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# Remarks on the Istratescu measure of noncompactness

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ABSTRACT

In this paper we give estimations of Istratescu measure of noncompactness $I(X)$ of a set $X \subset l^p(E_1, \dots, E_n)$ in terms of measures $I(X_j)$ ($j = 1, \dots, n$) of projections $X_j$ of $X$ on $E_j$. Also a converse problem of finding a set $X$ for which the measure $I(X)$ satisfies the estimations under consideration is considered.

## 1. Introduction

Let $E$ be a real Banach space with the norm $\| \ \|$. Given a nonempty subset $A$ of $E$ we say that it is $\varepsilon$- separated if for every pair $x, y$ in $A$ we have

$$\|x - y\| \geq \varepsilon .$$

For a nonempty and bounded subset $X$ of $E$ we shall consider the Istratescu measure of noncompactness of $X$, defined in the following way:

$$I(X) = \sup \left\{ \varepsilon > 0\colon \text{ there exists an infinite } \varepsilon\text{-separated subset } A \text{ of } X \right\} .$$

This classical measure was studied in many papers and has found many applications (cf. [1,2]).

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Particularly, in the geometry of Banach spaces the following "separation" constant, often called the Kottman constant, is considered:

$$K(E) = I(B(E))$$

where $B(E)$ denotes the unit ball of a space $E$.

As it was indicated by Papini (see [13]) the Kottman constant is strictly related to other significant notions, for example:

$$K(E) = \frac{2P(E)}{1 - P(E)} \quad \text{(cf. [10,12])},$$

where $P(E)$ is the packing constant of $E$, defined as follows:

$$P(E) = \sup\{r > 0\colon \text{ infinitely many balls of radius} \geq r \text{ can be packed in } B(E)\}.$$

(We say that a collection of balls $\{B(x_i, r_i)\}$ is packed in $B(E)$ if $B(x_i, r_i) \subseteq B(E)$ for every index $i$, and moreover the interiors of any two of the balls are disjoint they do not "overlap").

We have also:

$$K(E) \cdot J(E) \geq 2 \quad \text{(cf. [13])},$$

where $J(E)$ is the Yung constant of $E$, defined by

$$J(E) = \sup\{2r(A)/\delta(A);\ \ A \text{ is bounded, nonempty subset of } E\},$$

where $\delta(A)$ is the diameter of $A$, and $r(A) = \inf\limits_{x \in E} \sup\limits_{a \in A} \|x - a\|$.

We also know that $K(E)$ is continuous with respect to the Banach-Mazur distance in any isomorphism class (cf. [11]).

If we want to get some numerical results we may start from the Elton-Odell $(1 + \varepsilon)$-separation theorem (cf. e.g. [4]), which gives us:

$$1 < K(E) \leq 2$$

(the right inequality is trivial).

From the papers of Kottman, Papini, Domínguez Benavides and other authors it follows that:

i) $K(H) = 2^{1/2}$, if $H$ is a Hilbert space (cf. e.g. [5]),

ii) $K(l^p) = 2^{1/p}$, for $1 \leq p < \infty$ (cf. [10]),

iii) $K(c_o) = K(l^\infty) = 2$,

iv) $K(L^p) = \max\{2^{1/p}, 2^{1/q}\}$, for $1 \leq p < \infty, 1/p + 1/q = 1$ and $\mu$ not purely atomic measure (cf. [3,12]),

v) $K(l^p(E_1, E_2, \ldots)) = \max\{2^{1/p}, \sup_{i \in \mathbb{N}} K(E_i)\}$, where $E_i$ are Banach spaces for $i = 1, 2, \ldots$ (with a norm $\| \ \|_i$ respectively) and $l^p(E_1, E_2, \ldots)$, with $1 \leq p < \infty$, denotes the space of all sequences $\{x_i\}$, $x_i \in E_i$, with $\sum_{i=1}^{\infty} \|x_i\|_i^p < \infty$ ($l^p(E_1, E_2, \ldots)$ is a Banach space with the natural norm) (cf. [9,11]).

Recently some new interesting results concerning the Kottman constant in Orlicz and Musielak-Orlicz sequence spaces were obtained by H. Hudzik and others [6,7,8].

The result v) suggests the following question:

What can be said about Istratescu measure of a bounded subset of $l^p(E_1, E_2, \ldots)$ if measures of its projections on subspaces $E_i$ are known?

