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MANIFOLDS AS BRANCHED COVERS OF SPHERES

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Manifolds as branched covers of spheres

by

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by R. Piergallini

This notes are based on lectures I gave at the Departamento de Matematicas Fundamentales of U.N.E.D. at Madrid in February 1989.

I would like to thank all the members of the department and specially Antonio Costa, for their invitation to give the lectures and their hospitality.

My intention was only to expose some of the basic ideas and recent results on representing manifolds as branched covers of spheres, without any ambition of being exhaustive. Details and applications can be found in the references.

1. Branched coverings

By a *branched covering* we mean a non-degenerate PL map $p : M \longrightarrow N$ between PL n-manifolds which is a finite ordinary covering over the complement of a codimension 2 subpolyhedron of N.

The *singular set* of p is the subpolyhedron $S_p \subset M$ at which p fails to be a local homeomorphism, the *branching set* of p is the subpolyhedron $B_p = f(S_p) \subset N$, and the *pseudo-singular set* of p is the subpolyhedron $S_p' = \mathrm{Cl}\,(p^{-1}(B_p) - S_p) \subset M$. Finally, we put $L_p = p^{-1}(B_p) = S_p \cup S_p'$.

The following facts are well-known:

a) The branched covering p is uniquely determined by the ordinary covering $c_p : M - L_p \longrightarrow N - B_p$ induced by restriction. The monodromy $\omega_p : \pi_1(N - B_p) \longrightarrow \Sigma_{d(p)}$ of c_p, where $\Sigma_{d(p)}$ is the symmetric group of degree $d(p) = degree$ of p and the base-point is understood, is called *monodromy* of p .

b) The manifold M can be constructed from N , in the following way (cf. figure 1): 1) spit N along a $(n-1)$-subpolyhedron Q , such that $B_p \subset Q$ and ω_p is trivial on $\pi_1(N-Q)$; 2) take d copies of N split along Q ; 3) glue them together according to the monodromy ω_p .

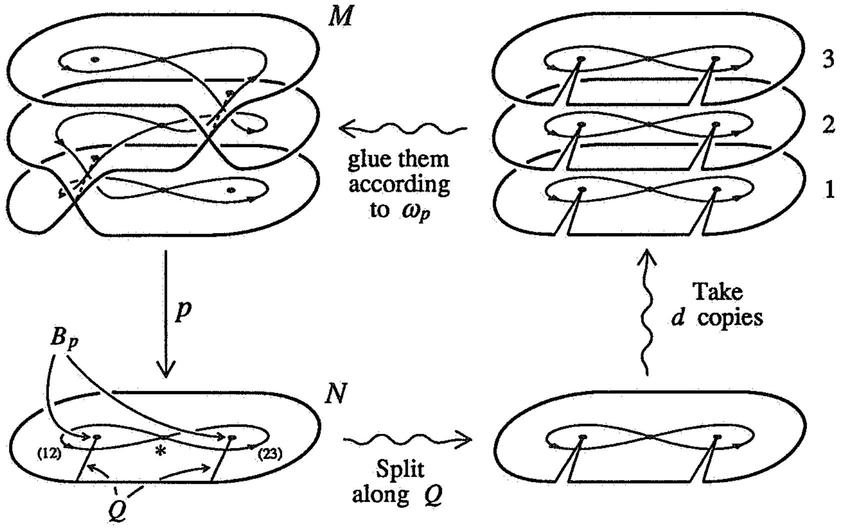

Figure 1.

c) L_p and B_p are homogeneously $(n-2)$-dimensional and meet the boundaries in homogeneously $(n-3)$-dimensional subpolyhedra, so that the restriction of p to the boundary is again a branched covering.

d) If B_p is a locally flat submanifold of N, then S_p and S_p' are locally flat disjoint submanifolds of M and the map $s : L_p \longrightarrow B_p$ induced by restriction is an ordinary covering; moreover, near to any point $x \in S_p$, p looks like the complex map $z \longrightarrow z^{d(p,x)}$ crossed by the identity of R^{n-2} , where $d(p,x) \geq 2$ is the *branching index* of p at x (cf. figure 2).

