Mathware & Soft Computing 1 (1994) 45-68

An analysis of MYCIN-like expert systems

Petr Hájek* & Julio J. Valdés**
*Institute of Computer Science, Academy of Sciences, Czech Republic
** Depto. Inteligencia Artificial, Inst. de Cibernetica, Matematica y Fisica, Acad. de Ciencias de Cuba. La Habana, Cuba

Abstract

The paper is a review of our theoretical analysis of uncertainty processing in a broad class of truth-functional expert systems similar to MYCIN and PROSPECTOR, main attention being paid to parallel combination of rules. Algebraic and probabilistic aspects are stressed. The role of Dempster-Shafer theory is investigated.

Keywords: Inference under uncertainty, truth-functional expert systems, ordered Abelian groups, graphical probabilistic models, Dempster-Shafer theory of evidence.

1 Introduction

(E)MYCIN and PROSPECTOR have been recognized as prototypes of first-generation rule-based expert systems, are mentioned and explained in monographs on uncertainty in expert systems [Pearl,1988, Neapolitan, 1990, Kruse-Schwecke-Heimsohn, 1991, Lopez de Mantaras, 1990], [Puppe 1993], but seem not to be further a matter of broad theoretical interest. Nevertheless, systems of this kind - let us call them MYCIN-like - are still offered, particularly as a component of various expert systems shells. In this note, we are going to survey the results of our long-term study of MYCIN-like systems and its relation to theoretical work of others. Our analysis seems to be more or less finished and is presented in full detail in Chap. VI - IX of the monograph [Hájek, Havránek, Jiroušek 1992]; here we summarize and review the main results in a condensed and hopefully transparent form. The paper is organized as follows: Section I contains main definitions, making our notion of MYCIN like systems precise (and distinguishing it from systems based on fuzzy logic). Section II presents an algebraic analysis based on the notion of an ordered Abelian groups; the main result says that the group of PROSPECTOR (or, equivalently, of EMYCIN) is universal in a very general sense. In Section III, we summarize a probabilistic analysis; here the main results says that the methods of local computation in general probabilistic models (as developed by Pearl, Lauritzen, Speigelhalter and others) may be applied to give a method of guarded use of MYCIN-like systems guaranteeing (partially and for a particular kind of these systems) a sort of probabilistic soundness. In Section IV we show that both the algebraic and the probabilistic investigations generalize to systems in which uncertainty is expressed using belief functions of Dempster-Shafer theory.

Various authors have investigated the inference mechanism of MYCIN-like systems; besides the basic monograph [Buchanan-Shortliffe, 1984], the papers [Heckerman, 1986], [Horvitz-Heckerman, 1986], [Johnson, 1986], [Cheng-Kyashap, 1989], [Prade, 1985], [Dubois-Prade, 1985] are of main relevance. [Zhang 1992] is a relevant recent paper; in Sect. 3 the author appears to rediscover some old results of [Hájek 1985] (and also presents several new results; no reference is given to our work). Our approach shows not to be covered by any of the mentioned works.

Our algebraic analysis (which is a joint work of the present authors) shows that MYCIN-like systems form a rather homogenous class; this is pleasing but does not answer the question, which intuitive notion of uncertainty underlies them. The presented probabilistic analysis (which is due to P. Hájek) shows, in way different from that of Heckerman and others, how far one can go in trying to understand the *results* of the system as conditional probabilities. Our final answer to the question “What can one do with MYCIN-like systems?” is: “Not too much but still more than you would believe.”

Section I. What are MYCIN-like systems

1.1. We assume the following formal structure: We have a finite set *Prop* of *propositional variables* (briefly propositions); a *rule* is a propositional formula of the form $E \Rightarrow H$ where H is a proposition and E is a formula not containing H. For simplicity we shall assume that E is an *elementary conjunction* , i.e. a conjunction of propositions and negated propositions in which each proposition occurs at most once. E is the *antecedent* and H the *succedent* of the rule. (Example: propositions $1, 2, \ldots 10,$ rules $1\& -2 \Rightarrow 4$, $1\& -2\&3 \Rightarrow 4$, $\Rightarrow 4$ [empty antecedent]). A *weighted rule* consists of a rule and its *weight* $w \in G$ where G is a linearly ordered set of weights. A *rule pattern* is an acyclic set of rules, i.e. a set B of rules such that there is no sequence $R_0 \ldots R_n$ of rules in B such that the succedent of each R_i occurs in the antecedent of $R_{i+1} (i < n)$ and the succedent of R_n occurs in R_0. (Example of a cycle: $1\&2 \Rightarrow 4, -4\&5 \Rightarrow 6, 6 \Rightarrow 1$). We shall assume that each proposition occurs in at least one rule. Observe that a rule pattern decomposes all propositions occuring in its rules into three disjoint sets: *Ques* - the set of *questions,* i.e. propositions occuring in no succedent, *Goal* - the set of all goals, i.e. propositions occuring no antecedent, and *intermediate* propositions. A *weighting* over B is a mapping $\kappa : B \rightarrow G$ associating to each rule its weight. A *rule base* is a pair $\Theta = (B, \kappa)$ where B is a rulle pattern and κ is a weighting. $\Theta(H|E) = w$ means that the rule $E \Rightarrow H$ is in B and $\kappa(E \Rightarrow H) = w$. A *questionnaire* is a mapping $q : Ques \rightarrow G$ assigning to each question its weight.

The following operations are assumed on the set of weights: truth-functional interpretations of logical connectives (generalized truth-tables, NEG and CONJ, say), a binary operation CTR computing the contribution of a rule (this is often called the function evaluating modus ponens) and a binary operation $\oplus$ for parallel combination of contributions of rules with the same succedent.

1.2. The *global weight* of a formula H is given a questionnaire q and the *contribution* of a rule R given q (determined by the rule base Θ and the structure of weights) defined as follows.

(1) $W_{\Theta,G}(H|q) = q(H)$ if H is a question,
(2) $W_{\Theta,G}(A\&B|q) =$ CONJ $(W_{\Theta,G}(A|q), (W_{\Theta,G}(B|q)),$

(3) $W_{\Theta,G}(\neg A|q) =$ NEG $(W_{\Theta,G}(A|q))$,

(4) $W_{\Theta,G}(H|q) = V_{\Theta,G}(R_1|q) \oplus \ldots \oplus V_{\Theta,G}(R_n|q)$ if $R_1, \ldots, R_n$ are all the rules in B whose succedent is H;

(5) $V_{\Theta,G}(E \Rightarrow H) =$ CTR$(W_{\Theta,G}(E|q), \kappa(E \Rightarrow H))$.