The aim of this paper is to answer this question for the finite product space $l^p(E_1, \ldots, E_n)$ which may be treated as the special case of the space $l^p(E_1, E_2, \ldots)$.

## 2. Notation, definitions and some auxiliary facts

This section is devoted to establish some auxiliary results which will be needed further on.

Let $E_j$ be a Banach space with a norm $\| \ \|_j$ for $1 \leq j \leq n$.

Let us recall that the product space $l^p(E_1, \ldots, E_n)$ is defined as the linear space $E_1 \times \ldots \times E_n$ with a norm:

$$\|x\| = \left( \sum_{j=1}^{n} \|x_j\|_j^p \right)^{1/p} \quad \text{if } 1 \leq p < \infty,$$

and

$$\|x\| = \max_{1 \leq j \leq n} \|x_j\|_j, \quad \text{for } p = \infty,$$

where $x = (x_1 \ldots, x_n) \in E_1 \times \ldots \times E_n$.

The basic properties of product spaces may be found in [9], for example.

In the sequel we will use the following fact concerning the Istratescu measure of noncompactness in product spaces.

**Lemma 1**

*If $X$ is a bounded and nonempty subset of the space $l^p(E_1, \ldots, E_n)$, $1 \leq p \leq \infty$, then*

$$I(X) \geq I(X_j) \text{ for } j = 1, \ldots, n,$$

*where $X_j$ denotes the projection of $X$ on $E_j$.*

*Proof.* Fix an arbitrary $j$, $1 \leq j \leq n$, and $\varepsilon > 0$. Let $\{x_{kj}\}$ be an $I(X_j) - \varepsilon$-separated sequence in $X_j$. Then there exists a sequence $\{x_k\}$ of elements of $X$ such that $x_{kj}$ is a projection of $x_k$ on the space $E_j$, for $k = 1, 2, \ldots$. It is easy to see that $\{x_k\}$ is $I(X_j) - \varepsilon$-separated, too. Indeed, for any $k, l \in \mathbb{N}$ we get

$$\begin{aligned} \|x_k - x_l\| = \left( \sum_{i=1}^{n} \|x_{ki} - x_{li}\|_i^p \right)^{1/p} &\geq \left( \|x_{kj} - x_{lj}\|_j^p \right)^{1/p} \\ &\geq I(X_j) - \varepsilon \text{ if } 1 \leq p < \infty, \end{aligned}$$

and

$$\begin{aligned} \|x_k - x_l\| = \max_{1 \leq i \leq n} \|x_{ki} - x_{li}\| &\geq \|x_{kj} - x_{lj}\| \\ &\geq I(X_j) - \varepsilon \text{ for } p = \infty. \end{aligned}$$

Thus for any $\varepsilon > 0$ we have

$$I(X) \geq I(X_j) - \varepsilon,$$

which ends the proof. □

Now, let us formulate a few properties of the Istratescu measure of noncompactness which will be used later on (cf. [1,2]).

For any nonempty and bounded subsets $X, Y$ of a Banach space $E$ we have

$$X \subset Y \Rightarrow I(X) \leq I(Y), \tag{1}$$

$$I(RX) = RI(X) \text{ for any } R > 0, \tag{2}$$

$$I(X + a) = I(X) \text{ for any } a \in E. \tag{3}$$

The next useful property of the Istratescu measure $I$ is formulated in the following lemma.

**Lemma 2**

*Let $X$ be a bounded and convex subset of a Banach space $E$ such that $0 \in X$, and let $f: E \to \mathbb{R} \cup \{\infty\}$ be a function defined by*

$$f(x) = \begin{cases} \inf\{t \geq 0 : x \in tX\} & \text{if the set is nonempty,} \\ \infty & \text{otherwise.} \end{cases}$$

*Then for any sequence $\{x_k\}$ of elements of $X$ and $\delta > 0$ there exist $0 \leq R \leq 1$ and a subsequence $\{y_k\}$ of $\{x_k\}$ such that:*

$$\lim_{k\to\infty} f(y_k) = R, \tag{4}$$

*and for any $k, l \in \mathbb{N}$:*

$$\|y_k - y_l\| \leq RI(X) + \delta.$$

*Proof.* Since $x_k \in X$ we get $0 \leq f(x_k) \leq 1$ for $k = 1, 2, \ldots$. Hence using Weierstrass theorem we can choose a subsequence $\{w_k\}$ of $\{x_k\}$ and $0 \leq R \leq 1$ satisfying (4). Put $\varepsilon = \delta/(I(X) + 1) > 0$.
From (4) it follows that $w_k \in (R + \varepsilon)X$ for $k \geq k_0, k_0 \in \mathbb{N}$.
Now, using (2) and Ramsey's theorem (cf. [4], for example) we claim that there exists a subsequence $\{y_k\}$ of $\{w_k\}$ such that

$$\|y_k - y_l\| \leq (R + \varepsilon)I(X) + \varepsilon = RI(X) + \delta \quad \text{for any} \quad k, l \in \mathbb{N}.$$

