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SOME ASPECTS AND PROBLEMS IN HOLOMORPHY

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0. Introduction.

Holomorphy or Complex Analysis in any (finite or infinite) dimensions has undergone a progress in the past 20 years or so which led to the publication of some expository books in 1969 |27|, 1970 |35|, 1973 |34|, 1974 |8|, 1980 |14|, 1981 |11|, 1982 |9|, 1984 |22|, 1985 |5|,|7|,|18|,|36| and 1986 |26|. In these lectures we propose only to describe some problems in Holomorphy in an as clear as possible way dealing with the following aspects: a holomorphic classification of locally convex spaces; topology on spaces of holomorphic mappings; holomorphic factorization; and holomorphic continuation.

1. Terminology and notation.
Topological vector spaces.

All topological vector spaces considered here will be complex and locally convex. If α is a seminorm on a vector space E, we denote by E_α the vector space E seminormed by α , and by $E/\alpha = E_\alpha/\alpha^{-1}(0)$ the associated normed space. We let CS(E) be the set of all continuous seminorms on a topological vector space E. We represent by wE the weak space associated with a topological vector space E, that is E endowed with its weak topology $\sigma(E,E')$. We refer to Köthe |20|, Horvath |17| and Jarchow |19| for the terminology and notation, particularly for the concepts of a Fréchet space, semi-Montel space, Montel space, Schwartz space, Köthe space, DFM space (Dual of a Fréchet-Mon tel space), and Silva space or equivalently DFS space (Dual of a Fréchet-Schwartz space).

Holomorphy.

Let E and F be locally convex spaces. We denote by P(mE;F) the vector space of all m-homogeneous polynomials of E to F for $m \in \mathbb{N}$. If U is an open nonvoid subset of E, we let $\mathcal{H}(U;F)$ be the vector space of all holomorphic mappings f of U to F. A mapping f of U to F is said to be finitely holomorphic when its restriction $f|(U \cap S)$ is holomorphic for every finite dimensional vector subspace S of E intersecting U, where S has its natural topology. We shall use on $\mathcal{H}(U;F)$ the compact-open topology τ_o besides other topologies τ_ω and τ_δ . When $F=\mathbb{C}$, we shall use the simpler notations P(mE) and $\mathcal{H}(U)$ to denote $P(mE;\mathbb{C})$ and $\mathcal{H}(U;\mathbb{C})$. We refer to Dineen |11|, Colombeau |9|, Barroso |5|, Chae |7| and Mujica |26| for the terminology and notation, particularly holomorphic mappings, and the topologies τ_o , τ_ω and τ_δ .

2. A holomorphic classification of locally convex spaces.

Let E and f be complex locally convex spaces, U be an open nonvoid subset of E, and $\mathcal{H}(U;F)$ be the vector space of all holomorphic mappings of U to F.

Definition 1. A given E is a holomorphically bornological space if, for every U and every F, we have that each mapping $f : U \longrightarrow F$ belongs to $\mathcal{H}(U;F)$ if (and always only if) f is finitelly holomorphic, and f is bounded on every compact subset of U.

Remark 2. Every holomorphically bornological space is a bornological space (see |20|, |17|, |19| for the concept of a bornological space).

Definition 3. A given E is a *holomorphically barreled space* if, for every U and every F, we have that each collection $\mathfrak{X} \subset \mathcal{H}(U;F)$ is amply bounded if (and always only if) $\mathfrak{X}$ is bounded on every finite dimensional compact subset of U.

Remark 4. Every holomorphically barreled space is a barreled space (see |20|, |17|, |19| for the concept of a barreled space).

Definition 5. A given E is a *holomorphically infrabarreled space* if, for every U and every F, we have that each collection $\mathfrak{X} \subset \mathcal{H}(U;E)$ is amply bounded if (and always only if) $\mathfrak{X}$ is bounded on every compact subset of U.

Remark 6. Every holomorphically infrabarreled is an infrabarreled space (see |20|, |17|, |19| for the concept of an infrabarreled space, also called quasibarreled space).

Definition 7. A given E is a *holomorphically Mackey space* if, for every U and every F, we have that each mapping $f : U \longrightarrow F$ belongs to $\mathcal{H}(U;F)$ if (and always only if) f belongs to $\mathcal{H}(U;wF)$.

