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LIFTING UNITS IN SELF-INJECTIVE RINGS AND AN INDEX THEORY FOR RICKART C*-ALGEBRAS

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In this paper we study the following question: If R is a right self-injective ring and I is an ideal of R, when can the units of R/I be lifted to units of R?.

We answer this question in terms of $K_o(I)$. For a purely-infinite regular right self-injective ring R we obtain an isomorphism between $K_1(R/I)$ and $K_o(I)$ which can be viewed as an analogue of the index map for Fredhlom operators. Moreover $K_o(I)$ is realized as a group of continuous functions.

By giving a purely algebraic description of the connecting map $K_1(A/I) \longrightarrow K_o(I)$ in the case where A is a Rickart C*-algebra, we are able to develop an index theory for Rickart C*-algebras. This extends Breuer's theory for W*-algebras cf.|4||5|.

REFERENCES

|1| P. Ara and P. Menal, On regular rings with involution. Arch. Math. Vol. 42, 126-130 (1984).

|2| H. Bass, Algebraic K-Theory, (Benjamin, New York, 1968).

|3| S.K. Berberian, Baer *-rings, Grundlehren Band 195 (Springer, Berlin and New York, 1972).

|4| M. Breuer, Fredholm theories in von-Neumann algebras I. Math. Ann. 178, 243-254 (1968).

|5| _____, Fredholm theories in von-Neumann algebras II. Math. Ann. 180, 313-325 (1969).

|6| R.G. Douglas, Banach algebra techniques in operator theory. Academic Press. New York and London. (1972).

|7| K.R. Goodearl, von Neumann regular rings. Pitman. London. (1979).

|8| _____, Directly finite aleph-nought-continuous regular rings. Pacific J. of Math. Vol. 100, nº 1,105-122,(1982).

|9| _____, Notes on Real and Complex C*-algebras. Shiva Math. Series 5, (1982).

|10| K.R. Goodearl and P. Menal, Preprint.

|11| D. Handelman, finite Rickart C*-algebras and their properties. Studies in Analysis Adv. in Math. Suppl. Studies. Vol. 4, 171-196, (1979).

|12| P. Menal and J. Moncasi, On regular rings with stable range 2. J. Pure and Appl. Alg. 24,25-40, (1982).

|13| _____, K_1 of von Neumann regular rings. J. Pure and Appl. Alg. 33, 295-312, (1984).

|14| J. Milnor, Introduction to algebraic K-Theory. Ann. of Math. Studies. 72, Princeton, (1971).

|15| C. Olsen, Index theory in von Neumann algebras. Memoirs of the Amer. Math. Soc. 294, (1984).

|16| M.A. Rieffel, Dimension and stable rank in the K-theory of C*-algebras. Proc. London Math. Soc.(3), 46, 301-333, (1983).

|17| L.N. Vaserstein, stable rank of rings and dimensionality of topological spaces. Funct. Anal. Appl. 5, 17-27 (translation 102-110)(1971).

|18| _____, Bass's first stable range condition. J. Pure and Appl. Alg. 34, 319-330, (1984).

|19| R.C. Walker, The Stone Cech compactification. Ergebnisse der Math. Band 83, Springer, Berlin and New York, (1974).