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Examples of Fuzzy Basic Proximities

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In this note examples of fuzzy basic proximities have been provided which contradict the following, proved in [2,Theorem 2.7]:

1. $c_\Pi = c_{\Pi^*}$ iff $\Pi(x_p) = \Pi^*(x_p)$, for every fuzzy point x_p in I^X;

2. $c_\Pi = c_{\Pi^*}$ iff $N(\Pi,x_p) = N(\Pi^*,x_p)$, for every fuzzy point x_p in I^X;

here Π and Π^* are fuzzy basic proximities on a nonempty set X.

Let X be a nonempty set and I be the closed unit interval [0,1] of the real line. A fuzzy set in X is an element of the set I^X of all functions from X to I. A fuzzy point x_p in I^X, $x \in X$, $0 < p \leqslant 1$, is defined by

$$x_p(y) = \begin{cases} p, & y = x \\ 0, & y \neq x. \end{cases}$$

We shall denote by λ, μ, μ', ν, the fuzzy sets in X. For $\alpha \in [0,1]$, the element $\lambda \in I^X$ defined by $\lambda(x) = \alpha$, for all $x \in X$, is denoted by 'α'.

A binary relation Π on I^X is a fuzzy basic proximity on X iff it satisfies:

FP1. $\Pi = \Pi^{-1}$,

FP2. $\Pi(\lambda) \equiv \{\mu \in I^X : (\lambda,\mu) \in \Pi\}$ is a fuzzy grill on X for all λ in I^X, and

FP3. $\lambda \wedge \mu \neq 0 \Rightarrow (\lambda,\mu) \in \Pi$ [1].

The pair (X,Π) is called a fuzzy basic proximity space. In a fuzzy basic proximity space (X,Π), the set $N(\Pi,\lambda)$ of all fuzzy proximal neighbourhoods of a fuzzy set λ is given by

$$N(\Pi,\lambda) = \{\mu : (1-\mu) \notin \Pi(\lambda)\}.$$

For $\lambda \in I^X$, the closure $c_\Pi(\lambda)$ of λ with respect to a basic proximity Π on X is defined by

$$c_{\Pi}(\lambda) = \vee\{x_p : x_p \in \Pi(\lambda)\}.$$

The function $c_{\Pi}(\lambda) : I^X \longrightarrow I^X$ is a Čech closure operator [2,3].

EXAMPLES. For $0 < \alpha < 1$, define a relation Π on I^X as follows:

$$\Pi \equiv \Pi_{\alpha} =$$
$$\{(\lambda,\mu) : \lambda \wedge \mu \neq 0\} \cup \{(\lambda,\mu) : \lambda \neq 0, \mu \neq 0 \text{ and } (\lambda \vee \mu)(x) > \alpha, \text{ for some } x \in X\}.$$

Then

(a) Π is a basic proximity on X;

(b) $c_{\Pi}(\lambda) = 1$, for $\lambda \in I^X-\{0\}$;

(c) for $1 \geqslant p > \alpha$ and $x \in X$, $\Pi(x_p) = I^X-\{0\}$ and $N(\Pi,x_p) = \{1\}$;

(d) for $0 < p \leqslant \alpha$ and $x \in X$, $\Pi(x_p) = \{\lambda : \lambda(x) \neq 0\} \cup \{\lambda : \lambda(y) > \alpha$, for some $y \in X\}$, and $N(\Pi,x_p) = \{\lambda \geqslant 1-\alpha : \lambda(x) = 1\}$.

Proof. (a) If $\mu' \geqslant \mu$, then $\lambda \wedge \mu' \geqslant \lambda \wedge \mu$ and $\lambda \vee \mu' \geqslant \lambda \vee \mu$. Hence $\mu \in \Pi(\lambda)$ implies $\mu' \in \Pi(\lambda)$. Next, let $\mu \vee v \in \Pi(\lambda)$. If $(\mu \vee v) \wedge \lambda \neq 0$, we are through. Suppose $((\mu \vee v) \wedge \lambda)(x) > \alpha$ for some $x \in X$. Since $(\mu \vee v) \vee \lambda = (\mu \vee \lambda) \vee (v \vee \lambda)$, either $(\mu \vee \lambda)(x) > \alpha$ or $(v \vee \lambda)(x) > \alpha$. Thus either $\mu \in \Pi(\lambda)$ or $v \in \Pi(\lambda)$. Hence, for all $\lambda \in I^X-\{0\}$, $\Pi(\lambda)$ is a fuzzy grill on X. By definition, Π satisfies conditions FP1 and FP3. Therefore Π is a basic proximity on X.

(b) It is sufficient to note that, for $\lambda \neq 0$, $x_1 \in \Pi(\lambda)$ for every $x \in X$.

(c) The first part is immediate. For the second part, consider $\lambda \neq 1$. Then $1-\lambda \in \Pi(x_p)$, $1 \geqslant p > \alpha$, i.e. $\lambda \notin N(\Pi,x_p)$. Hence $N(\Pi,x_p) = \{1\}$.

(d) Follows from the definition.

For $0 < \alpha, \beta < 1$, the following hold:

(i) $c_{\Pi_{\alpha}}(0) = c_{\Pi_{\beta}}(0) = 0$ and $c_{\Pi_{\alpha}}(\lambda) = c_{\Pi_{\beta}}(\lambda) = 1$, $\lambda \neq 0$;

(ii) If $\alpha < p < \beta$, then $\Pi_{\alpha}(x_p) \neq \Pi_{\beta}(x_p)$, provided X has at least two elements;

(iii) If $\alpha < p < \beta$, then $N(\Pi_{\alpha},x_p) \neq N(\Pi_{\beta},x_p)$, provided X has at least two elements;

According to (i), $c_{\Pi_{\alpha}} = c_{\Pi_{\beta}}$. Therefore (ii) contradicts (1) and (iii) contradicts (2). However the following holds:

$$c_{\Pi} = c_{\Pi^*} \quad \text{iff} \quad \Pi(x_1) = \Pi^*(x_1), \forall x \in X.$$

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