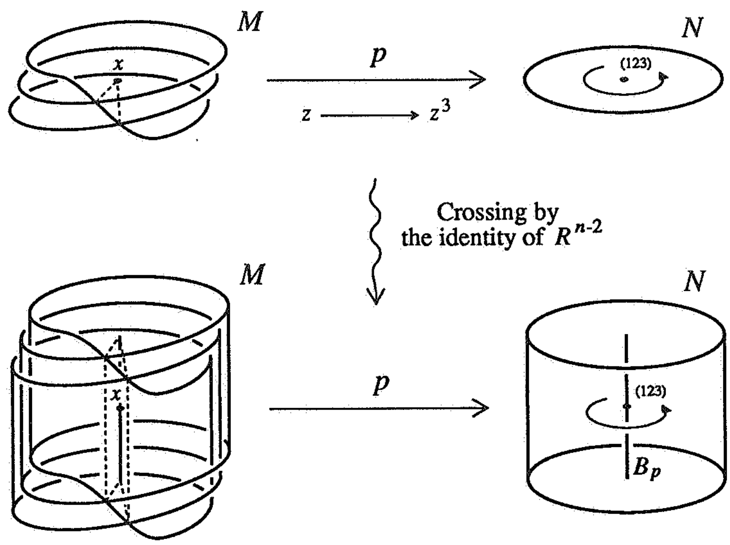

Figure 2.

By property *a*), a branched covering $p : M \longrightarrow N$ can be completely described in terms of its branching set $B = B_p \subset N$ and its monodromy $\omega = \omega_p$. Then, it makes sense to write $p = p_{B,\omega}$ and $M = M(B, \omega)$.

Another branched covering $p' : M' \longrightarrow N'$ is said to be *equivalent* to p iff there exist two PL homeomorphisms $h : N \longrightarrow N'$ and $k : M \longrightarrow M'$ making the following diagram commutative.

$$\begin{array}{ccc} M & \xrightarrow{\ k\ } & M' \\ {\scriptstyle p}\downarrow & & \downarrow{\scriptstyle p'} \\ N & \xrightarrow[\ h\ ]{} & N' \end{array}$$

Again by *a*) and standard facts about ordinary coverings, we have that: $p_{B,\omega}$ and $p_{B',\omega'}$ are equivalent iff they have the same degree d and there exists a PL homeomorphism $h : N \longrightarrow N'$, such that $h(B) = B'$ and $\omega' h_* = \omega$ up to conjugation in Σ_d, where h_* is the homomorphism induced by the suitable restriction of h. In this case, also the pairs (B, ω) and (B', ω') are called *equivalent.*

We note that not every pair (B, ω), where $B \subset N$ is a subpolyhedron satisfying property *c*) and $\omega : \pi_1(N - B) \longrightarrow \Sigma_d$ is a homomorphism, does correspond to a branched covering. This because the covering space $M(B, \omega)$ constructed by the procedure sketched in *b*) could not be a manifold. However, if B is a locally flat submanifold of N, then $M(B, \omega)$ is a manifold, for any ω which sends the meridians of B into non-trivial permutations.

2. Local models

Given a branched covering $p : M \longrightarrow N$ and a point $y \in N$, we call *local model of* p *at* y the restriction $p_y : D \longrightarrow C_y$ of p over a "sufficiently small" n-cell neighborhood of y. More precisely, if K_M and K_N are two triangulations which make p simplicial, we can take $C_y = \mathrm{St}(y, \beta K_N)$ where β denotes the barycentric subdivision.

The following facts can be easily proved:

e) p_y is uniquely determined (up to equivalence) by p and y.

f) D is the disjoint union of finitely many n-cells, in fact it is a regular neighborhood of the finite set $f^{-1}(y)$. Moreover, for each $x \in f^{-1}(y)$, the restriction $p_{x,y} : D_x \longrightarrow C_y$ where D_x is the component of D containing x, is equivalent to the cone of a branched covering of S^{n-1} onto itself.

We define the *local degree* $d(p,x)$ of p at x as the degree of the branched covering $p_{x,y}$. Of course the local degree is 1 out of S_p and coincides with the branching index at the locally flat $(n-2)$-manifold points of S_p.

g) Any branched covering $p : D \longrightarrow C$ satisfying the properties stated in *f*) for p_y is a local model of itself at the vertex of $C \cong$ Cone of S^{n-1}, so we call such a branched covering an (*abstract*) *local model*.

