EXTRACTA MATHEMATICAE 3, n.1, 23-25 (1988)

COMPACTOID SETS IN P-ADIC LOCALLY CONVEX SPACES

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A.M.S. 1980 Classification: 46P05

Several properties of compactoid sets in Locally convex spaces E over non-archimedean valued fields K are proved in this paper. As a consequence we derive partial afirmative answers to the following questions:

1. Let A be a compactoid set in E. Does there exist a compact set X in E such that A is contained in the closed absolutely convex hull of X? (GRUSON and VAN DER PUT [3], problem following 5.8).

2. Let A be a complete c'-compact set in E. Does it follows that A is the closed absolutely convex hull of some compact set X? (SCHIKHOF [6], problem following 1.7).

3. Is every weakly c'-compact set in E c'-compact? (SCHIKHOF [8], problem following 2.7).

A subset A of E is *compactoid* [10] if for each neighbourhood U of 0 in E there exists a finite set F $\subset$ E such that A $\subset$ U + C(F) (where C(F) denotes the absolutely convex hull of F); An absolutely convex set A $\subset$ E is *c'-compact* [7] if in the above we may choose F $\subset$ A.

First we give afirmative answers to problems 1 and 2 in this paper, for locally convex spaces with a *Schauder basis* (the concepts of Schauder an orthogonal basis we consider here are in [2]).

THEOREM 1. ([4], theorem 3.2) *Let E be a locally convex space with a Schauder basis such that E is $\sigma(E,E')$-sequentally complete and E' is $\sigma(E',E)$- sequentally complete.*

a) Assume that the basis on E is an orthogonal basis. Then, every compactoid subset of E is contained in the closed absolutely convex hull of a sequence (y_n) in E with $\lim y_n = 0$.

b) Assume that the valuation on K is dense. Then, every c'-compact and complete subset of E is the closed absolutely convex hull of a sequence (y_n) in E with $\lim y_n = 0$.

REMARKS 1. For an example showing that just "E has a Schauder basis" in part a) of theorem 1 it is not enough, see [6] note to theorem 2.1.

2. Observe that if $|K|$ is discrete, then problem 1 in this paper has an afirmative answer for every locally convex space over K. It is a direct consequence of [6], theorem 1.5.

Now we study problem 3 in this paper for locally convex spaces with duals of countable type. For that, we denote by E'_c the dual space E' endowed with the locally convex topology defined by the family of non-archimedean seminorms $\{||\cdot||_A$: A is weakly c'-compact in E $\}$ given by $||f||_A = \sup_{x\in A} |f(x)|$ ($f\in E'$, A weakly c'-compact). Then, in the same wein as theorem 8.3 of [5] we obtain,

THEOREM 2. *Let E be an strongly polar space* (see [5] for this concept) . *Then, the following properties are equivalent:*

i) E'_c is of countable type.

ii) Every weakly c'-compact subset of E is weakly metrizable.

iii) Every weakly c'-compact subset of E is metrizable and compactoid.

<u>Proof</u>. i)→ii) and iii)→i) follow from [9], theorem 5.1.

ii)→iii): similar to a)→b) in theorem 8.5 of [5].

COROLLARY. *Let E be an strongly polar space. Then, if E'_c is of countable type, every weakly c'-compact set in E is c'-compact.*

Furthermore,, if E is metrizable, then problem 3 in this paper has an afirmative answer for E if and only if E'_c is of countable type.

Also, as a consequence of proposition 3 of [1] we have an afirmative answer to our problem 3 in the following case:

THEOREM 3. *Let E be a weakly quasicomplete space* (e.g. a reflexive space) *over an spherically complete valued field K. Then, every weakly c'-compact subset of E is c'-compact.*

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