If not, there would exist a subsequence of $\{w_k\}$ for which the opposite inequality holds, which in turn contradicts the fact that

$$I((R + \varepsilon)X) = (R + \varepsilon)I(X).$$

This completes the proof. □

Finally, let us formulate a numerical lemma which is a simple consequence of Hölder's inequality.

**Lemma 3**

*Let $1 \leq p < q < \infty$, $R_j \geq 0$ and $m_j \geq 0$ for $j = 1, \ldots, n$, and*

$$R_1^q + \cdots + R_n^q \leq 1.$$

*Then*

$$\sum_{j=1}^{n} R_j^p m_j^p \leq \left(\sum_{j=1}^{n} m_j^{pq/(q-p)}\right)^{(q-p)/q}.$$

*The equality is attained for:*

$$\hat{R}_j = m_j^{p/(q-p)} \left(\sum_{i=1}^{n} m_i^{pq/(q-p)}\right)^{-1/q}, \quad \text{where } j = 1, \ldots, n.$$

*Proof.* Indeed, putting

$$x_j = R_j^p, \quad y_j = m_j^p, \quad s = \frac{q}{p}, \quad t = \frac{q}{(q-p)},$$

we have $s, t > 1$ and $\frac{1}{s} + \frac{1}{t} = 1$. Hence, applying Hölder's inequality we obtain

$$\begin{aligned}
\sum_{j=1}^{n} R_j^p m_j^p = \sum_{j=1}^{n} x_j y_j &\leq \left(\sum_{j=1}^{n} x_j^s\right)^{1/s} \left(\sum_{j=1}^{n} y_j^t\right)^{1/t} \\
&= \left(\sum_{j=1}^{n} R_j^q\right)^{p/q} \left(\sum_{j=1}^{n} m_j^{pq/(q-p)}\right)^{(q-p)/q} \\
&\leq \left(\sum_{j=1}^{n} m_j^{pq/(q-p)}\right)^{(q-p)/q}.
\end{aligned}$$

The remainder of the proof is an easy calculation and is therefore omitted. □

## 3. Main results

Let $E_j$ be a Banach space with a norm $\| \ \|_j$ for $j = 1, \ldots, n$. For the Istratescu measure of noncompactness in the product space $l^p(E_1, \ldots, E_n)$ we have the following estimations.

**Theorem 1**

*If $X$ is a bounded subset of $l^p(E_1, \ldots, E_n)$ then*

$$\max_{1 \leq j \leq n} I(X_j) \leq I(X) \leq \left(\sum_{j=1}^{n} [I(X_j)]^p\right)^{1/p}, \quad \text{if } 1 \leq p < \infty. \tag{5}$$

*and*

$$I(X) = \max_{1 \le j \le n} I(X_j), \quad \text{if } p = \infty, \tag{6}$$

*where $X_j$ denotes a projection of $X$ on $E_j$ for $j = 1, \dots, n$.*

The converse is also true, which is stated in the next theorem.

**Theorem 2**

*If $X_1, \dots, X_n$ are bounded and convex subsets of $E_1, \dots, E_n$ respectively, then for any number $\mu$ satisfying*

$$\max_{1 \le j \le n} I(X_j) \le \mu \le \left( \sum_{j=1}^{n} [I(X_j)]^p \right)^{1/p}, \quad \text{where } 1 \le p < \infty,$$

*there exists a subset $X$ of $l^p(E_1, \dots, E_n)$ such that $I(X) = \mu$ and projections of $X$ on each of $E_j$ coincide with $X_j$.*