In dimension 2, B_p is a discrete set, hence singular local models $p : D \longrightarrow B^2$ are described (up to equivalence) by: $B_p = \{\text{origin}\}$ and $\omega_p(\alpha) = \sigma$, where α is a generator of $B^2 - B_p \cong S^1$ and σ is any non-trivial permutation in Σ_d. Then, $p^{-1}(\text{origin})$ has one point for each orbit of σ, and the corresponding component of D is mapped by p onto B^2 as $z \longrightarrow z^n$, where n is the order of the orbit. (cf. figure 3)

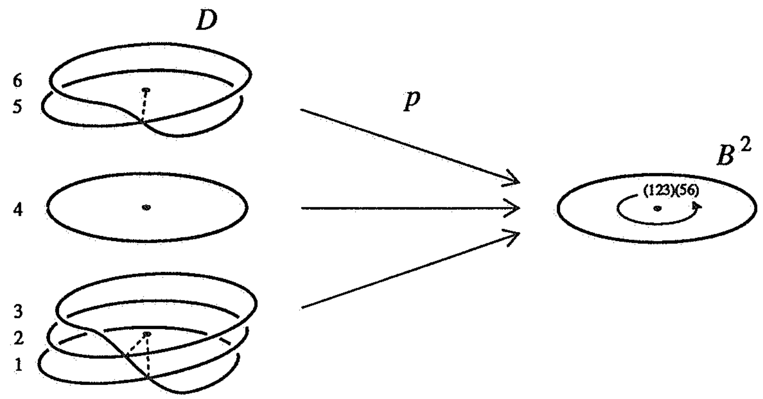

Figure 3.

By property *d*), the n-dimensional singular local models $p : D \longrightarrow B^n$ with locally flat branching set, are given by crossing the 2-dimensional ones by the identity of R^{n-2} (cf. figure 4).

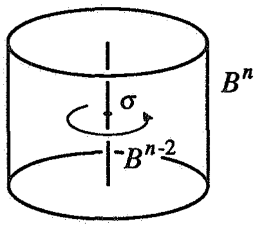

Figure 4.

In dimension 3, a local model with singular branching set can be obtained as the cone of the covering of S^2 given by gluing the two coverings of B^2 depicted in figures 1 and 2 (cf. figure 5). We can

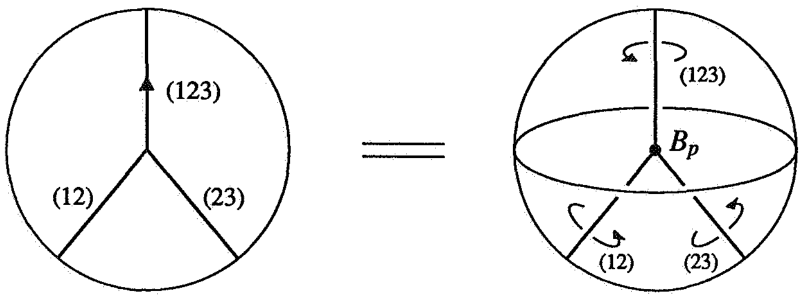

Figure 5.

represent such a covering, by labelling each oriented bridge of a diagram of the branching set, with the monodromy of the corresponding meridian (of

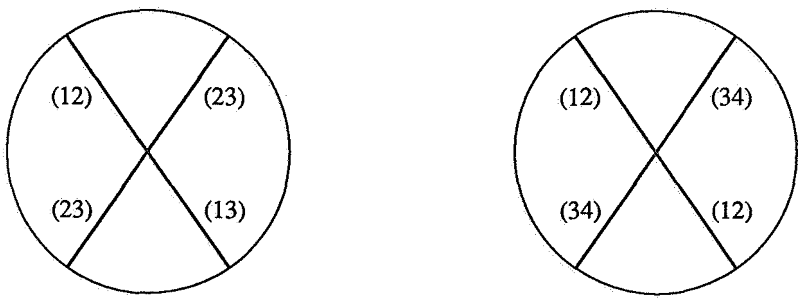

Figure 6.

course, the orientation can be omitted for transpositions).

The diagrams in figure 6 represent other examples of 3-dimensional local models with singular branching set.

Crossing by an interval the examples of figures 5 and 6, we get 4-dimensional local models, whose branching sets are singular along a line.

Branching sets with isolated singularities, can be obtained by making the cone of non-cyclic branched covers of S^3 by copies of itself. In figure 7 are represented 4-dimensional local models, whose branching set has respectively a node-like singularity (briefly a *node*) and a cusp-like singularity (briefly a *cusp*).

