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The martingale Smirnov class

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Abstract

The Smirnov class $N_*(\delta)$ of dyadic martingales is studied. Continuous linear functionals on this class and its Fréchet envelope are described. It is proved that, in contrast to the case of H^p-spaces, the space $N_*(\delta)$ is not isomorphic to the Smirnov class of holomorphic functions on the unit disc. Finally, atomic decompositions of elements of $N_*(\delta)$ are obtained.

0. Introduction

This paper has been motivated by the following result [3]:

For $0 < p < \infty$, the Hardy spaces $H^p(\mathbb{D})$ on the unit disc $\mathbb{D}$ and $H^p(\mathbb{T})$ on the unit circle $\mathbb{T}$, and the Hardy space $H^p(\delta)$ of dyadic martingales on the interval $[0, 1)$, are all isomorphic.

Since the Haar system is an unconditional basis of $H^p(\delta)$, it follows that there exists an unconditional basis for each of the spaces $H^p(\mathbb{D})$ and $H^p(\mathbb{T})$. Later on Wojtaszczyk [6] proved that the Franklin system is an unconditional basis of $H^1(\mathbb{T})$. He also constructed, using spline function, unconditional bases of $H^p(\mathbb{T})$ for each $0 < p < 1$. However, the problem of existence of an unconditional basis of the Smirnov class (or the Hardy algebra) $N_*(\mathbb{D})$ of the unit disc seems to be still open. One could try to solve this problem by studying the martingale Smirnov class $N_*(\delta)$ defined in a similar way as the martingale Hardy spaces.

In this paper, we first show that the martingale Smirnov classes defined in terms of square and dyadic maximal functions are isomorphic. From this we deduce that the Haar system is an unconditional basis in $N_*(\delta)$. We also obtain a representation of continuous linear functionals on $N_*(\delta)$, and construct the Fréchet envelope of $N_*(\delta)$. Unfortunately, this envelope is not isomorphic to the Fréchet envelope of the Smirnov class $N_*(\mathbb{D})$ of analytic functions on the unit disc. Consequently, $N_*(\delta)$ is *not* isomorphic to $N_*(\mathbb{D})$, and the existence of an unconditional basis for the latter space remains unresolved. In the last part of the paper, we obtain atomic decompositions of functions in $N_*(\delta)$.

1. Preliminaries

By a *dyadic interval* in $[0,1)$ we mean any interval of the form $[k/2^n,(k+1)/2^n)$, where $n=0,1,2,\dots$ and $0\le k<2^n-1$. The length of an interval I is denoted $|I|$. We write $\mathcal{D}$ for the family of all dyadic intervals, and $\preceq$ for the natural (linear) order in $\mathcal{D}$. That is, if $I,J\in\mathcal{D}$, then $J\preceq I$ iff $|I|<|J|$ *or* $|I|=|J|$ and J lies to the left of I.

For $I=[k/2^n,(k+1)/2^n)\in\mathcal{D}$, we denote by $I^-=[2k/2^{n+1},(2k+1)/2^{n+1})$ and $I^+=[(2k+1)/2^{n+1},(2k+2)/2^{n+1})$ its left and right half subintervals, respectively. Using this notation, the *Haar orthonormal system* $\{\chi_I : I\in\mathcal{D}\}$ in $L^2([0,1))$ is defined as follows:

$$\chi_{[0,1)}=1_{[0,1)}\quad\text{and}\quad \chi_I=|I|^{-1/2}(1_{I^-}-1_{I^+}),\quad\text{for}\quad I\in\mathcal{D},\quad I\ne[0,1).$$

We shall denote by H^0 the linear space of all (formal) Haar series $f=\sum_{I\in\mathcal{D}}c_I\chi_I$. For each such a series f, we define

$$S(f,t)=\left(\sum_I|c_I\chi_I(t)|^2\right)^{1/2},\qquad t\in[0,1)$$

the *square function* of f, and

$$f^*(t)=\sup\left\{\frac{1}{|I|}\left|\int_I f\right| : t\in I\in\mathcal{D}\right\},\qquad t\in[0,1)$$

the *dyadic maximal function* of f, where

$$\int_I f:=\sum_{J\in\mathcal{D}}\int_I c_J\chi_J(t)\,dt=\sum_{J\preceq I}\int_I c_J\chi_J(t)\,dt.$$

We identify any function $f \in L^1([0,1))$ with its Fourier-Haar series $f = \sum_I c_I(f)\chi_I$, where $c_I(f) = \int_{[0,1)} f(t)\chi_I(t)\,dt$ for every $I \in \mathcal{D}$.

