A NOTE ON P-SPACES

by

O. MARRERO

Department of Mathematics. Francis Marion College
Florence, South Carolina 29501 (U. S. A.)

A ring (all rings in this note are assumed to be associative) R is said to be *π-regular* if for all $a \in \mathrm{R}$ there exists a positive integer n (depending on a) and there exists $x \in \mathrm{R}$ such that $a^n x a^n = a^n$. The class of π-regular rings, which includes the class of regular (in the sense of von Neumann) rings, was introduced by McCoy in 1939 [2]; since then, π-regular rings have continued to receive the attention of several writers.

It is known [cf. 1, Problem 4J., pp. 62-63] that a completely regular topological space X is a P-space if and only if C (X) is a regular ring; it might be of some interest to observe that this characterization can be stated more generally, replacing «regular ring» by «π-regular ring», as in the theorem below.

A π-regular ring without non-zero nilpotent elements is regular; also, if X is a completely regular topological space, then C (X) has no non-zero nilpotent elements. Consequently, one has the following:

THEOREM.—*A completely regular topological space X is a P-space if and only if C (X) is a π-regular ring.*

REFERENCES

[1] GILLMAN, L. and JERISON, M.: *Rings of Continuous Functions.* Van Nostrand, Princeton, N. J., 1960.

[2] McCOY, N. H.: *Generalized regular rings.* «Bull. Amer. Math. Soc.», 45 (1939), 175-178.