In words: the global weight of a question is given by the questoinaire, composed formulas are evaluated using truth tables, the global weight of a proposition H which is intermediate or a goal is the $\oplus$-sum of contributions of all rules leading to H; the contribution of a rule is computed from the weight $\kappa(R)$ of the rule and the global weight of its antecedent using CTR. Note that in (5) a fixed order on the set of rules and a fixed bracketting is assumed; see also below.

1.3 It should be clearly stated that a questionnaire is understood as the system of the *beliefs* assigned by the user to the questions; second, the quantity $W_{\Theta,G}(H|q)$ is understood as the *global belief* on H given the knowledge Θ, data q and inference mechanism G. Note that we *have left open* what the weights of rules are; we speak just of *contributions.*

Until now, we have not made any assumptions on the operations on the set G of weights; we shall do it now and this will complete our specification of MYCIN like systems.

1.4 Assumptions on the structure of weights: Recall that G is assumed to bear a linear order $\leq$. We assume G to have a greatest element $\top$ (true), least element $\perp$ (false) and at least one more element o (no preference) weights $w > o$ are positive, $w < o$ are negative. Since we are interested in $\oplus$, we shall make the simplest possible assumptions on the other operations: NEG is antimonotone, NEG$(o) = o$, NEG(NEG$(x)) = x$; CONJ and CTR are definable from $\leq$ (e.g. CONJ$(x,y) = \min(x,y)$, CTR$(a,w) = o$ if $a \leq o$, CTR$(a,w) = \min(a,w)$ if $a > o, w \geq o$, CTR(a,w)=NEG(max(NEG$(a), w$)) if $a > o, w < o$).

$\top$ and $\perp$ are extremal elements; $\top \oplus w = w \oplus \top = \top$ for $w \neq \perp, \perp \oplus w = w \oplus \perp = \perp$ for $w \neq \top; \top \oplus \perp, \perp \oplus \top$ may either remain undefined or one choses a definition reflecting the work of an evaluating algorithm for $W_{\Theta,G}$ ($\top \oplus \perp = \top, \ \perp \oplus \top = \perp$). Non-extremal elements are closed under $\oplus$ (uncertainties cannot give certainty) and the following are the assumptions on the behaviour of $\oplus$ on non-extremals: associativity, commutativity (i.e. in (4) above, the order of summands and bracketting are immaterial), o is a neutral element ($o + w = w$), NEG(x) is the inverse ($w \oplus$ NEG(w) $= o$), monotonicity: $x \leq y$ implies $x \oplus z \leq y \oplus z$. This means (as observed first in [Hájek, 1982, 1985], that non-extremals form an *ordered Abelian group* (oag) cf [Fuchs, 1963]). Adding $\top$ and $\bot$ behaving as above we get an *extended oag.*

1.5. Definition. *A MYCIN-like system* is given by a set *Prop* of propositions, an extended ordered Abelian group G of weights and a rule base $\Theta = (B, \kappa)$, where B is a rule patterns and κ a weighting.

1.6. Caution. This *differs* from truth-functional systems based on fuzzy logic, as investigated e.g. by [Bonissone] and [Dubois-Prade 1991]; there $\bot$ is a neutral element and $\oplus$ is a conorm, thus $u \oplus v \geq u, v$ is always satisfied. Here $u \oplus v > u, v$ if u, v are positive but $u \oplus v < u, v$ if u, v are negative; and $u < u \oplus v < v$ if $u < o < v$. This corresponds to the understanding of $\oplus$ as the operation combining contributions - both positive and negative - to the global weight of a proposition.

Section II. Algebraic foundations

Here we sumarize main results of [Hájek, 1985], [Valdes, 1987], and [Hájek-Valdes 1990]. The canonical example of an ordered Abelian group (oag) is the additive group of reals **Re** with the usual ordering. An isomorphic copy of this group is the multiplicative oag **Pos** of positive reals: the mapping $f(x) = e^x$ is clearly an isomorphism of **Re** to **Pos**, i.e. preserves ordering and satisfies $f(x + y) = f(x) \cdot f(y)$. In PROSPECTOR (see [Duda et al., 1976]), one works with odds and the combining function is multiplication. Odds vary over positive reals and are related to probabilities p (from the interval $(0, 1)$) by the formula $o = p/(1 - p)$. The formula allows us to find an operation $\oplus$ on $(0, 1)$ such that the mapping $g(x) = x(1-x)$ becomes an isomorphism of **PP**$= ((0, 1), \oplus, \leq)$ to **Pos**; the result is

$$x \oplus y = \frac{xy}{xy + (1 - x)(1 - y)}.$$

PP is called *PROSPECTOR's group* on $(0, 1)$. (The reader may find an isomorphic operation on $(-1, 1)$. Clearly, **PP** is isomorphic to **Re**. We claim that **PP** is universal for MYCIN-like systems over any oag G. The rest of the section is mainly devoted to various formulations of this result. To this end we need the notion of an Achimedean oag.

An oag **G** with the neutral element o is *Archimedean* if for any a, b such that $o < a < b$ then is a natural number n such that $b < a \oplus \ldots \oplus a$. ($n$ summands).

Clearly, **Re** (and hence **PP**) is Archimedean. A natural example of a non-Archimedean oag is the group **Re**×**Re** of pairs of reals for which $\oplus$ is defined coordinatewise $(x, y) \oplus (u, v)\ H = (x + u, y + w)$ and $\leq$ is defined lexicographically: $(x, y) \leq (u, v)$ if either $x < u$ or $[x = u$ and $y \leq v]$.

Now we shall present this formulation of universality of **PP**.

Theorem. *Let* **G** *be an oag. (1) If* **G** *acts on the real interval* $(0, 1)$ *with the usual ordering then* **G** *is isomorphic to* **PP**. $(0, 1)$ *may be replaced by any other real interval* (a, b) *for* $a < b$.

(2) If **G** *is Archimedean then* **G** *is isomorphic to an ordered subgroup of* **PP**.