Remark 8. Every holomorphically Mackey space is a Mackey space (see |20|, |17|, |19| for the concept of a Mackey space).

Definitions 1,3,5, and 7 were introduced in |30|, |31| and developed in |3|. A variation of Definition 1 was given in |21|.

We recall that a subset K of E is said to be *fast compact* if there is a complex Banach space S which is a vector subspace of E and contains K, such that the inclusion mapping $S \longrightarrow E$ is continuous and K is compact in S, hence compact in E.

Definition 9. A given E is a holomorphically ultrabornological space if, for every U and every F, we have that each mapping f : U $\longrightarrow$ F belongs to $\mathcal{H}(U;F)$ if (and always only if) f is finitely holomorphic, and f is bounded on every fast compact subset of U.

Remark 10. Every holomorphically ultrabornological space is an ultrabornological space (see |20|, |17|, |19| for the concept of an ultrabornological space).

Definition 9 was introduced in |15|. It should be compared with the definition of a holomorphically bornological space given in |21|.

Proposition 11. Let us introduce the following abreviations for properties of a complex locally convex space: hub=holomorphically ultrabornological, hba=holomorphically barreled, hbo=holomorphically bornological, hib=holomorphically infrabarreled, hM=holomorphically Mackey. We have the following implications for the named properties:

$$\begin{array}{ccccc} & \nearrow & \text{hba} & \searrow & \\ \text{hub} & & & & \text{hib} \Rightarrow \text{hM} \\ & \searrow & \text{hbo} & \nearrow & \end{array}$$

Proposition 12. A Fréchet space and a Silva space (that is, a DFS space) are holomorphically ultrabornological.

Question 13. It is known that a DFM space is a holomorphically bornological space |10|. This contains the fact that a Silva space is a holomorphically bornological space (see the preceding Propositions 11 and 12) once a Silva space is a DFM space. It is not known if a DFM space is a holomorphically ultrabornological space. However, it is known that a DFM space is a holomorphically ultrabornological space if (and only if) it is a holomorphically barreled space |6|.

Question 14. If E_1 and E_2 are holomorphically Mackey spaces, is their cartesian product $E=E_1 x E_2$ a holomorphically Mackey space?. In the affirmative case, it follows that any cartesian product of holomorphically Mackey spaces is also a holomorphically Mackey space, as noted in |6|. Remark that $E=\mathbb{C}^{\mathbb{N}} x \mathbb{C}^{(\mathbb{N})}$ is a cartesian product of two holomorphically ultrabornological spaces; but E is not a holomorphically infrabarreled space |31|, |3|. However, it is known that E is a holomorphically Mackey space (this was stated without proof in |31|, and it is proved in |12|). More generally, it is known that, if E is a holomorphically infrabarreled space, then $\mathbb{C}^I x E$ is a holomorphically Mackey space for every set I (see |6|).

Question 15. If a holomorphically bornological space is complete in a suitable sense, must it be holomorphically ultrabornological?. This question is motivated by the remark that, if a bornological space is sequentially complete, then it must be an ultrabornological space. See |21| for a proof that certain quasicomplete holomorphically bornological spaces must be holomorphically barreled.

Question 16. For E to be a holomorphically barreled space it is necessary and sufficient that E be a holomorphically infrabarreled space, and moreover that E has the following Montel property: for every U and every F, we have that each collection $\mathcal{X} \subset \mathcal{H}(U;F)$ is relatively compact for τ_0 if (and always only if) $\mathcal{X}$ is bounded on every finite dimensional compact subset of U, and $\mathcal{X}(x)$ is relatively compact in F for every $x \in U$ (see |3|). On the other hand, for E to be a holomorphically infrabarreled space it is necessary that E be a holomorphically Mackey space, and moreover that E has the following infra-Montel property: For every U and every F, we have that each collection $\mathcal{X} \subset \mathcal{H}(U;F)$ is relatively compact for τ_0 if (and always only if) $\mathcal{X}$ is bounded on every compact subset of U, and $\mathcal{X}(x)$ is relatively compact in F for every $x \in U$ (see |3|). Is this necessary condition also sufficient?.

