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BALANCED BIG COHEN-MACAULAY MODULES AND FLAT EXTENSIONS OF RINGS

Santiago Zarzuela Armengou

Facultat de Matemàtiques
Universitat de Barcelona
Gran Via 585 08007 Barcelona
Spain

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Let (R,m) be a (commutative, Noetherian) local ring. A (not necessarily finitely generated) R-module is said to be a balanced big Cohen-Macaulay R-module (bbCM R-module) if any system of parameters for R is an M-sequence. A such R-module ever exists provided the ring R contains a field as a subring (see [Ho] Th.5.7 and [Ba-St]). Recently some questions concerning bbCM modules and flat extensions of rings have been settled. Let M be a bbCM R-module and $p \in \mathrm{Supp}_R(M)$ such that $p \in \mathrm{Ass}_R(M/(a_1, \ldots, a_r)M)$ for some M-sequence $a_1, \ldots, a_r$; then M is a bbCM R_p-module provided R is catenary (Foxby, private communication, Takeuchi [Ta]). On the other hand Ogoma has constructed a counterexemple for the above question in general ([Og] §5 II). It has also been proved that given a flat, integral extension of local rings $R \to S$ and M a bbCM R-module then $M \otimes_R S$ is a bbCM S-module and that the analogous result is not true when $S = \hat{R}$.

This paper deals with the following more general question: let $R \to S$ be a flat morphism of local rings (R,m), (S,n); let M be a bbCM R-module. When is $M \otimes_R S$ a bbCM S-module?. We obtain necessary and sufficient conditions for the above question have an affirmative answer. In fact, let M be a bbCM R-module; we say that a prime ideal of Spec(R) belongs to the little support of M ($\mathrm{supp}_R(M)$) if there exists some integer $i \geq 0$ such that $\mu_R^i(p,M) \neq 0$. By [Sh] Th.3.2 $p \in \mathrm{supp}_R(M)$ if and only if $p \in \mathrm{Ass}_R(M/(a_1, \ldots, a_r)M)$ for some M-sequence $a_1, \ldots, a_r$.

THEOREM 1.- The following are equivalent:

(i) $M \otimes_R S$ is a bbCM S-module

(ii) $n(M \otimes_R S) \neq M \otimes_R S$ and $\forall q \in \mathrm{supp}_S(M \otimes_R S)$

1) $h(q/pS) = \mathrm{depth}(C_{\bar{q}})$

2) $h(q) + \dim(S/q) = \dim(S)$, where $p = q^c$, $C = S/pS$ and $\bar{q} = qC$.

COROLLARY 2 (O'Carroll [O'Ca]).- Let $R \to S$ be a flat, integral extension of local rings and M a bbCM R-module. Then $M \otimes_R S$ is a bbCM S-mod.

COROLLARY 3 (Foxby, private communication, Takeuchi [Ta]).- Let R be a catenary local ring and M a bbCM R-module. Then $\forall p \in \mathrm{supp}_R(M)$ M_p is a bbCM R_p-module.

COROLLARY 4.- Let R be a local ring. Suposse that R is an homomorfic image of a Cohen-Macaulay ring and that M is a bbCM R-module. Then $M \otimes_R \hat{R}$ is a bbCM $\hat{R}$-module.

COROLLARY 5.- Let R be a local ring. Suposse that R satisfies the second chain condition and that M is a bbCM R-module. Then $M \otimes_R R^h$ is a bbCM R^h-module.

We also obtain the following counterexemples:

PROPOSITION 6.- Let R be a non catenary local domain containing a field. Then there exists a bbCM R-module such that $M \otimes_R \hat{R}$ is not a bbCM $\hat{R}$-module.

PROPOSITION 7.- Any counterexemple for the localization is a counterexemple for the completation.

PROPOSITION 8.- There exists a local catenary domain R satisfying the first chain condition such that for any bbCM R-module M $M \otimes_R R^h$ is not a bbCM R^h-module.

Finnally we remark that the family of rings whose bbCM modules localize is larger than catenary rings.

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