(3) If **G** *is an arbitrary oag (Archimedean or not) and* D *is a finite subset of* **G** *then there is a finite subset* D' *of* PP *such that* $(D, \oplus_G \leq_G)$ *is isomorphic to* $(D', \oplus_{PP}, \leq_{PP})$*, i.e. there is a one-one mapping* f *of* D *onto* D' *preserving the order and such that, for any* $x, y, z \in D$ $[x \oplus_G y = z$ *iff* $f(x) \oplus_{PP} f(y) = f(z)]$.

For (1) and (2) see e.g. [Fuchs, 1963] ((2) is called Hölder's theorem); (3) is proved in our paper [Hájek-Valdes , 1990] but appears to be known to specialists on oag's.

A trivial corollary of (1), observed already in [Hájek 85] is the fact that the oag of certainty factors of MYCIN is isomorphic to PROSPECTOR's oag **PP**. This fact has found some interest of researches in the domain of uncertainty; but, unfortunately, this appears to be the only lucky fact. Further considerations that we made have remained rather unnoticed. But especially (3) has an immense importance for MYCIN-like systems; we formulate the basic fact as the corollary below. First two definitions.

Let B be a knowledge pattern, let **G** be an oag and let $D \subseteq$ **G**. A weighting κ (questionnaire q) for B is *over* D if its range is included in D i.e. weights of rules (answers to questions) belong to D. D can be thought on as the finite set of weights *accessible to the user* (e.g. two-decimal-digit-numbers from $(0, 1)$ or so).

The *comparative result* of a run (B, κ, q, G) is the linear (quasi)order of all propositions according to the global weight, i.e. the ordering $\preceq$ defined as

$$i \preceq j \text{ iff } W_{B,\kappa,G}(i|q) \leq W_{B,\kappa,G}(j|q).$$

We write $CR(B, \kappa, q, G)$ for the ordering $\preceq$. It does not tell us precise weights assigned to propositions but only tells, for each pair of propositions, which of them is more believed (or that their beliefs are equal).

Corollary. *Let B be a knowledge pattern,* **G** *on oag and D a finite subset of* **G**. *Then there is an one-one mapping of D onto finite subset D' of* **PP** *such that, for each weighting κ and questionnaire q, both over D, the comparative result of $(B, \kappa, q, \mathbf{G})$ is the same as the comparative result of $(B, \kappa', q', \mathbf{PP})$. (Here $\kappa'(r) = f(\kappa(R)), q'(i) = f(q(i))$, i.e. κ' and q' results from κ, q by replacing the values in D by the corresponding value in* **PP***).*

We may express this briefly by saying that for each rule pattern B, oag **G** and a finite $D \subseteq \mathbf{G}$, there is a $D' \subseteq \mathbf{PP}$ such that $\mathbf{G}, D$ is comparatively equivalent to **PP**, D' for the rule pattern B: weightings and questoinnaire D, processed using **G**, give the same comparative results as corresponding weightins and questoinnaires over D', processed using **PP**.

There are several other results in [Hájek-Valdes, 1990] not reproduced here; the last corollary seems to be the most important. Note that investigations of [Heckerman-Horvitz, 1988] and [Cheng, Kyashap, 1989] are related to the point (1) of the above theorem: they show that, under some conditions (like continuity), all possible combining functions are isomorphic to real addition (or multiplication of positive reals). But they assume throughout that weights are reals and do not use algebraic considerations. Our last result seems not to have any analogies in the literature.

Section III. Probabilistic foundations

We survey main result of [Hájek, 1988, 1989]. The usual naive use of MYCIN-like systems in governed by the following two simple assumptions.

(i) Weights of rules are conditional beliefs (conditional probabilities, odds, likehood ratios etc.).

(ii) Global conditional beliefs $W(H|q)$ are computed using combining functions.

It was soon observed that the conjunction of (i) and (ii) is unrealistic and not tenable; cf. [Johnson, 1986], [Heckerman, 1988]. Clearly, one can give up (ii) - this leads to strictly probabilistic systems as those described in [Lauritzen-Spiegelhalter, 1988], [Shachter, 1986] and others. The possibility of giving up (i) and keeping (ii) was not much investigated; but let us mention the fact that [Heckerman-Horvitz, 1988] studies weights of makes as *belief updates,* changes of belief. Our approach systematically develops this last idea.

We make the following restricting assumptions: (a) Weights are reals from the unit interval $(0,1)$ and the group operation is PROSPECTOR's $\oplus$ (as described above). (b) There are no intermediate propositions (we have only two levels: questions and goals). (c) We consider only three-valued questoinnaires with range included in $\{0, 1/2, 1\}$; these questionnaires are in the obvious one-one correspondence with elementary conjunctions of questions (e.g. if $1, 2, 3, 4, 5$ are questions then $1\& - 3\&5$ represents the questionnaire q such that $q(1) = q(5) = 1, q(3) = 0, q(2) = q(4) = 1/2$). If $\beta = (R, \kappa)$ is a rule base (R is a rule pattern and κ is a weighting) then $\beta(H|E) = w$ means that the rule $E \Rightarrow H$ is in R and κ assigns the weight w to this rule. We need two more definitions:

Two rule bases β, β' (with the same questions and goals) are *compatible* if $\beta(H|E) = \beta'(H|E)$ whenever both sides are defined. A rule base β is *probabilistically sound* if there is a joint probability P on propositions such that $\beta(H|E) = P(H|E)$ whenever both sides are defined.

3.1 Distinguishing a knowledge base from MYCIN's rule base.

If a rule base β is understood as a knowledge base, i.e. a system of expert's conditional beliefs then it cannot be directly processed by a MYCIN-like inference engine: this leads to well-known defects. Indeed, in general, the global belief $(W_\beta(H|E)$ will be distinct from $\beta(H|E)$. Exercise: let H be 1 & 2 and let B contain rules $1 \Rightarrow H, 2 \Rightarrow H, 1 \ \& \ 2 \Rightarrow H$ with some weights. What is $W_\beta(H|E)$?) But assuming β to be probabilistically sound we can always change the weights in such a way that the new rule base β' will reproduce expert's beliefs as global conditional weights:

Theorem *(cf.[Hájek, 1984]). If $\beta = (R, \kappa)$ is probabilistically sound then* *there is a weighting κ' for R such that, for each E, H, such that $\beta(H|E)$ is defined, we have*

$$W_{(R,\kappa')}(H|E) = \beta(H|E).$$

The new rule base $\Theta = (R, \kappa')$ is called *Möbius transform* of (R, κ) and denoted by $M(\beta)$. Observe that for rule bases of the form studied the global weight satisfies

$$W_\Theta(H|E) = \oplus\{\Theta(H|E')|E' \subseteq E\}$$

(the global weight of H given E is the group-theoretic sum of weights of all rules whose succedent is H and whose antecedent is a subconjunction of E). This yields a method of direct computations of the new weights.