3. Topology on spaces of holomorphic mappings.

Let E and F be complex locally convex spaces, U be an open nonvoid subset of E, and $\mathcal{H}(U;F)$ be the vector space of all holomorphic mappings of U to F. We may consider three natural topologies $\mathcal{T}_c$, $\mathcal{T}_\omega$ and $\mathcal{T}_\delta$ on $\mathcal{H}(U;F)$. We have $\mathcal{T}_o \subset \mathcal{T}_\omega \subset \mathcal{T}_\delta$. If E is finite dimensional, then $\mathcal{T}_c = \mathcal{T}_\omega = \mathcal{T}_\delta$. The question arises as to when $\mathcal{T}_c = \mathcal{T}_\omega$ or $\mathcal{T}_\omega = \mathcal{T}_\delta$.

Definition 1. The compact-open topology $\mathcal{T}_o$ on $\mathcal{H}(U;F)$ is defined by the family of all seminorms $p_{K\beta}$ as K varies over all compact subsets of U and β varies over all continuous seminorms of F, where

$$p_{K\beta}(f)=\sup\{\beta[f(x)] \ ; \ x\in K\}$$

for all $f\in\mathcal{H}(U;F)$.

Definition 2. A seminorm p on $\mathcal{H}(U;F)$ is said to be ported by a compact subset K of U if there is a continuous seminorm β on F such that, to every neighborhood V of K in U there corresponds a real number $c(V)>0$ for which

$$p(f)\leqslant c(V)\sup\{\beta[f(x)] \ ; \ x\in V\}$$

for all $f\in\mathcal{H}(U;F)$. The ported topology $\mathcal{T}_\omega$ on $\mathcal{H}(U;F)$ is defined by the set of all seminorms on $\mathcal{H}(U;F)$ each of which is ported by some compact subset of U.

Definition 3. If I is a countable cover of U by open subsets of U and β is a continuous seminorm of F, we denote by $\mathcal{H}_{I\beta}(U;F)$ the vector subspace of $\mathcal{H}(U;F)$ of all $f\in\mathcal{H}(U;F)$ such that βf is bounded on every $V\in I$. We use on $\mathcal{H}_{I\beta}(U;F)$ the semimetrizable topology $\mathcal{T}_{I\beta}$ defined by the family of seminorms $p_{V\beta}$ as V varies in I, where

$$p_{V\beta}(f)=\sup\{\beta[f(x)] \ ; \ x\in V\}$$

for all $f\in\mathcal{H}_{I\beta}(U;F)$. We note that we have the union

$$\mathcal{H}(U;F)= \bigcup_I \mathcal{H}_{I\beta}(U;F)$$

for all β . We now define on $\mathcal{H}(U;F)$ the inductive limit topology $\mathcal{T}_{\delta\beta}$ corresponding to this union, namely the largest locally convex topology on $\mathcal{H}(U;F)$ such that each inclusion mapping $\mathcal{H}_{I\beta}(U;F) \longrightarrow \mathcal{H}(U;F)$ is continuous for every I, where β is fixed. Finally, we define on $\mathcal{H}(U;F)$ the <u>limit topology</u> $\tau_\delta = \bigcap_\beta \tau_{\delta\beta}$ as an intersection.

<u>Lemma 4</u>. If F is a Hausdorff space, F$\neq$0, and the topologies τ_0 and τ_ω coincide on $\mathcal{H}(U;F)$, then every bounded subset of F is precompact.

<u>Question 5</u>. Let F be given.

(a) When is it true that the topologies τ_0 and τ_ω coincide on $\mathcal{H}(U;F)$ for every U and every F?.

(b) Is the answer to (a) positive if E is a Fréchet-Montel space?.

(c) When is it true that the topologies τ_ω and τ_δ coincide on $\mathcal{H}(U;F)$ for every U and every F?.

We now indicate positive results in the direction of this Question 5.

The following Propositions 6 and 7 are due to Mujica |23|, |24|. Another related result due to Mujica |25| states that, for a nuclear Fréchet space E, the topologies τ_0 and τ_ω coincide on $\mathcal{H}(U)$ for every polynomially convex open nonvoid subset U of E. Note that none of these three results implies any of the other two.

<u>Proposition 6</u>. Let E be a Fréchet-Schwartz space. Then the topologies τ_0 and τ_ω coincide on $\mathcal{H}(U)$ for every balanced open nonvoid subset U of E.

<u>Proposition 7</u>. Let E be a Fréchet-Schwartz space with the bounded approximation property. Then the topologies τ_0 and τ_ω coincide on $\mathcal{H}(U)$ for every open nonvoid subset U of E.

