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ASYMPTOTIC ANALYSIS: BINGHAM FLUID FLOW THROUGH A PERFORATED WALL

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1. Introduction.

We submit a short summary of the work done on a mathematical problem related to the stationary flow of a Bingham fluid (DL 1) through a sieve.

The modelling of phenomena of this type in done within the framework of Asymptotic Analysis (ECK 1) (COL 1), that is: as problem depending on a small paramenter $\varepsilon > 0$. This parameter is the radius of the holes, which the fluid goes through, and the distance among holes, so that the whole set forms a network with periodic structure.

The formal results we give deal with the limit behavior of such a problem for $\varepsilon \to 0$.

Although the results are formal and no convergence theorem is stated the theorems we refer to, express the well-poseness of the terms of the outer expansions and inner expansion that will make up the formal limit problem.

The rigorous study of convergence is an open problem, and a not easy one; the usual methods to deal with that problem are the well-known methods of the Energy (LBP 1) (SP 1) and the methods of the Epi-convergence of functionals (AT 1).

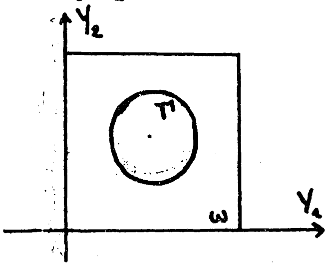
The study of this kind of problems for the linear case, acoustic and Stokes flow, have been done by (SH-SP 1), (SP 2), (CA 1).

From now on, we assume that the reader is familiar with the asymptotic analysis'techniques.

2. Definition of the problem.-

In the space R^3, of coordinates (x_1, x_2, x_3), we consider a bounded domaine Ω, with boundary $\partial\Omega$.- Let Σ be the intersecction with the plane $x_3 = 0$, and let Ω^+ and Ω^- be the parts of Ω in the half-space $x_3 > 0$ (resp. $x_3 < 0$).

In the auxiliary plane of coordinates (y_1, y_2) we consider a rectangle ω with its sides parallel to the axes, and vertex at the origin, and within ω, a hole T, symmetric with respect to the center; let $\Gamma = \omega - T$.



To symplify, we take T of area equal to 1.- We suppose ω and T periodically reproduced throughout the plane (y_1, y_2) Anyway, T, ω . will denote both the hole and rectangle of the figure and all the others obtained via periodic translations.

We introduce the paramenter $\varepsilon > 0$, that will tend to zero and for each ε, we call εT the homothetical hole to T of rate ε, on the plane (x_1, x_2).

Under these conditions, we consider the flow of a fluid in the domain Ω^ε.

(1) $$\Omega^{\varepsilon} = \Omega^{+} \cup \Omega^{-}\{\varepsilon\bar{T} \cup \Sigma\}$$

If $\underline{n}$ is the unit normal vector exterior to Ω , we define over the boundary $\partial\Omega$ a velocity field $\underline{b}(x)$ independent of ε and verifying:

(2) $$\int_{\partial\Omega} \underline{b}.\underline{n} \; dS = 0$$

(3) $$\int_{\partial\Omega - \partial\Omega} \underline{b}.\underline{n} \; dS = F = 0$$

Under these hypotheses, we consider the steady flow of a Bingham fluid in Ω^{ε} , with velocity $\underline{v}^{\varepsilon}$ and pressure p^{ε} , with plasticity threshold $g_{\varepsilon}=0(\frac{1}{\varepsilon})$, $g_{\varepsilon}= \frac{1}{\varepsilon} g$, $g > 0$ and viscocity $\mu = 0(1)$ $\mu > 0$.

Then the mathematical problem is:

(4) $$\frac{\partial}{\partial x_j}\sigma^{\varepsilon}_{ij} = 0 \qquad \text{in } \Omega^{\varepsilon}$$

(5) $$\sigma^{\varepsilon}_{ij} = -p^{\varepsilon}\delta_{ij} + \frac{1}{\varepsilon} g \frac{D_{ij}(\underline{v}^{\varepsilon})}{(D_{II}(\underline{v}^{\varepsilon}))^{\frac{1}{2}}} + 2\mu D_{ij}(\underline{v}^{\varepsilon}) \quad \text{if } D_{II}(\underline{v}^{\varepsilon}) \neq 0$$

$$\sigma^{\varepsilon}_{ij} \quad \text{indeterminate if } D_{II}(\underline{v}^{\varepsilon}) = 0, \; D_{II}(\underline{v}) = \frac{1}{2} D_{ij}(\underline{v}) \, D_{ij}(\underline{v})$$

(6) $$\text{div } \underline{v}^{\varepsilon} = 0 \text{ in } \Omega^{\varepsilon}$$

(7) $$\underline{v}^{\varepsilon}|_{\partial\Omega} = b \quad , \underline{v}^{\varepsilon}|_{\Sigma-\bar{T}_{\varepsilon}} = 0$$

$\sigma^{\varepsilon}_{ij}$ is the strengh tensor and $D_{ij}(\underline{v}^{\varepsilon})$ the velocity gradient tensor (DL 1).

3. Formal asymptotic analysis.

We seek regular asymtotic expansions in Ω^{+}, Ω^{-} and in a layer close to Σ.

We postulate outer expansions (ECK 1) in $\Omega^{\pm}$ of the form

(8) $$\underline{v}^{\varepsilon} = \underline{u}^{0\pm}(x) + 0(1)$$

(9) $$\sigma^{\varepsilon}_{ij} = \frac{1}{\varepsilon} \sigma^{\pm}_{ij}(x) + 0\left(\frac{1}{\varepsilon}\right)$$

We propose expansions in the boundary layer via the "two scale" techniques (SP 1), (COL 1)

(10) $$\underline{v}^{\varepsilon} = \underline{v}^{\varepsilon}(x_1^0, x_2^0, y_1, y_2, y_3) + 0(1)$$

(11) $$p^{\varepsilon} = \frac{1}{\varepsilon}(p^{0}(x_1^0, x_2^0, y_1, y_2, y_3) + 0(1)$$

with certain periodicity conditions in de variable $\underline{y}$ that are given by the geometric structure of the wall (L1).

REMARK 1.- The first term of the outer expansion $u^{0\pm}(x)$ fulfills a constitutive law, which is known as perfectly plastic, and a variational formulation of that problem can be obtained as the minimisation of a functional on spaces of the type $BD(\Omega^{\pm})$, see (T1) (L1) for the details of the statement and proof of the existence theorem.

REMARK 2.- The first term of the expansion in the boundary layer Σ , is the solution of a boundary problem on an infinite prism, of base ω , with periodicity conditions on the lateral faces, for the equations of a Bingham fluid with finite plasticity threshold.- This problem is well-posed in function spaces of type H^1 , see (LH1) for the details of the proof.

FINAL REMARK.- The matching between the outer and inner expansions take the following relased form:

(12) $u_3^{0\pm} = v_3^{0\pm}$ on Σ, for the normal componet

(13) $g|v_2^{\pm\infty} - u_2^{0\pm}| = (v_2^{\pm\infty} - u_2^{0\pm}).(\sigma^{\pm}.n_{\pm})$ on Σ, for the tangent component.

The conditions define the above problem in a precise way (T1). The usual uniqueness arguments, strict monotony, fail.

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