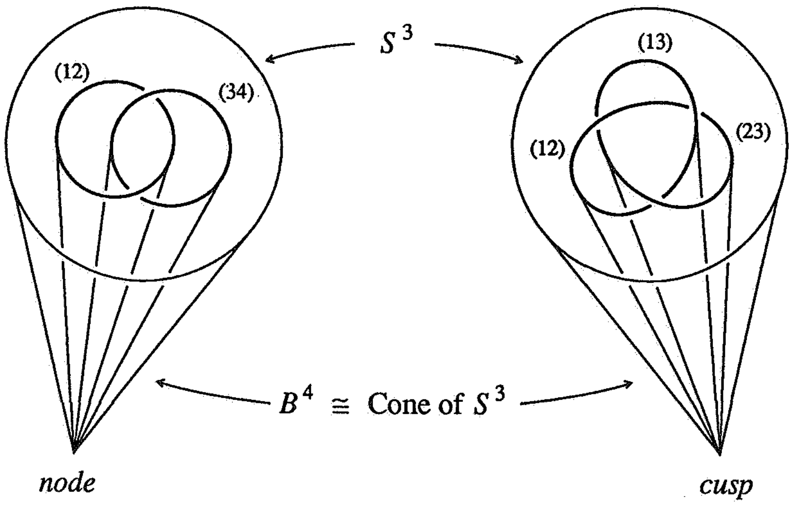

Figure 7.

3. The Alexander's theorem

The first result in the direction of representing manifolds as branched covers of spheres is the following theorem proved by J. W. Alexander [1] in 1920.

Theorem. Any closed orientable n-dimensional PL manifold M is a cover of S^n, branched over the $(n-2)$-skeleton of the standard n-simplex $\Delta^n \subset E^n \subset E^n \cup \{\infty\} \cong S^n$.

Sketch of proof. Let T be any triangulation of M, then the barycentric subdivision βT of T has a black and white chess-board coloration for its n-simplices (that is any two n-simplices having a common $(n-1)$-face have different colours). Then, the claimed covering can be easily obtained by sending the black n-simplices of βT onto the standard n-simplex $\Delta^n \subset E^n \subset E^n \subset \{\infty\} \cong S^n$ and the white ones onto $\mathrm{Cl}\,(S^n - \Delta^n)$.

The following problems naturally arise from the Alexander's theorem, in order to make branched covers of spheres an effective tool for representing and studying manifolds:

1) *Reducing degree.* Bernstein and Edmonds [4] established that at least n sheets are needed, in representing all the closed orientable PL n-manifolds. But, in general, the problem whether n sheets are sufficient or not in dimension n, is still open. For example, the n-dimensional torus $T^n \cong S^1 \times \ldots \times S^1$ is a n-fold branched cover of S^n (cf. [36]), but there is no branched covering of T^n onto S^n with degree $< n$ (cf. [4]).

2) *Reducing local degrees.* The best hope would be to reduce all local degrees at singular points to 2. However, we could be satisfied with the weaker condition that all the branching indices at locally flat points of the singular set are equal to 2 (we call *simple* a branched covering with this property). Of course, this last condition coincides with the previous one, when the singular set is locally flat.

3) *Reducing branching set singularities.* Of course, in this case the

best hope would be for locally flat branching sets. But, in general, this can not be obtained. For example, the Kummer complex surface K^4 can not be represented as a cover of S^4 branched over a locally flat surface (cf. corollary 3.5 of [4]).

4) *Recognition problem.* We would like conditions on two pairs (B, ω) and (B', ω') with $B, B' \subset S^n$, necessary and sufficient for having $M(B, \omega) \cong M(B', \omega')$. Of course, if (B, ω) and (B', ω') are equivalent, then the corresponding manifolds are homeomorphic, in fact in this case we have much more, namely the to coverings $p_{B,\omega}$ and $p_{B',\omega'}$ are equivalent.

We note that, by looking at the boundaries of local models, a solution of the recognition problem would give us also conditions on the pair (B, ω) at the points where B is not locally flat, in order to have that $M(B, \omega)$ is a manifold.

4. Representing surfaces

In the 2-dimensional case, there is no problem. In fact, any closed orientable surface F_g of genus g, can be represented as a 2-fold simple branched cover of S^2, as shown in figure 8. This representation is unique up to equivalence, in fact: any two simple coverings of S^2 by F_g of the same degree are equivalent (cf. [5]).