In Propositions 6 and 7, E is still restricted to being a Fréchet-Schwartz space. The following Propositions 8 and 9 are due to Ansemil and Ponte |1|. They also give another proof of Proposition 6, and state further related results.

<u>Proposition 8</u>. Let E be a Fréchet-Montel space, and U be a balanced open nonvoid subset of E. Then the following conditions

are equivalent:

(a) The topologies τ_0 and τ_ω coincide on $\mathcal{H}(U)$.

(b) The topologies τ_0 and τ_ω coincide on $P(^mE)$ for all $m \in \mathbb{N}$.

Proposition 9. Let E be a Fréchet-Köthe space which is a Fréchet-Montel space. Then the topologies τ_0 and τ_ω coincide on $\mathcal{H}(U)$ for every balanced open nonvoid subset U of E.

Remark 10. There are Fréchet-Köthe spaces that are Fréchet-Montel spaces, but that are not Fréchet-Schwartz spaces (see Köthe |20|, Jarchow |19|). Thus Proposition 9 gives an instance of a Fréchet-Montel space which is not a Fréchet-Schwartz space, such that the topologies τ_0 and τ_ω coincide on $\mathcal{H}(U)$ for every balanced open nonvoid subset U of E, a conclusion which does not follow from Proposition 6. A further such instance is given by Ansemil and Ponte |1| by using a construction due to Floret |13|.

Remark 11. Ansemil and Ponte |1| indicate a relationship between Question 5 (b) and the following open question which dates back to Grothendieck |16|: Is the completion of the projective tensor product of two Fréchet-Montel spaces also a Fréchet-Montel space?. It is known that the completion of the projective tensor product of two Fréchet-Schwartz spaces is also a Fréchet-Schwartz space |16|.

Remark 12. It is known that, if $E=\mathbb{C}^I$ where I is a nonvoid set, then the topologies τ_0 and τ_ω coincide on $\mathcal{H}(U;F)$ for every open nonvoid subset U of E and every F, provided I is countable; and conversely, τ_0 and τ_ω do not coincide on $\mathcal{H}(U;F)$ if F is a Hausdorff space not reduced to the origin, provided I is uncountable (see |4|). Thus it is not reasonable to ask Question 5 (b) for a Montel space E that is not a Fréchet space.

Proposition 13. Assume that every open nonvoid subset U of E has a countable base of compact subsets (that is, a sequence of compact subsets of U such that every compact subset of U is contained in some member of that sequence), and that E is a holomorphically infrabarreled space. Then the topologies τ_0 , τ_ω and τ_δ coincide on $\mathcal{H}(U;F)$ for every U and every F.

Remark 14. A DFM space E is an example of a case satisfying the conditions of Proposition 13. Thus it is a Montel space which is not necessarily metrizable giving an affirmative answer to Question 5 (a). This result had been proved by Barroso-Matos-Nachbin |2| for a DFS space, and it was extended by Dineen |10| to a DFM space.

Remark 15. Question 5 (c) as to when the topologies τ_ω and τ_δ coincide was considered by Dineen |11|.

4. Holomorphic factorization.

Definition 1. Let E, E_o and F be complex locally convex spaces, $\pi_o : E \longrightarrow E_o$ be a continuous linear mapping, U be an open nonvoid subset of E, and $f \in \mathcal{H}(U;F)$. We say that f factors holomorphically through π_o if there is a cover $\mathcal{C}$ of U by open nonvoid subsets of U such that, to every $V \in \mathcal{C}$ there corresponds an open nonvoid subset W of E_o with $\pi_o(V) \subset W$, and to every $V \in \mathcal{C}$ there corresponds $g \in \mathcal{H}(W;F)$ satisfying $f = g\pi_o$ on V.

Convention 2. Let $\pi_i : E \longrightarrow E_i$ be a continuous linear mapping between the complex locally convex spaces E and E_i $(i \in I)$, where I is a nonvoid set, such that we have the projective (also called inverse) limit representation $E = \varprojlim_{i \in I} E_i$ meaning that the topology given on E is the smallest topology on E for which every π_i $(i \in I)$ is continuous.

