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# LIE ALGEBRAS ALL OF WHOSE SUBALGEBRAS ARE SUPERSOLVABLE*

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A Lie algebra L is said to be minimal non supersolvable if all its subalgebras, except L itself, are supersolvable. The main results in this paper are summarized in:

THEOREM: For a solvable Lie algebra L, L is minimal non supersolvable if and only if one of the following conditions hold:

a) $L^2$ is the nilpotent radical of L, the Frattini ideal of L, $\Psi(L)$, equals $(L^2)^2$ and $L/\Psi(L)$ contains a basis $\{\bar{e}_1,\bar{e}_2,\dots,\bar{e}_r,\bar{x}\}$ (where $\bar{z} = z+\psi(L)$ and $r>1$) such that $[\bar{e}_i,\bar{x}] = \bar{e}_{i+1}$ $(i=1,\dots,r-1)$, $[\bar{e}_r,\bar{x}] = c_0\bar{e}_1 + \dots + c_{r-1}\bar{e}_r$, the polynomial $X^r - c_{r-1}X^{r-1} - \dots - c_1X - c_0$ is irreducible and $\operatorname{ad} x|_{\psi(L)}$ is split.

b) The characteristic of the ground field F is $p>0$, $F=\{t^p - t : t\in F\}$, every chief factor of L below $\psi(L)$ is one dimensional and $L/\psi(L)$ contains a basis $\{e_0,\dots,e_{p-1},x,y\}$ with $[e_i,y] = (\alpha+i)e_i$ ($\alpha$ a fixed scalar in F), $[e_i,x] = e_{i+1}$ (mod. p), $[x,y] = x$ and $[e_i,e_j] = 0$.

c) The characteristic of the ground field F is $p>0$, F is perfect when $p=2$, every chief factor below $\psi(L)$ is one dimensional and $L/\psi(L)$ contains a basis $\{e_0,\dots,e_{p-1},x,y,z\}$ with $[e_i,z] = e_i$ $\forall i$, $[e_i,y] = ie_{i-1}$ $\forall i$, where $e_{-1} = 0$, $[e_i,x] = e_{i+1}$ $\forall i$, where $e_p = 0$, $[x,y] = z$ and $[x,z] = [y,z] = [e_i,e_j] = 0$.

This result provides examples of minimal dimension of solvable Lie algebras over fields of characteristic p in which the derived sub-

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algebra is not nilpotent (which is not possible in the characteristic zero case).

As a consequence of the last Theorem we get:

COROLLARY: Let L be a solvable Lie algebra over an algebraically closed field F. Then L is elementary (that is, the Frattini subalgebra of every subalgebra of L is trivial) if and only if L has got a basis $\{b_1, \ldots, b_m, c_1, \ldots, c_n\}$ with $[b_i, c_j] = \lambda_{ij} b_i$, where $\lambda_{ij} \in F$, and the other products are zero.

## REFERENCES

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