Moreover, for any branched covering $p = p_{B,\omega} : F_g \longrightarrow S^2$, the Hurwitz formula gives $\chi(F_g) = d(p)\chi(S^2) - \Sigma\,(d(p,x) - 1)$, where the summation extends over all $x \in S_p$. By observing that $\Sigma_{p(x)=b}\, d(p,x)$ coincides with the number $o(\omega_b)$ of orbits of the monodromy around $b \in B$, we get the formula $g = 1 - d + |B|\,d/2 - \Sigma_{b \in B}\, o(\omega_b)/2$, which enables us to classify $F_g = M(B, \omega)$ in terms of B and ω.

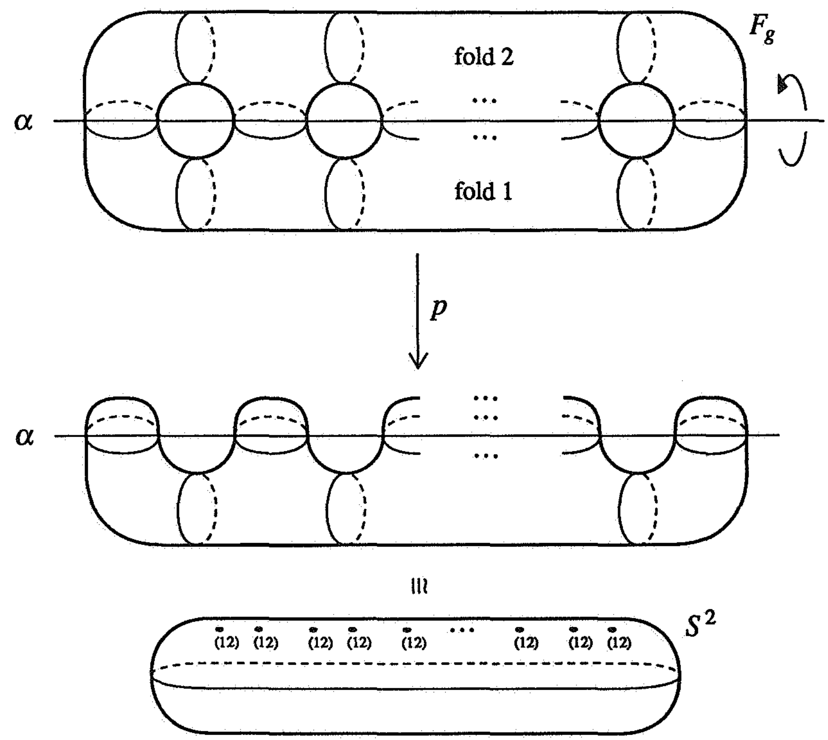

Figure 8.

5. Representing 3-manifolds

In dimension 3, the following theorem, proved independently by H. M. Hilden [26] , U. Hirsch [38] and J. M. Montesinos [42], completely answer the first three problems of section 3.

Theorem. Any closed orientable 3-manifold M is a 3-fold simple cover of S^3 branched over a knot.

Sketch of proof. First of all, we consider the 3-fold simple branched covering of B^3 by H_g = handlebody of genus g , shown in figure 9.

