EXTRACTA MATHEMATICAE Vol. 7, Núm. 2-3, 129 – 130 (1992)

Global Minimun Point of a Convex Function [1]

J.M. SORIANO

Departamento de Análisis Matemático, Universidad de Sevilla
41080 *Sevilla, Spain*

AMS *Subject Class.* (1991): 58C30, 65H10. *Received November* 26, 1992

With the invention of high–speed computers large–scale problems from diverse fields as economics, agriculture, military planning, and flows in networks, became at least potentially solvable, being a lot of them extremum problems.

The great importance of extremum problems in applied mathematics leads us to the general study of extremum of functions from $\mathbb{R}^n$ to $\mathbb{R}$. It is not easy to know the extremum points, neither for differentiable functions, because it is not always possible to solve the equation $\nabla f(x) = 0$ to calculate critical points. Convex functions have a particularly simple extremal structure [2], and there exist algorithms to calculate extremum points, supposing its existence. However it is not easy to prove the existence of extremum even in the case of convex differentiable functions [2], [3]. Therefore it is very important to give sufficient conditions to guarantee this existence.

Given a strictly convex function f from $\mathbb{R}^n$ to $\mathbb{R}$, we prove in this paper the existence of a unique global minimum point for f if the following condition is verified:

$$\lim_{X\to\infty} \frac{\partial f(X)/\partial X_i}{X_i} > 0 \qquad \text{for } i = 1,\dots,n. \tag{1}$$

This proof is founded on the continuation method and the method can serve to determinate numerically that point as we have showed [5], [6]; see also [1] and [7].

THEOREM. *Let $f:\mathbb{R}^n \longrightarrow \mathbb{R}$, $f \in C^3(\mathbb{R}^n)$ be a strictly convex function verifying* (1). *Then there exists a unique minimum point for f.*

[1] This paper is to appear in Applied Mathematics and Computation.

REFERENCES

1. GARCÍA, C.B. AND ZANGWILL, W.I., "Pathways to Solutions, Fixed Points and Equilibria", Prentice–Hall Series in Computational Mathematics, Prentice–Hall, London, 1981.
2. ROBERTS, A.W. AND VARBERG, D.E., "Convex Functions", Pure Appl. Math., Vol. 57, Academic Press, New York – London, 1973.
3. ROCKAFELLAR, R.T., "Convex Analysis", Princeton Math. Ser. n. 28, Princeton University Press, Princeton, N.J., 1970.
4. PETROVSKI, I.G., "Ordinary Differential Equations", Dover Publications, INC., New York, 1966.
5. SORIANO, J.M., Sobre la existencia y el cálculo de ceros de funciones regulares, *Rev. Real Acad. Cienc. Exact. Fis. Natur.* **LXXXII** (3 – 4) (1988), 523 – 531.
6. SORIANO, J.M., A special type of triangulations in numerical nonlinear analysis, *Collect. Math.* **XLI** (1) (1990), 45 – 48.
7. SORIANO, J.M., Global minimum point of a convex function, to appear in *Applied Mathematics and Computation.*
8. ZEIDLER, E., "Nonlinear Functional Analysis and its Applications", Springer–Verlag, New York – Berlin – Tokyo, 1985.