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UNIFORM INTEGRABILITY AND CONVERGENCE IN THE pTH-MEAN OF RANDOMLY WEIGHTED SUMS

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Let X be a separable Banach space with norm $\|.\|$, and (Ω,B,P) a probability space. Let $\{V_n\}$, $n \geq 1$, be a sequence of random elements in X, and denote by $\{A_{nk}\}$, $n,k \geq 1$, a double array of random variables (random elements in R) defined from the same probability space (Ω,B,P). In this note, we study some limit theorems for the sequence of sums $S_n = \sum_{k\geq 1} A_{nk}V_k$, $n \geq 1$.

The particular case of constants weights, when $\{A_{nk}\}$ is a double array of real numbers, has been considered by a lot of researchers, which began studying these problems under the condition of identical distribution of the random elements $\{V_n\}$. The later researchers were gradually weakening this condition and they were obtaining interesting results under the following conditions:

a) uniform boundedness of moments: there exists $r > 0$ such that $\sup_n E\|V_n\|^r < \infty$

b) uniform domination: there exists a random variable V on Ω such that for each $a > 0$:

$$P\left[\|V_n\| \geq a\right] \leq P\left[|V| \geq a\right] \quad \text{for every } n \geq 1.$$

The case of random weights is less treated in the literature, and in general the results cannot be directly carry from the case of constant weights to the case of random weights.

In this note, we present two results about the convergence in the pth-mean of $S_n = \sum_{k\geq 1} A_{nk} V_k$ under the condition of uniform integrability of the random variables $\|V_n\|^r$ for some $0 < r < 1/2$. This condition is weaker than the uniform domination, and it is related with the uniform boundedness of moments (see 3.).

Our first result is the following:

THEOREM 1.- Let $\{V_n\}$, $n \geq 1$, be a sequence of random elements in a separable Banach space X, such that $\{\|V_n\|^r\}$, $n \geq 1$, is uniformly integrable for some $0 < r < 1/2$.

Let $\{A_{nk}\}$, $n,k \geq 1$, be a double array of random variables such that:

a) $\lim_{n\to\infty} (\max_k E|A_{nk}|^r) = 0$

b) $\sum_{k\geq 1} E^r |A_{nk}|^r \leq C$ for each n, where C is a constant.

Then, $S_n \longrightarrow 0$ in the r/2th-mean.

<u>Proof</u>: The completeness of X and the absolute convergence of S_n implies the convergence of S_n with probability one.

The uniform boundedness and the uniform continuity of $\{E\|V_n\|^r\}$, as well as the truncation method yield the result.

We also obtain, in a similar way, the following result, which, compared with Theorem 1, provides the convergence in the pth-mean with a smaller p ($0 < r < 1/2 \Rightarrow r^2 < r/2$), but we require conditions for smaller moments of the weights A_{nk} ($0 < r < 1/2 \Rightarrow r^2/(1-r) < r$).

THEOREM 2.- Let $\{V_n\}$, $n\geq 1$, be a sequence like in Theorem 1.

Let $\{A_{nk}\}$, $n,k\geq 1$, be a double array of random variables such that:

a) $\lim_{n\to\infty} (\max_k E|A_{nk}|^{r^2/(1-r)}) = 0$

b) $\sum_{k\geq 1} E^r|A_{nk}|^{r^2/(1-r)} \leq C$ for each n, where C is a constant.

Then $S_n \longrightarrow 0$ in the r^2th-mean.

REFERENCES

1. ROHATGI, V.K. (1971). Convergence of weighted sums of independent random variables. Proc. Cambridge Philos. Soc. 69, 305-307.
2. TAYLOR, R.L. (1978). Stochastic Convergence of Weighted Sums of Random Elements in Linear Spaces. Lecture Notes in Mathematics, Vol. 672, Springer-Verlag, Berlin.
3. WANG, X.C. & BHASKARA RAO, M. (1985). A note on convergence of weighted sums of random variables. Internat. J. Math. & Math. Sci. Vol.8 No.4, 805-812.
4. WANG, X.C. & BHASKARA RAO, M. (1987). Some results on the convergence of weighted sums of random elements in separable Banach spaces. Studia Math. 86, 131-153.

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