*Proof of Theorem 2.* Let $1 \le p < \infty$, $E_1, \dots, E_n$ and $X_1, \dots, X_n$ be as in the statement of the theorem. Keeping in mind (3), without loss of generality we may assume that $0 \in X_j$ for $j = 1, \dots, n$. Denote $m_j = I(X_j)$.
We define $f_j : E_j \to \mathbb{R} \cup \{\infty\}$ $(j = 1, \dots, n)$ similarly as in Lemma 2, namely:

$$f_j(x) = \begin{cases} \inf\{t \ge 0 : x \in tX_j\} & \text{if the set is not empty,} \\ \infty & \text{otherwise}. \end{cases}$$

Consider the sets

$$D_q = \left\{ (x_1, \dots, x_n) \in l^p(E_1, \dots, E_n) : \sum_{j=1}^{n} [f_j(x_j)]^q \le 1 \right\}$$

for $1 \le q < \infty$, and

$$D_\infty = X_1 \times \dots \times X_n.$$

It is easy to see that

(7) the projection of $D_q$ onto the space $E_j$ coincides with $X_j$ for any $1 \le q \le \infty$ and $j = 1, \dots, n$.

Now we will calculate the Istratescu measure of noncompactness of the sets $D_q$ in the space $l^p(E_1, \ldots, E_n)$.

We have:

$$I(D_q) = \max_{1 \leq j \leq n} I(X_j) \quad \text{for } 1 \leq q \leq p, \tag{8}$$

$$I(D_q) = \left( \sum_{j=1}^{n} [I(X_j)]^{pq/(q-p)} \right)^{(q-p)/pq} \quad \text{for } p < q < \infty, \tag{9}$$

$$I(D_q) = \left( \sum_{j=1}^{n} [I(X_j)]^{p} \right)^{1/p} \quad \text{for } q = \infty. \tag{10}$$

To prove (8) fix $p$ and $q$, where $1 \leq q \leq p < \infty$. The inequality

$$I(D_q) \geq \max_{1 \leq j \leq n} I(X_j)$$

follows immediately from (7) and Lemma 1.

To prove the opposite, since $D_q \subseteq D_p$ for $1 \leq q \leq p$, taking into account (1) it is sufficient to show that

$$I(D_p) \leq \max_{1 \leq j \leq n} I(X_j).$$

Suppose to the contrary that there exists an $\alpha + \varepsilon$-separated sequence $\{x_k\}$ contained in $D_p$, where $\alpha = \max_{1 \leq j \leq n} I(X_j)$ and $\varepsilon > 0$. Denote $x_k = (x_{k1}, \ldots, x_{kn})$ for $k = 1, 2, \ldots$. Put $\delta = (\alpha + \varepsilon)^p - \alpha^p > 0$. Applying consecutively Lemma 2 to each of $X_j$ and taking a subsequence of $\{x_k\}$ in place of $\{x_k\}$ if necessary, we may assume that:

$$\lim_{k \to \infty} f_j(x_{kj}) = R_j, \quad \text{where } 0 \leq R_j \leq 1 \text{ for } j = 1, \ldots, n, \tag{11}$$

and

$$\|x_{kj} - x_{lj}\|_j^p \leq R_j^p m_j^p + \frac{\delta}{2^j} \quad \text{for } l, k \in \mathbb{N}. \tag{12}$$

Moreover it is easy to see that

$$\sum_{j=1}^{n} R_j^p \leq 1 \tag{13}$$

(if not, due to (11), $x_k$ with sufficiently large $k$ would not belong to $D_p$).
Now, using (12) and (13) we get

$$\|x_k - x_l\|^p = \sum_{j=1}^{n} \|x_{kj} - x_{lj}\|_j^p < \sum_{j=1}^{n} R_j^p m_j^p + \delta$$
$$\leq \alpha^p (\sum_{j=1}^{n} R_j^p) + \delta \leq \alpha^p + \delta = (\alpha + \varepsilon)^p$$

which contradicts the fact that $\{x_k\}$ is $\alpha + \varepsilon$-separated. This implies (8).