3.2. The problem of extrapolation. In the preceeding paragraph we started with a probabilistically sound rule base β and constructed a new rule base Θ with the same rule pattern such that W_Θ is compatible with β, in other words, the rule base Θ *reproduces beliefs β as global weights W_Θ*. But W_Θ is *total:* $W_\Theta(H|E)$ is defined for each H, E. Is W_Θ probabilistically sound? Unfortunately, not in general. (This is unfortunate since $W_\Theta(H|E)$ is the advice of the expert system concerning H, if user's data are E; the system of advices should be as much probabilistically sound as possible). The reason is, roughly, that β may contain too little information on the background probability. This leads us to the following *task:*

Find a method of improving (extending) the starting knowledge base β to a reasonably richer $\hat{\beta}$ such that for some probability P and for $\Theta = M(\hat{\beta})$,

$$W_\Theta(H|E) = P(H|E)$$

for *most* arguments H, E (if not for all).

We shall present such a method below. First we have to make a digression.

3.3 Graphical probabilistic models. One can take a lesson from probabilistic expert systems and their methods of local computations and factorization of probabilities. We confine ourselves to the descriptions of a certain algorithm (in 3.4); to prove its properties one needs facts on factorizations of probabilities over cliques of a graphs, log-linear probabilistic models and collapsibility.

Basic notions concerning graphs are supposed to be known; we recall some of them using simple examples. Fig.1 contain three undirected graphs G_1, G_2, G_3 all having four vertices a, b, c, d.

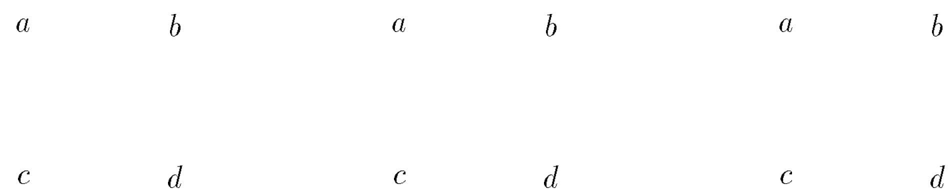

Fig.1

G_1 and G_2 and G_3 are *connected,* G_2 is not. No of G_1, G_2, G_3 is *complete,* G_2 and G_3 are *triangulated,* G_1 is not. *Cliques* of $G_1 : \{a,b\},\{a,c\}\{b,d\}\{c,d\}$. Cliques of $G_2 : \{a\},\{c,d\},\{b,d\}$. Cliques of $G_3 : \{a,b,d\},\{a,c,d\}$.

A graph G *collapses* to a set $A \subseteq G$ of vertices of the following condition holds: whenever two distinct elements a, b of A can be connected in G by a path

$$a = x_o - x_1 - \ldots - x_{n-1} - x_n = b$$

whose interior $\{x_1, \ldots, x_{n-1}\}$ is disjoint from A then (a, b) is an edge in G. For example, G_3 collapses to $\{a,b,d\}$ but not to $\{b,c,d\}$.

Last definition: the *closure* of an element $g \in G$ consists of g and of all elements h such that (g, h) is an edge in G. For example, in $G_1, cl(a) = \{a,b,c\}$; in $G_3, cl(a) = \{a,b,c,d\}$.

3.4. The method of guarded use of the MYCIN-like inference.

A probabilistically sound rule base $\beta = (R, \kappa)$ (knowledge base) is assumed to be given.

(a) Construct the *graph* $G(\beta)$: for each rule r in R, join each pair of propositions occuring in r by an edge; thus the set of all propositions occuring in r becomes a complete subgraph. For example, if $1\ \&\ -2 \Rightarrow 7$ is in R then we get undirected edges $(1,2),(1,7),(2,7)$.)

(b) Improve $G(\beta)$: by adding (possibly few) edges among questions, produce a graph $\hat{G}(\beta)$ such that, for each goal $H, cl(H)$ is triangulated and $\hat{G}(\beta)$ collapses to $cl(H)$.

(c) Using $\hat{G}(\beta)$, improve β: for each rule r not in R, if the propositions occuring in r from a complete subgraph of $\hat{G}(\beta)$ then add r to R and define its weight. The resulting rule base is $\hat{\beta} = (\hat{R}, \hat{k})$.

(d) The final rule base is $\beta^* = M(\hat{\beta})$ (Möbius).

Theorem. *If β^* results from β by the above construction then there is a probability P on propositions such that, for each goal H and each elementary conjunction E satisfying $prop(E) \subseteq cl(H)$:*

$$\text{if } \hat{G} \text{ collapses to } prop(E) \cup \{H\} \text{ then } W_{\beta^*}(H|E) = P(H|E).$$

(Clearly, $prop(E)$ is the set of all propositions occuring in E.)

The moral of this theorem reads: If you guard the construction of the rule base to be processed by the MYCIN-like inference engine then you can guard its use: you gain control over the probabilistic soundness of the results. After a run of the engine you can look at the graph $\hat{G}$ and either say "the result in the conditional probability $P(H|E)$" or say "I cannot be sure on the probabilistic meaning of the result; you have to give definite meaning of the result; you have to give definite answer to more questions (or disregard some answered given)."

An implementation of the method of guarded use is a part of the integrated environment for uncertainty management using graphical models described in [Valdes 92].

Section IV. Relations to Dempster-Shafer theory of evidence

The present section is based on [Hájek-Valdes 87], [Hájek-Valdes 91] and [Hájek 91]. Dempster-Shafer theory of evidence (briefly, DS-theory) was created by Dempster [Dempster 67, 68] and Shafer [Shafer 78] and originally was not related to AI; a milestone for its application in rule-based systems was the paper [Gordon and Shortliffe 84] and presently there exists a huge literature in DS-theory and its applications in expert systems. The reader is recommended to consult survey papers [Smets 88] and [Hájek-Harmanec 92].

