EXTRACTA MATHEMATICAE 5, n.1, 21-22 (1990)

# ON THE EVALUATION MAP

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1980 A.M.S. Classification 55P62.

The evaluation map of an augmented differential graded algebra (DGA) $A$ was first defined in [2] as a natural vector map (over a field $\mathbf{K}$ of characteristic $0$),

$$ev_A : ext_A(\mathbf{K}, A) \longrightarrow H(A).$$

which assigns to each element $[f] \in Ext_A(\mathbf{K}, A)$ represented by a cycle $f : P \rightarrow A$ ($P$ semifree resolution of $\mathbf{K}$), the class $[f(p)] \in H(A)$ where $p$ is a cycle in $P$ representing $1$.

The evaluation map of a pointed topological space $S$ is, by definition, the evaluation map of the DGA $C^*(S; \mathbf{K})$.

"*Having non trivial evaluation map*" is a property which has geometric consequences: In fact, any pointed space $T$ of the form $T = S \cup_f e^{n+1}$, in which the characteristic class of $e^{n+1}$ in $H^{n+1}(T; \mathbf{K})$ is non zero (we shall refer to it as a space with a terminal cell), has non trivial evaluation map [1,prop.1.6]. Moreover, given a c-connected DGA $A$, "$ev_A \neq 0$" is an intermediate property between *"having a cohomology class represented by an element which annihilates $A^+$ "* and *"having a cohomology class which annihilates $H^+(A)$ "*.

We use this property to obtain other interesting geometric results:

Our first goal is a characterization of 1-connected *rationally elliptic* spaces in terms of their evaluation map. A 1-connected space $S$ is rationally elliptic if the vector spaces $\pi_*(S) \otimes \mathbf{Q}$ and $H^*(S; \mathbf{Q})$, are finite dimensional [1]. The homogeneous spaces are classical examples of such spaces. We prove:

THEOREM A: *Let $S$ be a 1-connected pointed space with $\pi_*(S) \otimes \mathbf{Q}$ finite dimensional. Then, the following statements are equivalentes:*

*(i)* $H^*(S; \mathbf{Q})$ *is finite dimensional.*

*(ii)* $ev_S$ *(over $\mathbf{Q}$) is different from zero.*

Observe that the finiteness of the dimension of $\pi_*(S) \otimes \mathbf{Q}$ is necessary as is shown in the following, EXAMPLE: The space $\mathbb{C}P^{\infty} \vee S^n$ has infinite dimensional rational cohomology and its evaluation map is different from zero, since it has a terminal cell, .

Theorem A is a first aproximation to the main result:

THEOREM B: *Let $F \to E \xrightarrow{\rho} B$ be a fibration of simply connected spaces.*

*(i) If $H^*(F;\mathbb{Q})$ is finite dimensional and satisfies Poincare Duality,*

$$ev_B \neq 0 \qquad \textit{implies} \qquad ev_E \neq 0.$$

*(ii) if $\pi_*(\rho) \otimes \mathbb{Q}$ is surjective,*

$$ev_E \neq 0 \qquad \textit{implies} \qquad ev_F \neq 0.$$

In particular, in view of Theorem A, we deduce the following,

COROLLARY: *Given a fibration $F \to E \xrightarrow{\rho} B$ of simply connected spaces in which $\pi_*(F) \otimes \mathbb{Q}$ is finite dimensional and $\pi_*(\rho) \otimes \mathbb{Q}$ is surjective,*

$$ev_E \neq 0 \qquad \textit{implies} \qquad \dim H^*(F;\mathbb{Q}) < \infty.$$

REMARK: As we said before, that holds if, for example, $E = S \cup_f e^{n+1}$ has a terminal cell.

All prerequisites in Sullivan's theory of minimal models and its connection with rational homotopy theory can be found in [1], [4] and [8]. We follow the notation in [1] and [4].

## References


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