To prove (9) let $1 \leq p < q < \infty$. Denote $\beta \doteq (\sum_{j=1}^{n} [I(X_j)]^{\frac{pq}{q-p}})^{\frac{q-p}{pq}}$.
First we will prove that $I(D_q) \leq \beta$. Suppose to the contrary that there exists a sequence $\{x_k\}$ of elements of $D_q$ which is $\beta + \varepsilon$-separated, where $\varepsilon > 0$. Denote $\delta = (\beta + \varepsilon)^p - \beta^p > 0$. Using the same argumentation as above we may assume without loss of generality that for the sequence $\{x_k\}$ the properties (11), (12) and $R_1^q + \cdots + R_n^q \leq 1$ hold.
Now, taking into account (12) and applying Lemma 3 we obtain

$$\|x_k - x_l\|^p = \sum_{j=1}^{n} \|x_{kj} - x_{lj}\|_j^p < \sum_{j=1}^{n} R_j^p m_j^p + \delta$$
$$\leq \beta^p + \delta = (\beta + \varepsilon)^p$$

which contradicts the fact that the sequence $\{x_k\}$ is $\beta + \varepsilon$-separated. Thus we have proved that $I(D_q) \leq \beta$,

To prove the opposite inequality let $\varepsilon > 0$ and $\delta = \beta^p - (\beta - \varepsilon)^p$. Consider $\hat{R}_1, \ldots, \hat{R}_n$ such as in Lemma 3. We have:

$$\sum_{j=1}^{n} \hat{R}_j^q = 1, \quad \sum_{j=1}^{n} \hat{R}_j^p m_j^p = \beta^p \quad \text{and} \quad \hat{R}_j \geq 0 \quad \text{for} \quad j = 1, \ldots, n.$$

From (2) it follows that

$$I(\hat{R}_j X_j) = \hat{R}_j m_j \quad \text{for} \quad j = 1, \ldots, n.$$

Thus we can choose a sequence $\{x_{kj}\}_{k \in \mathbb{N}}$ of elements of $\hat{R}_j X_j$ such that

$$\|x_{kj} - x_{lj}\|_j^p \geq \hat{R}_j^p m_j^p - \frac{\delta}{n} \quad \text{for} \quad j = 1, \ldots, n \text{ and } k \neq l.$$

Let

$$x_k = (x_{k1}, \dots, x_{kn}) \quad \text{for} \quad k = 1, 2 \dots .$$

Since $x_{kj} \in \hat{R}_j X_j$ we have $f_j(x_{kj}) \leq \hat{R}_j$ for $j = 1, \dots, n$ and

$$\sum_{j=1}^{n} [f_j(x_{kj})]^q \leq \sum_{j=1}^{n} \hat{R}_j^q = 1.$$

Hence $x_k \in D_q$ for $k = 1, 2, \dots$ Moreover we have:

$$\begin{aligned} \|x_k - x_l\|^p &= \sum_{j=1}^{n} \|x_{kj} - x_{lj}\|_j^p \geq \sum_{j=1}^{n} \hat{R}_j^p m_j^p - \delta \\ &= \beta^p - \delta = (\beta - \varepsilon)^p \quad \text{for} \quad k, l \in \mathbb{N}, k \neq l. \end{aligned}$$

Thus for an arbitrary $\varepsilon > 0$ there exists a sequence of elements of $D_q$ which is $\beta - \varepsilon$-separated. So $I(D) \geq \beta$. This ends the proof of (9).

To prove (10) let $q = \infty$. Arguing similarly as above it is easy to check that:

$$I(D_\infty) \geq \gamma, \quad \text{where} \quad \gamma = \left( \sum_{j=1}^{n} m_j^p \right)^{1/p} .$$

Suppose that the opposite inequality does not hold. Then there exists a sequence $\{x_k\}$ of elements of $D_\infty$ which is $\gamma + \varepsilon$-separated, where $\varepsilon > 0$. Let $x_k = (x_{k1}, \dots, x_{kn})$ where $x_{kj} \in X_j$ for $k = 1, 2, \dots$ Put $\delta = (\gamma + \varepsilon)^p - \gamma^p$.
Using Ramsey's theorem we can choose a subsequence of the sequence $\{x_k\}$ (without loss of generality all $\{x_k\}$) such that:

$$\|x_{kj} - x_{lj}\|_j^p \leq I(X_j)^p + \frac{\delta}{2n} \quad \text{for} \quad j = 1, \dots, n, \quad k, l \in \mathbb{N}.$$

Therefore

$$\|x_k - x_l\|^p = \sum_{j=1}^{n} \|x_{kj} - x_{lj}\|_j^p \leq \sum_{j=1}^{n} m_j^p + \frac{\delta}{2} < \gamma^p + \delta = (\gamma + \varepsilon)^p .$$

The obtained contradiction proves (10).

