WAHT IS MATHEMATICAL PROOF?

by

S. D. Bajpai

Department of Mathematics, College of Science and Technology,
Port Harcourt (Nigeria)

1. Mathematical Proof: The word proof has many meanings, Ordinarily, the act or process of proving in any sense; specifically, the establishment of a fact by evidence or truth by other truths is called proof. In science, the scientists form a hypothesis, perform a number of experiments, then conclude whether his hypothesis is true or not. A proof by performance is not a mathematical proof. A mathematical proof is a proof by deduction. The mathematician must prove his hypothesis by means of deductive logic before it can be accepted as true. The mathematical proof consists of showing that the statement to be proved called the conclusion, is a logical consequence of the given premises, called the hypotheses. A theorem can easily assume the form of an implication "$H \longrightarrow C$", where H is the hypothesis and C is the desired conclusion. Another way of stating this is "If H, then C", or "H implies C". Many theorems are given in the "If — then" form. However, theorems that are not given in the "if — then" form can be reduced to that form. One of the advantages of putting a theorem in the form "$H \longrightarrow C$" or "if — then" is that the hypothesis and the conclusion of the theorem are clearly distinguished.

Process of induction involves in observing a series of statements and then stating a conclusion about all similar future statements. In mathematical proof we avoid the process of induction and we do not prove a proposition by listing examples for which the proposition is true unless the proposition can be verified for all possible cases. In 1742, Christian Goldbach, a Russian mathematician stated that every even number is a sum of two prime numbers: for example;

$$8 = 3 + 5; \quad 10 = 3 + 7; \quad 12 = 5 + 7 \ldots$$

No mathematician has ever been able to prove deductively that his statement is correct and on the other hand no mathematician has ever been able to find an even number that is not the sum of two prime numbers. Consequently, Goldbach Statement is accepted only as a conjecture, not as a theorem. It seems in this conjecture Goldbach assumed unity as a prime number. However, a specific case sufficient to disprove some proposition about the whole class of statements is called a counter example. Since we can list as many odd prime numbers as desired, therefore some one might be inclined to state that all prime numbers are odd. But the prime number 2 is a counter exemple of the above statement. Although 2 is the only counter example, but we conclude that it is not true that all prime numbers are odd.

Since the deductive reasoning involves showing that a statement is logically true. Therefore in deductive reasoning rules to be used are the laws of logic, i.e. the law of substitution, the law of detachment and law of the syllogisms, etc.

2. Direct Proof: In direct proof we start with the hypothesis, establish a chain of implications and in the end obtain the conclusion. Each and every step in the proof is justified by premises, defined and undefined terms, axioms, previously proved theorems, or laws of logic. Although the law of substitution, the law of detachment and other laws of logic are frequently used in establishing the direct proof, but the most important basis of the direct proof is the use of the law of syllogisms, which is some times termed as the transitive property of implication. If we can show

$$H \longrightarrow C_1, \quad C_1 \longrightarrow C_2, \quad C_2 \longrightarrow C_3, \ldots, \quad C_n \longrightarrow C,$$

then we can conclude that $H \longrightarrow C$.

The techniques employed in proving theorems can be learned to a great extent by reading and following proofs that others have already given about different theorems.

One of the simpliest examples of a direct proof is that of solving an equation.

Let us solve the equation

$$4x - 7 = x + 8,$$

We have, $4x - 7 = x + 8$,

$$4x = x + 15,$$

$$3x = 15$$

$$x = 5.$$

This equation can be considered as the theorem "If $4x - 7 = x + 8$, then $x = 5$"

To prove this theorem follow ing implications have been used:

If $4x - 7 = x + 8$, then $4x = x + 15$.

If $4x = x + 15$, then $3x = 15$.

If $3x = 15$, then $x = 5$.

Therefore, if $4x - 7 = x + 8$, then $x = 5$.

If is important to note that each step in finding the solution is justified by a premise, a definition, and an axiom, or a previously proved fact or a law of logic.

If we interchange the hypothesis and the conclusion of a theorem, then the result obtained is called the converse of the original theorem. For example.

$$\text{If } x = 5, \text{ then } 4x - 7 = x + 8,$$

is the converse of the given equation.

If the steps of a direct proof are reversible, then the theoren and converse both are true. If $H \longrightarrow C$ and $C \longrightarrow H$, then write $H \longleftrightarrow C$ and say "H if and only if C". If is important to note that the converse of some theorems is not true.

3. Indirect Proof: It is not always easy to establish the direct proof because sometimes the direct method of proof is quite difficult. The indirect method of proof is called proof by contradiction or reductio ad absurdum. In the indirect proof, we accept hypothesis and assume that the negation of the conclusion, rather than the original conclusion itself, to be true. With this "assumed true" negation and other propositions already assumed or proved true we reason until we arrive at a contradiction of the hypothesis or some other proposition already known to be true, The logical basis of this type of proof is that we cannot have two contradictory true statements in one system and a true hypothesis cannot imply a false conclusion. Therefore we conclude that our assumption that the negation is true must be infact incorrect.