It must be stressed that the application of DS-theory in expert systems are by no means restricted to MYCIN-like systems (see e.g. [Shenoy-Shafer] for local computations in DS-theory); here we resctrict ourselves to the meaning of DS-theory for these systems.

The first thing to note on DS-theory is that it deals with *belief functions*; there are some functions assigning beliefs $bel(A)$ to propositions A; $bel(A)$ is a real from the interval [0,1] and $bel(A) + bel(\neg A)$ is always ≤ 1 but not necessarily equal to 1. We briefly recall basic definitions but have to omit discussion on motivation.

4.1. Let $V \neq \emptyset$ be a finite set called *frame of discernment*. A *basic belief assignment* (bba), over V is a mapping m associating to each $A \subseteq V$ a real $m(A) \in [0,1]$ such that $\sum_{A \subseteq V} m(A) = 1$ and $m(\emptyset) = 0$. The *belief function* given by m is $bel(A) = \sum_{B \subseteq A} m(B)$.

Dempster's rule of combination associates to each pair m_1, m_2 of bba's over V its combination m satisfying

$$m(A) = \frac{\sum_{B \cap C = A} m(B).m(C)}{\sum_{B \cap C \neq \emptyset} m(B).m(C)} = (m_1 \oplus m_2)(A),$$

assuming that the nominator is non-zero.

The corresponding belief function $bel = bel_1 \oplus bel_2$ satisfies

$$bel(A) = \frac{\sum_{B \cap C \subseteq A} m(B).m(C)}{\sum_{B \cap C \neq \emptyset} m(B).m(C)}.$$

4.2. For the particular case $V = \{0,1\} = \{false, true\}$ a bba m is given by two numbers $a = m(\{1\})$ and $b = m(\{0\})$ (since $m(\emptyset) = 0$ and $m(\{0,1\}) = 1 - a - b$). The pair (a, b) satisfies $a, b, \geq 0$ and $a + b \leq 1$. (a is the belief assigned to "yes", b the belief assigned to "no".) Each pair satisfying this is called a *Dempster pair* (or d-pair); D is the set of all d-pairs.

Dempster rule for bba's gives the following rule for d-pairs:

$$(a,b) \oplus (c,d) = (1 - \frac{(1-a)(1-c)}{1-(ad+bc)}, 1 - \frac{(1-b)(1-d)}{1-(ad+bc)}).$$

This is defined always except one of the summands is (1,0) and the other is (0,1); these two d-pairs are *extremal* and correspond to "cetainly yes" and "certainly no".

4.3. Allgebraically, $D - \{(0,1),(1,0)\}$ with $\oplus$ is a *commutative semigroup* ($\oplus$ is commutative and associative), (0,0) is a neutral element, i.e. $(0,0) \oplus (a,b) = (a,b)$; this means that $(0,0)$ is complete ignorance: no belief is assigned to "yes", no belief to "no". We extend $\oplus$ to extremal elements as we did with oag's. A d-pair is *Bayesian* if $a+b=1$; d-pairs form a subgroup **G** of **D** isomorphic to **PP** (PROSPECTOR's oag); its zero is $(.5,.5) = 0'$. For each d-pair $\gamma, \gamma \oplus 0'$ is Bayesian; the mapping $h(\gamma) = \gamma \oplus 0'$ is a homomorphism of **D** onto **G**.

S is the set of all d-pairs (a,a) $(0 \leq a \leq .5)$; S with $\oplus$ is a subsemigroup **S** of **D**. The mapping f associating with each d-pair $\gamma = (a,b)$ the d-pair $\gamma \oplus -\gamma = (a,b) \oplus (b,a)$ is a homomorphism of D onto S. For each d-pair γ, the pair $(h(\gamma), f(\gamma))$ determines γ uniquely; $h(\gamma)$ is called the *certainty* and $f(\gamma)$ the *crispness* of γ.

Define $(a,b) \leq (c,d)$ if $[h(a,b) \leq h(c,d)$ (in the group ordering of G) or $h(a,b) = h(c,d)$ and $a \leq c]$. This makes $D - \{(0,1),(1,0)\}$ to an *ordered semigroup*, i.e. for non-extremal α, β, γ are have

$$\alpha \leq \beta \rightarrow (\alpha \oplus \gamma \leq \beta \oplus \gamma).$$

4.4. The above suggest an obvious generalization of the MYCIN-like inference machine: put

$$\begin{aligned} NEG(\alpha) &= -\alpha \\ CONJ(\alpha,\beta) &= \min(\alpha,\beta) \end{aligned}$$

(where for $\alpha = (a,b)$ we set $-\alpha = (b,a)$, and min concerns the ordering $\leq$ just defined),

$$\begin{aligned} CTR(\alpha,\gamma) &= (0,0) \text{ if } \alpha \leq (0,0), \\ &= \min(\alpha,\gamma) \text{ if } \alpha \geq (0,0) \text{ and } \gamma \geq (0,0), \\ &= -\min(\alpha,-\gamma) \text{ if } \alpha \geq (0,0) \text{ and } \gamma \geq (0,0). \end{aligned}$$

Allow weights of rules and answers in a questionnaire to be d-pairs and define the *global weight* $W_\Theta(P,q)$ (of a proposition P given a rule base Θ and questionnaire q) and *contribution* $V_\Theta(R,q)$ (of a rule R given Θ, q) as above.

If h is the homomophism of **D** to **PP** described above, Θ is a rule base over **D** and q a questionnaire over **D** then we way define $h(\Theta)$ to be the rule base resulting from Θ by replacing in each rule $A \to S(\gamma)$ its weight γ by $h(\gamma)$; and define $h(q)$ to be the questionnaire q' over **PP** such that $q'(i) = h(q(i))$ for each question i.

4.5. *Homorphism theorem.* Under the above notation,

$$h(W_\Theta(P,q)) = W_{h(\Theta)}(P, h(q)) \text{ and } h(V_\Theta(R,q)) = V_{h(\Theta)}(R, h(q));$$

thus running the consultation over **D** and then projecting the result to **PP** is the same as first projecting the rule base and the questionnaire to **PP** nd then running the consultation. Hence the work over **D** is a refinement of the work over **PP**.