Finally, observe that

$$\lim_{q \to p} \left( \sum_{j=1}^{n} [I(X_j)]^{pq/(q-p)} \right)^{(q-p)/pq} = \max_{1 \leq j \leq n} I(X_j)$$

and

$$\lim_{q \to \infty} \left( \sum_{j=1}^{n} [I(X_j)]^{pq/(q-p)} \right)^{(q-p)/pq} = \left( \sum_{j=1}^{n} [I(X_j)]^p \right)^{1/p} ,$$

which in conjunction with (8), (9) and (10) completes the proof of Theorem 2. □

Since the proof of (10) is valid for any $1 \leq p < \infty$ and bounded (not necessarily convex) sets we obtain the following corollary.

**Corollary 1**

*If $X_1, \ldots, X_n$ are bounded and nonempty subsets of Banach spaces $E_1 \ldots, E_n$ respectively, then for the measure $I$ in the space $l^p(E_1, \ldots, E_n)$, where $1 \leq p < \infty$, we have*

$$I(X_1 \times \ldots \times X_n) = \left( \sum_{j=1}^{n} [I(X_j)]^p \right)^{1/p}.$$

*Proof of Theorem 1.* Let $X$ be a bounded subset of $l^p(E_1, \ldots, E_n)$. Observe, that in view of Lemma 1 the inequality

$$I(X) \geq \max_{1 \leq j \leq n} I(X_j) \tag{14}$$

holds for any $1 \leq p \leq \infty$, where $X_j$ denotes a projection of $X$ on $E_j$ for $j = 1, \ldots, n$.

On the other hand, we have $X \subseteq X_1 \times \ldots \times X_n$. Thus, using (1) and Corollary 1 we obtain:

$$I(X) \leq I(X_1 \times \ldots \times X_n) = \left( \sum_{j=1}^{n} [I(X_j)]^p \right)^{1/p},$$

for $1 \leq p < \infty$, which in conjunction with (14) gives (5).

Now, we will show that for $p = \infty$:

$$I(X) \leq \max_{1 \leq j \leq n} I(X_j).$$

Given $\varepsilon > 0$ let $\{x_k\}$ be an $I(X) - \varepsilon$-separated sequence of $X$. Denote $x_k = (x_{k1}, \ldots, x_{kn})$, for $k = 1, 2, \ldots$ We have

$$\|x_k - x_l\| = \max_{1 \leq j \leq n} \|x_{kj} - x_{lj}\|_j \geq I(X) - \varepsilon, \quad \text{for } k, l \in \mathbb{N}, k \neq l.$$

From Ramsey's theorem it follows that for at least one of the indices $j$, $1 \leq j \leq n$, there exists a subsequence $\{y_k\}$ of $\{x_k\}$ such that

$$\|y_k - y_l\| = \|y_{kj} - y_{lj}\|_j \quad \text{for all } k, l \in \mathbb{N}.$$

The sequence $\{y_{kj}\}$ of elements of $X_j$ is therefore $I(X) - \varepsilon$-separated and we have

$$I(X) - \varepsilon \leq I(X_j) \leq \max_{1 \leq i \leq n} I(X_i),$$

for any $\varepsilon > 0$, which completes the proof of (6) and Theorem 1. □

As an immediate consequence of Theorems 1 and 2 we obtain the following corollary concerning the Kottman constant of product spaces.

**Corollary 2**

*Let $E_j$ be a Banach space for $j = 1, \dots, n$ and let $1 \leq p \leq \infty$. Then:*

$$K\big(l^p(E_1, \dots, E_n)\big) = \max_{1 \leq j \leq n} K(E_j). \tag{15}$$

*Proof.* For $p = \infty$ the statement simply follows from the equality (6) of Theorem 1. Indeed, putting $X = B(l^\infty(E_1, \dots, E_n))$ in (6) we have:

$$I\big(B\big(l^\infty(E_1, \dots, E_n)\big)\big) = \max_{1 \leq j \leq n} I\big(B(E_j)\big)$$

which is (15).

If $1 \leq p < \infty$ then putting in (8) of the proof of Theorem 2 $X_j = B(E_j)$ for $1 \leq j \leq n$, and using the same notation, we have:

$$f_j(x) = \|x\|_j \quad \text{for} \quad x \in E_j,$$

and

$$D_p = B(l^p(E_1, \dots, E_n)),$$

so (15) is an immediate consequence of (8). This completes the proof. □

Finally, let us mention that analogous estimations as in Theorems 1 and 2 may be obtained for the space $l^p(E_1, E_2, \dots)$, however they are not satisfactory since the series which appears on the right hand side of (5) need not be convergent.

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