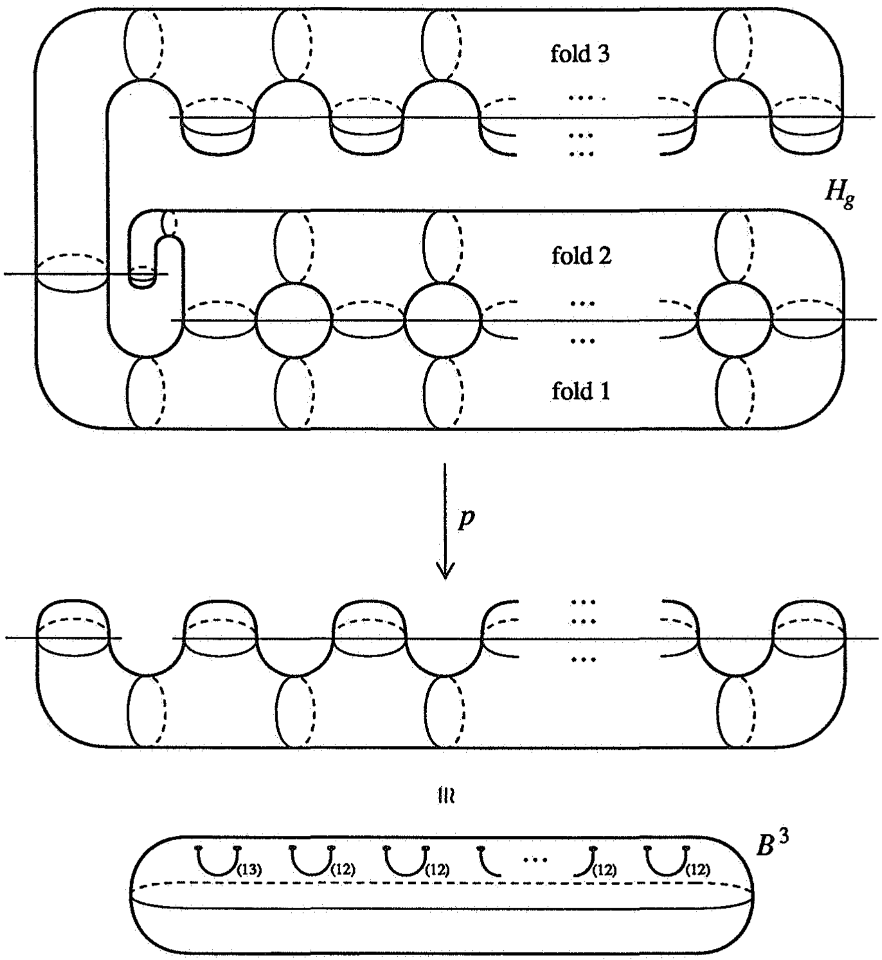

Figure 9.

The restriction of this branched covering to $F_g = \mathrm{Bd}\, H_g$ has the following property: any homeomorphism $k : F_g \longrightarrow F_g$ is (up to isotopy) the lifting of a homeomorphism $h : S^2 \longrightarrow S^2$ (cf. [5]). Then, by considering a Heegaard splitting of M , we get a 3-fold simple branched covering $M \cong H_g \cup_k H_g \longrightarrow B^3 \cup_h B^3 \cong S^3$ (cf. figure 10).

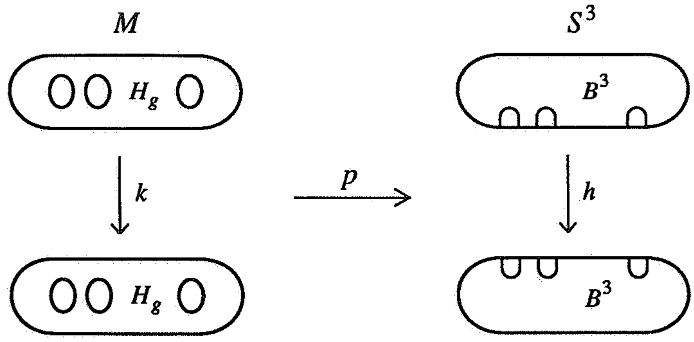

Figure 10.

The branching set of this last covering is a link in S^3 , which can be represented by a plat, labelled as shown in figure 11. Finally, such a link can be made into a knot, by using moves of the type I described in figure 12.

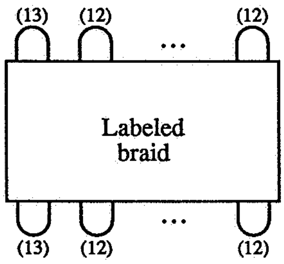

Figure 11.

By this theorem, any closed orientable 3-manifold can be represented as a cover $M(L, \omega)$ of S^3 , where we can think of (L, ω) as a *labelled link* (in fact a labelled knot), that is a diagram of the link $L \subset S^3$, whose bridges are labelled by transpositions (cf. section 2).

The recognition problem for 3-manifolds was long ago posed by Montesinos (cf. [41]), more or less in the following way: find a set of moves relating any two labelled links representing the same 3-manifold.