Definition 3. Following Convention 2, we say that holomorphic factorization holds for the given projective limit representation when every locally bounded $f \in \mathcal{H}(U;F)$ factors holomorphically through π_i for some $i \in I$, for every connected open nonvoid subset U of E and every complex locally convex space F.

Definition 4. Following Convention 2, we say that $V \subset E$ is uniformly open in the given projective limit representation when there are $i \in I$ and a open subset $W_i \subset E_i$ such that $V = \pi_i^{-1}(W_i)$. The definition of a projective limit representation of E means that the uniformly open subsets of E in that projective limit representation form a subbase of all open subsets of E. We say that the projective limit representation of E is basic when all uniformly open subsets of E in that projective limit representa tion form a base of all open subsets of E.

Proposition 5. In order that holomorphic factorization should hold for a projective limit representation it is necessary that it be basic.

Definition 6. Following Convention 2, we say that the projective limit representation is open when all $\pi_i : E \to E_i$ $(i \in I)$ are open surjective mappings.

Proposition 7. Holomorphic factorization holds for every open basic projective limit representation.

Proposition 8. Let the complex locally convex space E be given. The following conditions are equivalent:

(1) Holomorphic factorization holds for the projective limit representation $E=\varprojlim_{\alpha \in CS(E)} E_\alpha$.

(2) Holomorphic factorization holds for the projective limit representation $E=\varprojlim_{\alpha \in CS(E)} E/\alpha$.

(3) Holomorphic factorization holds for some projective limit representation $E=\varprojlim_{i \in I} E_i$ with complex seminormed spaces E_i $(i \in I)$.

(4) Holomorphic factorization holds for all basic projective limit representations $E=\varprojlim_{i \in I} E_i$ with complex locally convex spaces E_i and $\pi_i(E)=E_i$ $(i \in I)$.

Definition 9. We say that holomorphic factorization holds for a given complex locally convex space E when it holds for the standard projective limit representations $E=\varprojlim_{\alpha \in CS(E)} E_\alpha$ or equivalently $E=\varprojlim_{\alpha \in CS(E)} E/\alpha$, or equivalently for the remaining two situations in Proposition 8.

Example 10. We shall give an example of a complex locally convex space E for which holomorphic factorization does not hold. Let $E=\mathcal{H}(\mathbb{C};\mathbb{C})$ have the compact-open topology. Fix $a \in \mathbb{C}$. Then $f \in \mathcal{H}(E;\mathbb{C})$ defined by $f(u)=u[u(a)]$ for $u \in E$ does not factor holomorphically in the sense of Definition 3 if we consider the standard projective limit representations as in Definition 9.

Definition 11. We say that the openness condition holds for a complex locally convex space E when the set COS(E), of all $\alpha\in CS(E)$ such that the quotient mapping $E\longrightarrow E/\alpha$ is open, defines the topology of E and is directed.

Remark 12. Valdivia |37| has shown that COS(E) is not necessarily directed when it defines the topology of E.

Proposition 13. Holomorphic factorization holds for every complex locally convex space satisfying the openness condition.

Note that Proposition 13 follows from Proposition 7.

Proposition 14. Holomorphic factorization holds for every complex locally convex space E satisfying the following conditions:

(1) For every sequence V_n $(n\in\mathbb{N})$ of neighborhoods of 0 in E, there are $r_n>0$ $(n\in\mathbb{N})$ such that

$$V=\bigcap_{n\in\mathbb{N}} r_n V_n$$

still is a neighborhood of 0 in E.

(2) From every open cover of every open subset U of E we can extract a countable subcover of U.

For examples of complex locally convex spaces for which holomorphic factorization holds in view of Propositions 13 or 14 we refer to |33|.

Problem 15. Do we change definition 3 if we restrict F to being an arbitrary complex normed space (instead of being any complex locally convex space) in which case every $f\in\mathcal{H}(U;F)$ is locally bounded?.

Problem 16. Find necessary and/or sufficient conditions for holomorphic factorization to hold for a given projective limit representation, in particular for a given complex locally convex space. If E is a metrizable complex locally convex space for which holomorphic factorization holds, does the openness condition hold for E (that is, the converse to Proposition 13 then true)?.