For each $I \in \mathcal{D}$, the equality $P_I f = \sum_{J \preceq I} c_J \chi_J$ defines the natural projection of H^0 onto its subspace spanned by the functions $\{\chi_J : J \preceq I\}$. It is easily seen that for $P_I f$, as a function on $[0,1)$, we have

$$P_I(f,t) = \begin{cases} \frac{1}{|I^-|}\int_{I^-} f & \text{if} \quad t \in I^- \\ \frac{1}{|I^+|}\int_{I^+} f & \text{if} \quad t \in I^+ \end{cases} \qquad \text{for} \quad I \neq [0,1). \tag{1.1}$$

This implies

$$f^* = \sup_I |\, P_I f \,|, \tag{1.2}$$

$$f^* = \sup_I (\, P_I f \,)^*. \tag{1.3}$$

Moreover, we have

$$S(f) = \sup_I S(P_I f) \tag{1.4}$$

for each $f \in H^0$.

2. The dyadic Smirnov class

The *dyadic Smirnov class* $N_*(\delta)$ is defined to be the space of all $f \in H^0$ such that

$$\|\, f \,\|_* = \int_{[0,1)} \log(1 + f^*(t))\,dt < \infty.$$

It is easily seen that $\|\cdot\|_*$ is an F-norm so that the metric $d_*(f,g) = \|\, f - g \,\|_*$ defines on $N_*(\delta)$ a vector topology. Also, the space $(N_*(\delta), \|\cdot\|_*)$ is complete, i.e. it is an F-space.

Indeed, it is clear that the weak topology σ defined on H^0 by the coefficient functionals $f \mapsto c_I(f)$, $I \in \mathcal{D}$, is complete. Using (1.2) we see that

$$|I| \log\big(1 + |c_I(f)|\,|I|^{-1/2}\big) \leq \int_{[0,1)} \log(1 + |P_I(f,t) - P_{I-1}(f,t)|)\,dt \leq \|\, 2f \,\|_* \tag{2.1}$$

for each $I \in \mathcal{D} \setminus \{[0,1)\}$, where $I - 1$ is the interval directly proceeding I. It follows that the topology induced on $N_*(\delta)$ by σ is weaker than the F-norm topology defined by $\|\cdot\|_*$. Now, applying (1.3) one can verify that the $\|\cdot\|_*$-closed balls are σ-closed in H^0. In consequence, the F-norm topology of $N_*(\delta)$ is complete.

We now show that the class $N_*(\delta)$ can also be defined using square functions.

Theorem 2.1

For every $f \in H^0$, f belongs to $N_*(\delta)$ if and only if

$$\|f\|_{sq} = \int_{[0,1)} \log(1 + S(f,t))\, dt < \infty.$$

Moreover, the F-norm $\|\cdot\|_{sq}$ defines the same vector topology as $\|\cdot\|_*$.

For the proof we need the following two lemmas.

Lemma 2.2

For each positive measurable function g on a measure space (Ω, Σ, μ),

$$\int_\Omega \log(1+g)\, d\mu = \int_0^\infty \lambda_g(t) \frac{dt}{1+t},$$

where λ_g is the distribution function of g,

$$\lambda_g(t) = \mu(\{\omega \in \Omega : g(\omega) > t\}).$$

Proof. Indeed, we have

$$\int_\Omega \log(1+g)\, d\mu = \int_0^\infty \lambda_{\log(1+g)}(t)\, dt = \int_0^\infty \lambda_g(e^t - 1)\, dt = \int_0^\infty \lambda_g(t) \frac{dt}{1+t}. \ \square$$

Lemma 2.3 ([2], p. 98 and p. 100).

