Continuity of Derivations on Semi-Prime Banach Algebras

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(Presented by J.A. Navarro)

Let A be a complex Banach algebra. A derivation on A is a linear mapping $D : A \longrightarrow A$ such that $D(xy) = D(x)y + xD(y)$, for all x and y in A.

The question of continuity of derivations on Banach algebras has been considered by several authors, mainly in the case of commutative Banach algebras, semi-simple Banach algebras. (See [1], [5], [6], [7], [8], [9]).

We shall be concerned here with the problem as to whether every non-zero derivation on more general class of non-commutative Banach algebras is continuous. It is well-known that the class of semi-prime algebras includes the class of semi-simple algebras, the class of prime (in particular, primitive) algebras. We state the following theorems concerning continuity of derivations on semi-prime Banach algebras.

Let $\ell(\text{Socl } A)$ be denoted the left annihilator ideal of the socle of semi-prime algebra A.

THEOREM 1. *Let A be a prime Banach algebra which contains a compact element. Then every derivation $D : A \longrightarrow A$ is continuous.*

THEOREM 2. *If A is a semi-prime Banach algebra with dense socle, then every derivation $D : A \longrightarrow A$ is continuous.*

THEOREM 3. *If A is a semi-prime Banach algebra such that $\ell(\text{Socl } A) = \{0\}$, then every derivation $D : A \longrightarrow A$ is continuous.*

Detailed information and related results can be found in recent author's papers. (See [2], [3], [4]).

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