4.6. *Remark.* An expert system shell called EQUANT-PC implementing the inference machine for rule base and qustionnaires over **D** and stressing comparative results has been proposed and implemented (see e.g. [Hájek, Hájková 1990]). The system intends to be an *enfant terrible* among MYCIN-like systems and to encover (and partially overcome) weaknesses of the MYCIN-like approach. Another shell based on our investigations was implemented in Cuba.

4.7. In an analogy to **PP** (PROSPECTOR's oag) as a distinguisted member of the class of all ordered abelian groups we may understand Dempster's semigroup as a distinguisted member of a class of algebras called *dempsteroids*. A dempsteroid is an algebra $(E, \oplus, -, 0, 0', \leq)$ such that

(i) $(E, \oplus, \leq, 0)$ is an ordered commutative semigroup with the neutral element 0,

(ii) $-(-x) = x$ and $-(x \oplus y) = -x \oplus -y$ for all $x, y \in E$,

(iii)$0 \leq 0'$,

(iv) for each $x \in E, 0 \leq x \leq 0'$ iff $x \oplus 0' = 0'$ iff $x = -x$; let S be the set of all x such that $x = -x$.

(v) For each $x, y \in S$ such that $x \leq y$ there is a z such that $x \oplus z = y$.

This class of algebras was investigated in [Valdes 87] and [Hájek, Valdes 91]; one can show that in each dempsteroid **E** there are homomophisms h

and f, h projecting $\mathbf{E}$ to the subgroup of all elements x such that $x \oplus 0' = x$ $(h(x) = x \oplus 0')$ and projecting E to S $(f(x) = x \oplus -x)$. Some natural subclasses of the class of all dempsteroids are studied, in dependence of the properties of S (prominent cases:$S - \{0'\}$ is the non-negative part of on oag, e.g. $\mathbf{D}$, or S is an MV-algebra an the sense of [Chang, 1959]). The definition of a rule base and questionnaire over a dempsteroid and of the corresponding global weight is evident.

We call the reader's attention to the paper [Daniel 1994] where all isomorphisms of Dempster's semigroup are presented.

4.8. The algebraic analysis presented up to now has been complemented by a probabilistic analysis using local computations and graphical models (see [Hájek, Havránek, Jiroušek 1992] Chap.9 Sect.3). We shall not go into any details but we only state that are the investigations of Section III. above naturally generalize to rule bases with d-pairs as weights and we obtain a variant of the method of guarded use *provided* we are willing to replace Dempster's rule of combination by another rule of pointwise product: put $c = 1 - a - b, f = 1 - d - e$, then

$$(a, b) \oplus (d, e) = (\frac{ad}{ad + be + cf}, \frac{be}{ad + be + cf}).$$

The reason for the change of the operation is technical: Dempster's rule is too little invertible, which makes Möbius transform impossible. This should not be understood as a serious criticism of Dempster' rule; just for this particular method it is not suitable.

4.9. On the other hand, the question remains how to interpret d-pairs as weights. It is tempting but unjustified to understand a d-pair as an interval probability (saying: the probability of "yes" is $\geq a$ and the probability of "no" is $\geq b$), since neither Dempster's rule nor the modified rule admits this interpretation. A possible interpretation is suggested in the connection with the above-mentioned notion of d-soundness; we refer to [Hájek 1991] and [Hájek, Havránek, Jiroušek 1992] for details.

Conclusion.

One possible explanation of the apparent ability of MYCIN-like systems to produce reasonable results, inspite of all their weaknesses, might be the possibility of tuning them, by adding and deleting rules and by changing weights of rules, together with the emphasis to comparative rather than quantitative (numerical) results.The theory surveyed in the present paper offers systematic foundations and a mathematical model of such a tuning: the more or less heuristic and ad hoc process of incrementally improving and testing the rule base can be seen as an intuitive approximation of an ideal procedure of guarded conctruction of a rule base. And the algebraic insight shows the richness of possibilities one has and the central position of PROSPECTOR's group - with respect to comparative results. To close let us stress explicitly that our aim has been to *understand* MYCIN-like systems, not to advocate them.

References

[1] Adams J. B.: A probability model of medical reasoning and the MYCIN model, Mathematical Biosciences Vol.32, 1976 pp. 177-186

[2] Bonissone P. P. : Summarizing and propagating uncertain information with triangular norms, Int. J. Approx. Reasoning 1987, 1, 71-101

[3] Buchanan B. G., Duda R. O.: Principles of rule-based expert systems In: Advances in computers, New York: Academic Press Vol. 22, 1983 pp.164-216

[4] Buchanan B. G., Shortliffe E. H.: Rule based expert systems. MYCIN experiments of the Stanford Heuristic Programming Project, Reading, Mass.: Addison-Wesley Pub. Comp., 1984 739 p.

[5] Chang C.: A new proof of the completeness of Lukasewicz logic, Trans. A.M.S. Vol.93, 1959 pp.74-80

[6] Chang C.: Algebraic analysis of Lukasiewicz logic, Trans A.M.S. Vol.88, 1958 pp.467-490

[7] Cheng, Y., Kashyap, R. L.: A study of associative evidential reasoning, IEEE Trans. on Pattern Recogn.and Mach. Intell. 11, 623, 1989.