There is an absolute constant C such that, for every $f \in L^1[0,1)$ and $t > 0$, $Mathematics and Computer Science$,

$$\lambda_{f^*}(t) \le C\lambda_{S(f)}(t) + Ct^{-2} \int_{\{x : S(f,x) \le t\}} S^2(f,x)\, dx$$

and

$$\lambda_{S(f)}(t) \le C\lambda_{f^*}(t) + Ct^{-2} \int_{\{x : f^*(x) \le t\}} (f^*)^2(x)\, dx. \tag{2.3}$$

Proof of Theorem 2.1. Let $X = \{ f \in H^0 : \| f \|_{sq} < \infty \}$. Using the same argument as in the case of $N_*(\delta)$, one can verify that the F-normed space $(X, \|\cdot\|_{sq})$ is complete. Next, as is well known, the sublinear operators $f \mapsto f^*$ and $f \mapsto S(f)$ are of weak type (1,1) ([2] Append. 2 Theorem 2). Thus, L^1 is a subspace of both $N_*(\delta)$ and X. Applying Lemma 2.2 and inequality (2.2), we obtain

$$\begin{aligned}\| f \|_* &= \int_0^\infty \lambda_{f^*}(t)\frac{dt}{1+t} \\ &\le C\int_0^\infty \lambda_{S(f)}(t)\frac{dt}{1+t} + C\int_0^\infty \frac{1}{t^2(1+t)}\int_{\{S(f,x)\le t\}} S^2(f,x)\,dx\,dt \\ &\le C\int_0^1 \log(1+|S(f,x)|)\,dx + C\end{aligned}$$

for every $f \in L^1$ and an absolute positive constant C. This, (1.3), (1.4), and the dominated convergence theorem imply

$$\| f \|_* \le C\,\| f \|_{sq} + C \qquad \text{for every } f \in H^0.$$

Consequently, $X \subset N_*(\delta)$. The same argument as above (with (2.3) instead of (2.2)) shows that $N_*(\delta) \subset X$. Thus X and $N_*(\delta)$ are equal as vector spaces. Moreover, their complete metric vector topologies are stronger than the weak topology σ described above. Therefore, by the closed graph theorem, the identity operator is a topological isomorphism between X and $N_*(\delta)$. The proof is complete. □

Corollary 2.4

The Haar system is an unconditional Schauder bases in $N_*(\delta)$.

Corollary 2.5

Every Haar series $f = \sum_I c_I\chi_I \in N_*(\delta)$ converges almost everywhere.

Proof. Since $\sum_I |c_I|^2|\chi_I(x)|^2 = S^2(f,x) < \infty$ for a.a. $x \in [0,1)$, the series $\sum_I c_I\chi_I$ is convergent a.e. (cf. [2], Chap. III, Thm. 13). □

It is easily seen that if $f = \sum_I c_I\chi_I \in N_*(\delta)$, then

$$\text{(HCE)} \qquad |c_I| \le \exp\left(\| f \|_{sq}|I|^{-1}\right) \qquad \text{for each } I \in \mathcal{D}.$$

This leads us to a study of the vector space $F_*(\delta)$ consisting of all $f = \sum_I c_I\chi_I \in H^0$ such that

$$||| f |||_m = \sup_I |c_I| \exp(-(m|I|)^{-1}) < \infty \qquad \text{for each } m \in \mathbb{N}.$$

As easily seen, the space $F_*(\delta)$ equipped with the locally convex topology defined by the sequence of norms $\{|||\cdot|||_m : m \in \mathbb{N}\}$ is a Fréchet space.

Theorem 2.6

(a) $N_*(\delta)$ is a dense subspace of $F_*(\delta)$, with the inclusion mapping being continuous.
(b) For every family $\gamma = \{\, \gamma_I : I \in \mathcal{D}\}$ of scalars satisfying

$$\text{(DS)} \qquad \sup_I |\gamma_I| \exp(r|I|^{-1}) < \infty \qquad \textit{for some} \quad r > 0,$$

the formula

$$\text{(DLF)} \qquad T_\gamma f = \sum_I c_I(f)\gamma_I$$

defines a continuous linear functional on $F_*(\delta)$.
(c) For every continuous linear functional T on $N_*(\delta)$ there is a family $\gamma = \{\, \gamma_I : I \in \mathcal{D}\}$ satisfying (DS) such that $T = T_\gamma$.