The move I of figure 12 was considered a possible answer to this

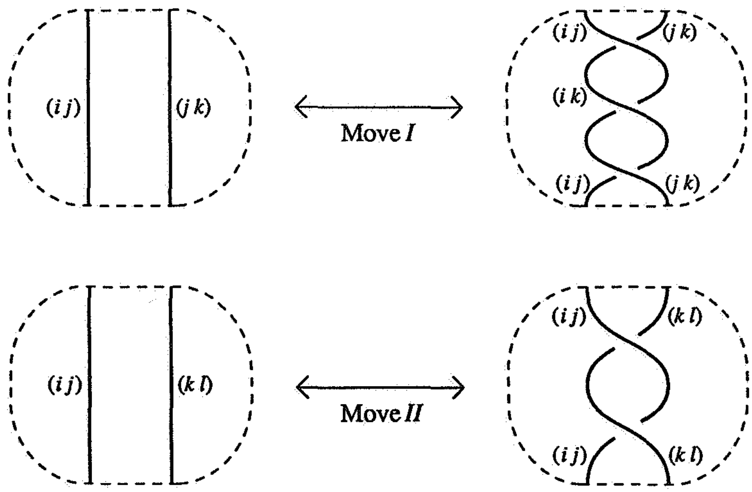

Figure 12.

problem for a long time (cf. [47]). A complete set of moves was recently given in [53], but the new moves was not completely satisfactory because of their complexity and non-local character.

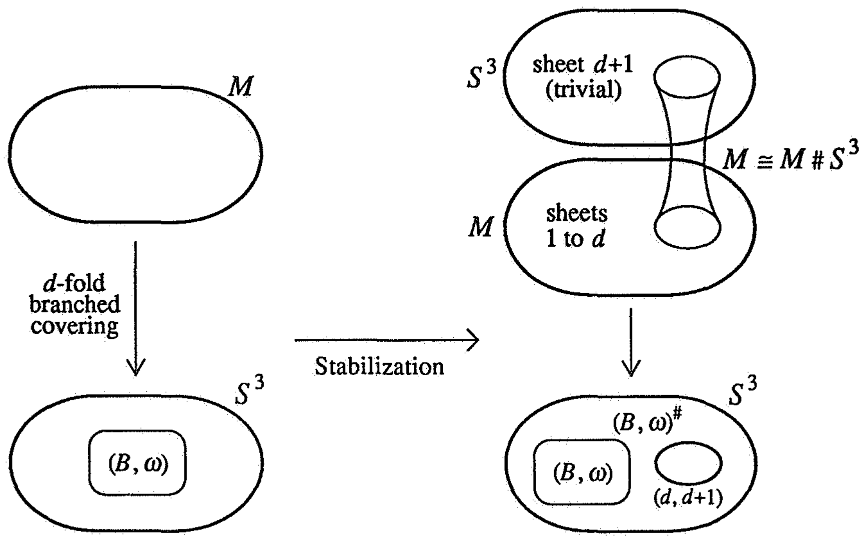

Figure 13.

Finally, in [54] is proved that such inconvenient can be avoided by stabilizing the coverings as shown in figure 13 (i.e. the covering in figure 8 stabilizes to the one in figure 9). Namely, we have the following theorem.

Theorem. Let (L, ω) and (L', ω') labelled links representing simple 3-fold branched covers of S^3. Then $M(L, \omega) \cong M(L', \omega')$ iff the stabilizations $(L, \omega)^{\#}$ and $(L', \omega')^{\#}$ are related (up to equivalence) by a finite sequence of moves *I* and *II*.

Sketch of proof. First of all, the moves *I* and *II* do not change the covering manifolds, since the 3-cells that they involve are covered by disjoint unions of 3-cells.

On the other hand, let (L, ω) and (L', ω') be two labelled links representing the same manifold M as simple 3-fold cover of S^3. We can assume up to isotopy that they are plats as in figure 11, in such a way that they induce two Heegaard splittings of M. Now, the move *I* allows us to realize a stable equivalence between these splittings, in order to get two new labelled links inducing the same splitting homeomorphism (cf. section 2 of [53]). Finally, the stabilizations of these new labelled links, can be related by showing that, in presence of a fourth trivial sheet, moves *I* and *II* together generate all the braids representing the identity homeomorphism of F_g (cf. section 3 of [53], and [54]).

The following question remains still open: are the moves *I* and *II* sufficient in order to relate (up to stabilization and equivalence) any two labelled links representing the same 3-manifold as simple 4-fold (n-fold) branched cover of S^3?

6. Representing 4-manifolds

By using the recognition theorem for branched coverings of S^3, in [54] was proved the following representation theorem for 4-manifolds, which gives the best possible answer to the first three problems stated in section 3.