Problem 17. Consider projective limit representations

(1) $E=\varprojlim_{i\in I} E_i$

(2) $E_i = \lim\limits_{\longleftarrow} {}_{j \in J_i} E_{ij}$ $(i \in I)$

with respect to the families of continuous linear mappings

$\pi_i : E \longrightarrow E_i$ $(i \in I)$ and

$\pi_{ij}: E_i \longrightarrow E_{ij}$ $(i \in I, j \in J_i)$. Introduce the composition of these projective limit representations

(3) $E = \lim\limits_{\longleftarrow} {}_{(i,j) \in I \times J_i} E_{ij}$

which is a projective limite representation with respect to the family of continuous linear mappings $\pi_{ij}\pi_i : E \longrightarrow E_{ij}$ $(i \in I, j \in J_i)$. Do we have transitivity of holomorphic factorization, in the sense that holomorphic factorization holds for that composition (3) if it holds for all given projective limit representations (1) and (2)?. Note that the answer is affirmative if every projective limit representation (2) is basic and open.

5. Holomorphic continuation.

Definition 1. Let F be a given Hausdorff complex locally convex space. We say that F is confined if, for every complex locally convex space E, we have that $f^{-1}(F)=U$ (or equivalently $f(U) \subset F$) whenever U is a connected open nonvoid subset of E and $f \in \mathcal{H}(U;\hat{F})$ is such that $f^{-1}(F)$ has a nonvoid interior, where $\hat{F}$ is a completion of F. To check this requirement on F, it suffices to take U as the open disc of center 0 and radius 1 in $E=\mathbb{C}$, to assume that $f \in \mathcal{H}(U;\hat{F})$ and that 0 is interior to $f^{-1}(F)$, and to conclude that always $f^{-1}(F)=U$, that is, to conclude that always $f(U) \subset F$ if there is a neighborhood V of 0 in U such that $f(V) \subset F$.

Lemma 2. If F is sequentially complete, then F is confined. Moreover wF is confined if and only if F is confined.

We recall that, if E and F are complex locally convex spaces and U is an open nonvoid subset of E, we introduce the vector space H(U;F) formed by every $f : U \rightarrow F$ such that $f \in \mathcal{H}(U;\hat{F})$ when f is considered as having its values in a completion $\hat{F}$ of F. It is clear that H(U;F) is independent of the choice of $\hat{F}$, that $\mathcal{H}(U;F) \subset H(U;F)$, and that we have $\mathcal{H}(U;F)=H(U;F)$ if F is complete.

Definition 3. Let U,V and W be connected open nonvoid subsets of a complex locally convex spaces E with $W \subset U \cap V$. If F is a complex locally convex space, we say that V is a holomorphic F-valued continuation of U via W when for every $f \in H(U;F)$ there exists $g \in H(V;F)$ such that f=g on W.

Definition 4. Let E be a given complex locally convex space. We say that weak holomorphy plus slight holomorphy imply holomorphy on E if, for every complex locally convex space F, we have that $f \in \mathcal{H}(V;F)$ whenever V and W are connected open nonvoid subsets of E with $W \subset V$ so that $f \in \mathcal{H}(V;wF)$ and $f|W \in \mathcal{H}(W;F)$.

Remark 5. Weak holomorphy plus slight holomorphy imply holomorphy on E in two noteworthy cases:

(1) E is a holomorphically Mackey space, because then weak holomorphy alone implies holomorphy on E.

(2) E is a Zorn space in the sense that, for every complex locally convex space F, we have that $f \in \mathcal{H}(V;F)$ whenever V is a connected open nonvoid subset of E and $f:V \longrightarrow F$ is finitely holomorphic such that there is some open nonvoid subset $W \subset V$ for which $f|W \in \mathcal{H}(W;F)$.

Proposition 6. Let E and F be given complex locally convex spaces. Assume that weak holomorphy plus slight holomorphy imply holomorphy on E, that F is confined and $F \neq 0$. Let U,V and W be connected open nonvoid subsets of E with $W \subset U \cap V$. Then V is a holomorphic F-valued continuation of U via W if and only if V is a holomorphic $\mathbb{C}$-valued continuation of U via W.

For details, see |29|.

Question 7. Is it true that, for any complex locally convex space E, weak holomorphy plus slight holomorphy always imply holomorphy on E?.

Question 8. Does Proposition 6 hold in general for an arbitrary E without assuming that weak holomorphy plus slight holomorphy imply holomorphy on E?.

In view of Proposition 6, an affirmative answer to Question 7 implies an affirmative answer to Question 8.

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