Proof. (a) This is a simple consequence of (HCE) and the fact that the space of all finite Haar series is dense in both $N_*(\delta)$ and $F_*(\delta)$.

(b) Fix a family $\gamma = \{\, \gamma_I : I \in \mathcal{D}\}$ with $\sup_I |\gamma_I| \exp(r|I|^{-1}) = C < \infty$ for some $r > 0$. Choose $m \in \mathbb{N}$ so that $2 < rm$. Then for each $f = \sum_I c_I \chi_I \in F_*(\delta)$ we have

$$\begin{aligned} \Big| \sum_I c_I \gamma_I \Big| &\le C \sum_I |||\, f\, |||_m \exp\Big(-\frac{r}{2}|I|^{-1}\Big) \\ &\le C|||\, f\, |||_m \sum_{n=0}^{\infty} \sum_{k=0}^{2^n-1} \exp(-r2^{k-1}) \le C' \, |||\, f\, |||_m, \end{aligned}$$

where $C' > 0$ is a constant. Thus the functional T_γ defined by (CLF) is continuous on $F_*(\delta)$.

(c). Let T be a continuous linear functional on $N_*(\delta)$. Fix $\varepsilon > 0$ such that

$$\text{(2.4)} \qquad |Tf| \le 1 \qquad \text{for all} \;\; f \in N_*(\delta) \;\; \text{with} \;\; \|\, f\, \|_{sq} \le \varepsilon.$$

Set $\gamma_I = T\chi_I$ for $I \in \mathcal{D}$, and let $\gamma = \{\gamma_I\}$. Since $\{\chi_I : I \in \mathcal{D}\}$ is a Schauder bases in $N_*(\delta)$, $Tf = \sum_I c_I(f)\gamma_I = T_\gamma f$ for every $f \in N_*(\delta)$. Thus it remains to verify (DS).

Define $f_{r,I} = r\exp(r|I|^{-1})\chi_I$ for each $r > 0$ and $I \in \mathcal{D}$. It is easily seen that $\inf_r \sup_I \|\, f_{r,I}\, \|_{sq} = 0$. Fix $r > 0$ such that $\|\, f_{r,I}\, \|_{sq} < \varepsilon$ for all I. Applying (2.4) we finally get

$$\sup_I |\, \gamma_I \,| \exp(r|I|^{-1}) = \sup_I |\, Tf_{r,I}\, | \le 1. \ \Box$$

Let us now recall that if $X = (X, \tau)$ is an F-space whose topological dual X' separates the points of X, then the *Fréchet envelope* $\hat{X}$ of X is defined to be the completion of the space (X, τ^c), where τ^c is the strongest locally convex topology on X that is weaker than τ. It is known (see e.g. [5]) that τ^c is equal to the Mackey topology of the dual pair (X, X'). Moreover, every metrizable locally convex space E is a Mackey space, i.e. its topology coincides with the Mackey topology of the dual pair (E, E'). In consequence, the Fréchet envelope $\hat{X}$ of X is defined uniquely up to isomorphism by the conditions: (a) $\hat{X}$ is a Fréchet space, (b) there exists a continuous embedding j of X onto a dense subspace of $\hat{X}$ such that the map $(\hat{X})' \ni T \mapsto T \circ j \in X'$ is a linear isomorphism of $(\hat{X})'$ onto X'. Thus, as a simple consequence of Theorem 2.6 we get the following.

Corollary 2.7

$F_*(\delta)$ is the Fréchet envelope of $N_*(\delta)$.

The reader is referred to [1] for information on power series spaces.

Corollary 2.8

$F_*(\delta)$ is isomorphic to the nuclear power series space $\Lambda_1(m)$.