Theorem. Any closed orientable PL 4-manifold is a simple 4-fold cover of S^4 branched over a transversally immersed surface.

Sketch of proof. Let M a closed orientable PL 4-manifold. By [44] and using handlebody decomposition, we can write $M = M_0 \cup_{\mathrm{Bd}} M_1$ where both M_0 and M_1 are simple 3-fold covers of B^4 branched over locally flat surfaces. Looking at the boundaries, we have two simple 3-fold branched coverings of S^3 by the same 3-manifold $\mathrm{Bd}\, M_0 = \mathrm{Bd}\, M_1$. By

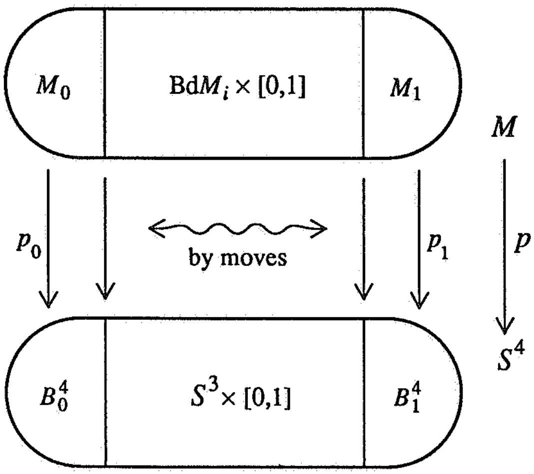

Figure 14.

stabilizing these two coverings and relating them by moves, we get a simple 4-fold branched covering $p : M \longrightarrow S^4$ (cf. figure 14).

The branching set of p is a surface $F \subset S^4$, whose only singularities are node and cusp points coming from the moves I and II as suggested in figure 15.

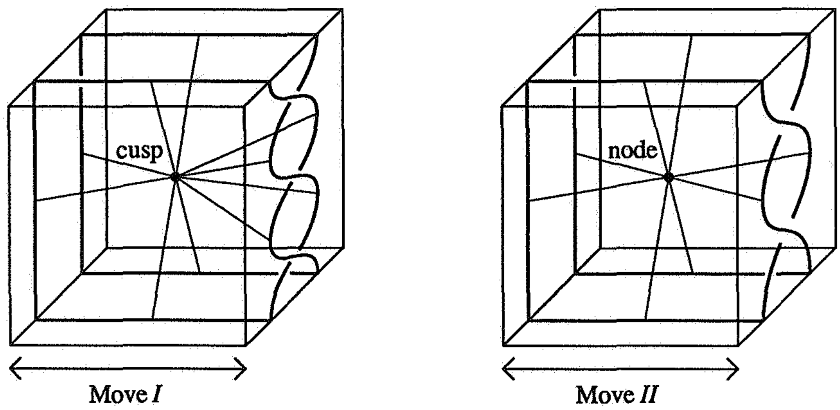

Figure 15.

Finally, by using branched covering cobordism (cf. [29]), we can remove all the cusps of F, in order to make it transversally immersed.

We observe that we cannot require the orientability of the branching surface F in the theorem. In fact, denoting by χ the Euler-Poincaré characteristic, we have $\chi(M) = 8 - \chi(F)$. Then F must be non-orientable if $\chi(M)$ is odd.

We conclude by remarking that, as far as we know, the recognition problem in dimension 4, as well as all the four problems stated in section 3 in higher dimensions, remains still open.

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These notes collect some of the talks given in the Seminario del Departamento de Matemáticas Fundamentales de la U.N.E.D. in Madrid. Up to now the following titles have appeared:

1 **Luigi Grasselli,** Crystallizations and other manifold representations.
2 **Ricardo Piergallini**, Manifolds as branched covers of spheres.
3 **Gareth Jones**, Enumerating regular maps and hypermaps.
4 **J.C.Ferrando, M.López-Pellicer**, Barrelled spaces of class N and of class χ_0
5 **Pedro Morales**, Nuevos resultados en Teoria de la medida no conmutativa.
6 **Tomasz Natkaniec**, Algebraic structures generated by some families of real functions.
7 **Gonzalo Riera**, Algebras of Riemann matrices and the problem of units.
8 **Lynne D. James**, Representations of Maps.
9 **Grzegorz Gromadzki**, On supersoluble groups acting on Klein surfaces.
10 **Maria Teresa Lozano**, Flujos en 3-variedades.