[8] Cohen P. R., Shafer G., Shenoy P. P.: Modifiable combining functions In: Proc. 3rd Workshop on Uncertainty in AI, Seattle, 1987 pp.10-21

[9] Daniel M.: Some algebraic results concerning Dempster's semigroup, in: Proceedings IPMU'94, Paris

[10] Davis R., Buchanan B. G., Shortliffe E. H.: Production rules as a representation for knowledge-based consultation system, Artificial Intelligence Vol.8, 1977 pp.15-45

[11] Dempster A. P.: Upper and lower probabilities induced by a multivalued mapping, Ann. Math. Statistics Vol.38, 1967 pp. 325-339

[12] Dempster A. P.: A generalization of Bayesian inference, J. Royal Statist. Soc., ser. B Vol.30, 1968 pp.205-247

[13] Dempster A. P.: Covariance selection, Biometrica Vol.28, 1972 pp.157-175

[14] Dempster A. P., Kong A.: Uncertain evidence and artificial analysis, Journal of Statisticial Planning and Inference Vol.20, 1988 pp.355-368

[15] Dubois D., Prade H.: Combination and propagation of uncertainty with belief functions - a reexamination In: Proc. 9th Int. Joint. Conf. on AI (IJCAI) Vol.1, 1985 pp.111-113

[16] Dubois D., Prade H.: Fuzzy rule-based systems and applications to decision and control, Rappport IRIT/91-92/R, Toulouse 1991

[17] Duda R. O., Hart P. E., Barnett P., Gaschnig J., Konolige K., Reboh R., Slocum J.: Development of the PROSPECTOR consultant system for mineral exploration Final report for SRI Projects 5821 and 6915, SRI International Artificial Intelligence Center, 1978

[18] Duda R. O., Hart P. E., Nilsson N. J.: Subjective Bayesian methods for rule-based inference systems, Proc. Natl. Comp. Conf. (AFIPS) Vol.45, 1976 pp.1075-1082

[19] Duda R. O., Hart P. E., Nilsson N. J., Sutherland G. L.: Semantic network representations in rule-based inference systems In: Pattern-Directed Inference Systems, 1978 pp.203-221

[20] Fuchs L.: Partially ordered algebraic structures, Pergamon Press, 1963 229 p.

[21] Ginsberg M. L.: Non-monotonic reasoning using Dempster's rule In: Proc. 3rd Natl. Conf. on AI (AAAI 84), Austin, 1984 pp. 126-129

[22] Glymour C.: Independence assumptions and Bayesian updating, Artificial Intelligence Vol.25, 1985 pp.95-99

[23] Goldman S. A., Rivest R. L.: Making maximum entropy computations easier by adding extra constraints In: Proc. 6th Ann. Workshop on Max. Entropy and Bayesian Methods in App. Statistics, Seattle, 1986

[24] Golumbic M.C.: Algorithmic Graph Theory and Pentect Graphs, Academic Press, London , 1980

[25] Goodman I. R., Nguyen H. T.: Uncertainty models for knowledge based systems, Amsterdam: North Holland, 1985

[26] Goodman I. R., Nguyen H. T., Rogers G. S.: On the scoring approach to admissibility of uncertainty measures in expert systems, J. Math. Anal. and Appl. (to appear), 1989 57 p.

[27] Gordon J., Shortliffe E.: The Dempster-Shafer theorie of evidence. In: (Buchanan, Shortliffe, ed.) Rule based expert systems. MYCIN experiments., Addison-Wesley Pub. Comp., 1984

[28] Guth M. A. S.: Uncertainty analysis of rule-based expert systems with Dempster-Shafer Mass Assignments, International Journal of Intelligent Systems Vol.3, 1988 pp.123-139

[29] Hájek P.: Combining functions for certainty factors in consulting systems In: Artificial Intelligence and Information-Control Systems of Robots, Smolenice, 1982 pp. 107-110

[30] Hájek P.: Combining functions in consulting systems and dependence of premisses - a remark, in: Artificial Intelligence and Information-Control Systems of Robots, I. Plander, ed., North-Holland 1984, 163-167

[31] Hájek P.: Combining functions for dertainty degrees in consulting systems, Int. J. Man-Machine Studies 22, 1985, 59-67

[32] Hájek P.: Logic and plausible inference in expert systems In: Inductive reasoning: managing empirical information in AI-systems, Roskilde, 1987

[33] Hájek P.: Towards a probabilistic analysis of MYCIN-like expert systems In: COMPSTAT 88, Kopenhagen, Physica-Verlag Heidelberg, 1988

[34] Hájek P.: Towards probabilistic analysis of MYCIN-like systems II In: Artificial Intelligence and Information-Control Systems of Robots (Plander, ed.), North-Holland, 1989

[35] Hájek P.: A note of belief functions in MYCIN-like systems In: AI'90, CSVTS Prague 1990 pp.19-26

[36] Hájek P.: Towards a probabilistic analysis of MYCIN-like expert systems - a postscript, in: Proc. WUPES'91, UTIA CSAV 1991

[37] Hájek P., Hájková M.: The expert system shell EQUANT-PC - philosophy, structure and implementation, Com. Stat. Quaterly Vol.4, 1990 pp.261-267

[38] Hájek P., Hájková M., Havránek T., Daniel M.: The expert system shell EQUANT-PC - brief information, Kybernetika Vol.25, 1988

[39] Hájek P., Harmanec D.: On belief functions (present state of Dempster-Shafer theory), in Proc. of a summer school in AI (Prague 1992), to appear in Springer-Verlag 1992

[40] Hájek P., Havránek T.: A note on the independence assumption underlying subjective Bayesian updating In: Artificial Intelligence and Information-Control Systems of Robots (Plander, ed.), North-Holland, 1987

[41] Hájek P., Havránek T., Jirouáek R.: Processing uncertain information in expert systems, CRC Press Inc., 1992

[42] Hájek P., Valdes J.: Algebraical foundation of uncertainty processing in rule based expert systems I (extended abstract) In: Sistemy obrabotki znanij i izobrazhenij, Smolenice, 1987

[43] Hájek P., Valdes J.: Algebraical foundations of uncertainty processing in rule-based expert systems II (extended abstract) In: AI'87 (Maá!k, ed.), 1987 áSVTS Prague

[44] Hájek P., Valdes J.: Algebraic foundations of uncertainty processing in rule-based expert systems, preprint No. 28, Mathematical Institute, Czechoslovak Academy of Sciences

[45] Hájek P., Valdes J.: Algebraic foundations of uncertainty processing in rule-based expert systems (group-theoretical aproach), Computers and Artificial Intelligence 9, 1990, 325-334

[46] Hájek P., Valdes J. J.: Generalized algebraic foundations of uncertainty processing in rule-based expert systems (dempsteroids), Computers and Artificial Intelligence 10, 1991, 29-42

[47] Hart P. E., Duda R. O., Einaudi M. T.: PROSPECTOR - A computer-based consultation system for mineral exploration, Mathematical Geology Vol.10, 1978 pp.589-610

[48] Havránek T.: Rule-based systems ruled out? In: International Conference Evolving Knowledge in Natural Science and Artificial Intelligence, University of Reading, 1989 24 p.