Proof. Define a map $\psi : \mathbb{Z}_+ \to \mathcal{D}$ by

$$\psi(m) = [k/2^n, (k+1)/2^n) \qquad \text{if } \; m = 2^n + k,\ n = 1, 2, \ldots,\ 0 \le k < 2^n - 1.$$

Then $1 \le m|\psi(m)| \le 2$ for every $m \in \mathbb{Z}_+$. Thus, by Theorem 2.6 and ([4], Prop. 3.4), $F_*(\delta)$ is isomorphic to $\Lambda_1(m)$. $\Box$

It is well known and easily seen that Fréchet envelopes of isomorphic F-spaces are isomorphic. It was proved by Yanagihara [8] that the Fréchet envelope (or the containing Fréchet space) of the Smirnov class $N_*(\mathbb{D})$ of holomorphic functions on the unit disc $\mathbb{D}$ is isomorphic to the nuclear power series space $\Lambda_1(\sqrt{m})$. Thus, in contrast to the case of dyadic and holomorphic Hardy spaces H^p, $0 < p < \infty$ (see [7]), we have

Corollary 2.9

The spaces $N_*(\delta)$ and $N_*(\mathbb{D})$ are not isomorphic.

3. Atomic decompositions

$N_*(\delta)$ can be treated both as a space of measurable functions (by Corollary 2.5) and as a sequence space, if we identify each f in $N_*(\delta)$ with the family $\{c_I(f) : I \in \mathcal{D}\}$. However, there is no natural way of expressing the F-norms $\| f \|_*$ or $\| f \|_{sq}$ in terms of the Haar coefficients of f. Nonetheless, in this section we show that $N_*(\delta)$ has a nice "atomic definition".

An *atom* is either the function $1_{[0,1)}$ or any measurable function a such that $|a(t)| \leq 1$ for all $t \in [0,1)$ and $\int_{[0,1)} a(t)\, dt = 0$. We denote by $\mathcal{A}$ the family of all atoms. For each $a \in \mathcal{A}$ we define

$$w(a) = \inf\{ |I| \,:\, I \in \mathcal{D},\ \operatorname{supp} a \subset I \}.$$

By an *atomic decomposition* of a measurable function f on $[0,1)$ we mean a pair (α, a), where $\alpha = (\alpha_n)$ is a sequence of scalars and $a = (a_n)$ is a sequence of atoms such that

$$f = \sum_{n=1}^{\infty} \alpha_n a_n \qquad \text{a.e. on } [0,1),$$

and we let $A(f)$ stand for the family of all atomic decompositions of f.

Proposition 3.1

If $(a_i) \subset \mathcal{A}$ and a sequence of scalars (α_i) satisfy

$$\sum_{i=1}^{\infty} \log(1 + |\alpha_i|) w(a_i) < \infty, \tag{3.1}$$

then the series $\sum_{i=1}^{\infty} \alpha_i a_i$ converges both in $N_*(\delta)$ and a.e. to a function f such that

$$\| f \|_* \leq \sum_{i=1}^{\infty} \log(1 + |\alpha_i|) w(a_i). \tag{3.2}$$

Proof. Clearly, $\mathcal{A} \subset N_*(\delta)$, and $\| \alpha_i a_i \|_* \leq \log(1 + |\alpha_i|) w(a_i)$. Hence the series $\sum_i \alpha_i a_i$ is absolutely convergent in the F-space space $N_*(\delta)$, and obviously (3.2) holds. Moreover, the sequence of functions $g_n = \sum_{i=1}^{n} |\alpha_i a_i|$ is increasing, and $\int_{[0,1)} \log(1 + g_n(t)) dt \leq \sum_{i=1}^{\infty} \|\alpha_i a_i\|_* < \infty$. Hence, by Fatou's lemma, $\int_{[0,1} \log(1 + \sup_n g_n(t)) dt < \infty$, and it follows that $\sum_{i=1}^{\infty} |\alpha_i a_i(t)| < \infty$ for a.a. $t \in [0,1)$. $\square$

Theorem 3.2

There is a positive constant C such that each $f \in N_*(\delta)$ has an atomic decomposition $((\alpha_i),(a_i))$ with

$$\sum_{i=1}^{\infty} \log(1+|\alpha_i|)w(a_i) \le C\| Cf \|_*. \tag{3.4}$$

Proof. Fix $f \in L^1$ and first assume $\int_{[0,1)} f = 0$. Define $U_k' = \{ x \in [0,1) : f^*(x) > \beta_k \}$, where $\beta_k = \frac{1}{2}(\exp 2^k - 1)$ and $k \in \mathbb{Z}$. Each U_k' can be represented as the union $\bigcup_i I_k^i$, where each I_k^i is a maximal dyadic interval contained in U_k'. Thus, for every k, the intervals $I_k^1, I_k^2, \dots$ are pairwise disjoint. Now let J_k^i be the dyadic interval containing I_k^i and twice as long as I_k^i. Since $J_k^i \not\subset U_k'$, the absolute value of

$$m_{J_k^i}(f) = \frac{1}{|J_k^i|}\int_{J_k^i} f$$

is not greater than β_k.

