $X = \{\text{logic, algebra, functional analysis, differential equations and numerical analysis}\}$, and the set Y is formed by the 13 students.

Finally, the relation between one element $x \in X$ and another $y \in Y$ is

$$\underset{\sim}{R}(x, y) = (5 - t_{xy})/4 \in L$$

where t_{xy} are the values of the table. (We are making the following assignation of values 1-1,2- 0.75,3-0.5,4-0.25,5-0.)

The concept lattice that we construct with this method has 15 concepts $(\underset{\sim}{A}_i, \underset{\sim}{B}_i)$ shown in the following table. The values in keys represent the membership of the elements to X and Y respectively with the defined order.

1	$\left\{ \begin{array}{l} \{0, 0.25, 0, 0, 0\} \\ \{0.75, 0.75, 0.75, 1, 1, 0.75, 1, 1, 1, 1, 1, 1\} \end{array} \right\}$
2	$\left\{ \begin{array}{l} \{0, 0.5, 0, 0, 0\} \\ \{0.5, 0.5, 0.75, 1, 1, 0.5, 1, 1, 1, 1, 1, 1\} \end{array} \right\}$
3	$\left\{ \begin{array}{l} \{0.25, 0.25, 0.25, 0.25, 0.25\} \\ \{0.75, 0.75, 0.75, 0.75, 0.75, 0.75, 0.75, 0.75, 0.75, 0.75, 0.75, 0.75\} \end{array} \right\}$
4	$\left\{ \begin{array}{l} \{0.25, 0.25, 0.25, 0.5, 0.25\} \\ \{0.75, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.75, 0.75, 0.5, 0.75\} \end{array} \right\}$
5	$\left\{ \begin{array}{l} \{0.25, 0.25, 0.5, 0.25, 0.25\} \\ \{0.5, 0.75, 0.75, 0.5, 0.75, 0.5, 0.75, 0.75, 0.5, 0.5, 0.5, 0.5\} \end{array} \right\}$
6	$\left\{ \begin{array}{l} \{0.25, 0.5, 0.25, 0.25, 0.25\} \\ \{0.5, 0.5, 0.75, 0.75, 0.75, 0.5, 0.75, 0.75, 0.75, 0.75, 0.75, 0.75, 0.75\} \end{array} \right\}$
7	$\left\{ \begin{array}{l} \{0.25, 0.5, 0.25, 0.5, 0.25\} \\ \{0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.75, 0.75, 0.5, 0.75\} \end{array} \right\}$
8	$\left\{ \begin{array}{l} \{0.25, 0.5, 0.5, 0.25, 0.25\} \\ \{0.5, 0.5, 0.75, 0.5, 0.75, 0.5, 0.75, 0.75, 0.5, 0.5, 0.5, 0.5\} \end{array} \right\}$
9	$\left\{ \begin{array}{l} \{0.5, 0.25, 0.25, 0.25, 0.25\} \\ \{0.75, 0.75, 0.5, 0.75, 0.5, 0.75, 0.5, 0.5, 0.75, 0.5, 0.5, 0.75, 0.75\} \end{array} \right\}$
10	$\left\{ \begin{array}{l} \{0.5, 0.25, 0.25, 0.5, 0.25\} \\ \{0.75, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.75\} \end{array} \right\}$
11	$\left\{ \begin{array}{l} \{0.5, 0.5, 0.25, 0.25, 0.25\} \\ \{0.5, 0.5, 0.5, 0.75, 0.5, 0.5, 0.5, 0.75, 0.5, 0.5, 0.75, 0.75\} \end{array} \right\}$

$$\begin{array}{l}
 12 \quad \left\{ \begin{array}{l} \{0.5, 0.5, 0.25, 0.5, 0.25\} \\ \{0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.75\} \end{array} \right. \\
 13 \quad \left\{ \begin{array}{l} \{0.5, 0.5, 0.5, 0.5, 0.5\} \\ \{0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5\} \end{array} \right. \\
 14 \quad \left\{ \begin{array}{l} \{0.75, 0.75, 0.75, 0.75, 0.75\} \\ \{0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25, 0.25\} \end{array} \right. \\
 15 \quad \left\{ \begin{array}{l} \{1, 1, 1, 1, 1\} \\ \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\} \end{array} \right.
 \end{array}$$

We can represent this *L – Fuzzy* concept lattice graphically as follows:

In general, to interpret an *L – Fuzzy* concept, we look at the membership of the objects and attributes which stand out from the others.

For example, if we look at the *L – Fuzzy* concept 4, then we can say that students 1, 10, 11 and 13 like the differential equations. Furthermore, if we compare them with the *L – Fuzzy* concept 10, then we see that also some of these students have preference for the logic; they are numbers 1 and 13.

These results are deductible through a meticulous observation of the table; the advantage of our theory is that it proposes an algorithm for calculating them. Also, if we look at concepts 7,8,10,11 and 12 we can see the combination of areas preferred by the students and their acceptance:

We can deduce from concepts 8 and 11 that the most popular pair of areas are algebra-functional analysis and logic-algebra. Moreover, we deduce from concept 12 that the combination logic-algebra-diff.equations is the most interesting one for the students. In some way, the *L – Fuzzy* concepts allow us in this case to group the areas. This is shown in the above graph, where we observe the hierarchies between the concepts. For example, if we are interested in knowing the opinion of a student about logic, on its own or with other areas, then we can analyze the left part of the graph where the concepts corresponding to that area are located.

5 Conclusions.

The L – Fuzzy concept theory allows us to see in a clearer and more concise way some results that could be visualized directly from the table, and additionally to extract more elaborate conclusions through an algorithmic process.

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