[49] Havránek T., Soudská O.: Using an expert system shell for setting statistical package parameters, Computational Statistics Quaterly Vol.3, 1988 pp.159-171

[50] Heckerman D.: Probabilistic interpretations for MYCIN's certainty factors In: Uncertainty in artificial intelligence, Amsterdam: North Holland, 1986 pp.167-196

[51] Heckerman D.: An axiomatic framework for belief updates, in: Uncertainty in Artificial Intelligence 2, North Holland 1988, pp. 11-22

[52] Heckerman D., Horvitz E. J.: The myth of modularity in rule-based systems for reasoning wiht uncertainty, in: Uncertainty in Artificial Intelligence 2, North-Holland 1988, pp.23-34

[53] Horvitz E. J., Heckerman D.: The inconsistent use of measures of certainty in artificial intelligence research In: Uncertainty in artificial intelligence, Amsterdam: North Holland, 1986 pp.137-151

[54] Johnson R. W.: Independence and Bayesian updating methods In: Uncertainty in Artificial Intelligence, North-Holland, 1986 pp.197-201

[55] Kim J. H., Pearl J.: A computational model for combined causal and diagnostic reasoning in inference systems In: Proc. 8th Intl. Joint Conf. on AI (IJCAI 85), Los Angeles, 1983 pp. 190-193

[56] Kruse, R., Schwecke,E., Heimsohn J.: Uncertainty and vagueness in knowledge based systems, Springer-Verlag 1991.

[57] Lauritzen S. L., Spiegelhalter D. J.: Fast manipulation of probabilities with local representations - with applications to expert systems In: Proc. Inductive Reasoning: managing empirical information in AI-systems, Roskilde (Denmark), 1987

[58] Lauritzen S. L., Spiegelhalter D. J.: Local computations with probabilities on graphical structures and their applications to expert systems, J. Royal Statist. Soc., ser. B Vol.50, 1988 pp.157-224

[59] Lauritzen S.L.: Propagation of probabilities, means and variances in mixed association models, Institute for Electronic Systems, Aalborg Vol.90, 1990 No.18 25 p.

[60] Lopez de Mantaras, R.: Approximate reasoning models, Ellis Horwood 1990.

[61] Melle van W.: A domain independent system that aids in constructing knowledge-bases consultation programs, PhD dissertation, Rept STAN-CS-80-820, Dept. of Comp. Sc. Stanford Univ., 1980

[62] Neapolitan R.E.: Probabilistic Reasoning in Expert Systems - Theory and Algorithms, J. Wiley and sons, New York, 1990

[63] Pearl J.: Fusion, propagation and structuring in belief networks, Artificial Intelligence Vol.29, 1986 pp.241-288

[64] Pearl J.: Distributed revision of composite beliefs, Artificial Intelligence Vol.33, 1987 pp.173-215

[65] Pearl J.: Evidential reasoning using stochastic simulation of causal models, Artificial Intelligence Vol.32, 1987 pp. 245-257

[66] Pearl J.: Probabilistic reasoning in intelligent systems: Networks of plausible inference, Morgan-Kaufmann Publishers, 1988

[67] Pendault E. P. D., Zucker S. W., Muresan L. V.: On the independence assumption underlying subjective Bayesian updating, Artificial Intelligence Vol.16, 1981 pp.215-222

[68] Prade H.: A synthetic view of approximate reasoning techniques In: Proc. 6th Intl. Joint Conf. on AI (IJCAI 83), Karlsruhe, W. Germany, 1983 pp.130-136

[69] Prade H.: A computational approach to approximate and plausible reasoning with applications to expert systems, IEEE Trans. Pattern Anal. and Machine Intelligence vol. PAMI-7, 1985, pp. 260-283 (corrections pp. 747-748)

[70] Puppe F.: Systematic introduction to expert systems, Springer-Verlag 1993

[71] Shachter R. D.: Intelligent probabilistic inference In: Uncertainty in Artificial Intelligence, North Holland, 1986 pp. 371-382

[72] Shafer G.: A mathematical theory of evidence, Princeton: Princeton University Press, 1976

[73] Shafer G.: The combination of evidence, Intl. Journal of Intelligent Systems Vol.1, 1986 pp.155-179

[74] Shafer G.: Belief functions and possibility measures In: Analysis of fuzzy information (vol. 1: Mathematics and logic), Boca Raton, Fla.: CRC Press, 1987 pp.51-84

[75] Shafer G.: Probability judgements in artificial intelligence and expert systems, Statistical Science, 1987 pp. 3-16

[76] Shafer G., Shenoy P.: Probability propagation In: Preliminary Papers of the Second International Workshop on Artificial Intelligence and Statistics, Florida, 1989 pp. 33.1-33.10

[77] Shenoy P. P., Shafer G.: Propagating belief functions with local computations, IEEE Expert Vol.1, 1986 pp.43-52

[78] Shenoy P.P., Shafer G.: An axiomatic framework for Bayesian and belief-function propagation, working paper 199, School of Business, the University of Kansas, Lawrence, 1988

[79] Shortliffe E. H.: Computer-based medical consultation: MYCIN, New York: Elsevier, 1976

[80] Shortliffe E. H., Buchanan B. G.: A model of inexact reasoning in medicine, Mathematical Biosciences Vol.23, 1975 pp. 351-379

[81] Smets P.: Belief functions (with discussion) In: Smets et al. [1988], 1988 pp.253-286

[82] Smets P., Mamdani A., Dubois D., Prade H. (eds.): Non-standard logics for automated reasoning, London: Academic Press, 1988

[83] Steve G.: Probabilistic inferential engines in expert systems: how should the strength of rules be expressed? In: Uncertainty in Knowledge-Based Systems (Lecture Notes in Computer Science 286), Springer-Verlag, 1987 pp.179-188

[84] Valdes J. J.: Algebraic and logical foundations of uncertainty processing in rule-based expert systems of artificial intelligence thesis, Czechoslovak Academy of Science 1987

[85] Valdes J.J.: An integrated environment for uncertaninty processing using graphical models. Proc. IPMU'92, Mallorca 1992

[86] Zhang C.: Cooperation under uncertainty in distributed expert systems, Artificial Intelligence 56 (1992), 21-69