Let $U_k = \bigcup_i J_k^i$. By passing to a subsequence, we can assume that the dyadic intervals $J_k^1, J_k^2, \dots$ are pairwise disjoint. Consider a Calderon-Zygmund decomposition of f:

$$f = g_k + b_k, \qquad \text{where} \quad g_k = (1 - 1_{U_k})f + \sum_i m_{J_k^i}(f) 1_{J_k^i}.$$

We have $|g_k| \le \beta_{k+1}$, $\operatorname{supp} b_k \subset U_k$ and $\int_{J_k^i} b_k = 0$. Moreover, $g_k \to f$ a.e. as $k \to \infty$, and $g_k \to 0$ a.e. as $k \to -\infty$. In consequence,

$$f = \sum_{-\infty}^{\infty}(g_{k+1} - g_k) = \sum_{-\infty}^{\infty}(b_k - b_{k+1}),$$

where the series are convergent a.e. Next, $|b_k - b_{k+1}| = |g_{k+1} - g_k| \le 2\beta_{k+1}$. Moreover, since each interval J_{k+1}^i is contained in some interval J_k^j and $\int f = 0$, it follows that $\int (b_k - b_{k+1})1_{J_k^i} = 0$. Thus,

$$a_k^i = \frac{2}{\beta_{k+1}}(b_k - b_{k+1})1_{J_k^i}$$

is an atom and

$$f = \sum_k \sum_i 2\beta_{k+1} a_k^i \quad \text{a.e.}$$

Finally, we have

$$\begin{aligned}
\sum_k \sum_i \log(1+|2\beta_{k+1}|)w(a_k^i) &\le \sum_k \sum_i 2^{k+1}|J_k^i| \\
&\le 4\sum_k \sum_i 2^k |I_k^i| = 4\sum_k 2^k m(U_k') \\
&\le 8\sum_k 2^k \big[m(U_k') - m(U_{k+1}')\big] \\
&\le 8\sum_k 2^k m\big(\{\, x : \beta_k \le f^* < \beta_{k+1} \,\}\big) \\
&\le 8\sum_k 2^k m\big(\{\, x : 2^k \le \log(1+2f^*) < 2^{k+1} \,\}\big) \\
&\le 8\int \log(1+2f^*)dt.
\end{aligned}$$

This proves the result when $\int f = 0$. In order to extend this to an arbitrary $f \in L^1$, it is enough to note that $f = (f - \alpha 1_{[0,1)}) + \alpha 1_{[0,1)}$, where $\alpha = \int_{[0,1)} f$ and $|\alpha| \le f^*$.

Let now $f \in N_*(\delta)$. Recall that, for every $g \in N_*(\delta)$, we have $P_I g \to g$ in $N_*(\delta)$ and a.e., and $\| P_I g \|_* \le \| g \|_*$ (see Corollaries 2.4 and 2.5, and (1.1)). Therefore, we can find by induction a sequence (f_k) in L^1 such that $f_0 = 0$ and

$$\| f_{k+1} \|_* \le \| f - (f_0 + \ldots + f_k) \|_* \le 2^{-k} \| f \|_*, \qquad k = 0, 1, \ldots$$

Thus, the series $\sum_{k=1}^{\infty} f_k$ converges to f in $N_*(\delta)$ and in measure. Now, for every k, we can find an atomic decomposition $f_k = \sum_j \alpha_{jk} a_{jk}$ with $\sum_j \log(1 + |\alpha_{jk}|) w(a_{jk}) \le \| f_k \|_*$. Then $f = \sum_k \sum_j \alpha_{jk} a_{jk}$ in measure and (3.4) holds. Finally, by Proposition 3.1, $f = \sum_k \sum_j \alpha_{jk} a_{jk}$ a.e. $\square$

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