DENSITY OF THE TRANSVERSALITY ON MANIFOLDS WITH CORNERS

by

JUAN MARGALEF ROIG AND ENRIQUE OUTERELO DOMINGUEZ

ABSTRACT

In this paper we establish a new concept of transversality on manifolds with corners, which is more restrictive than, and not equivalent to, the classical one.

Then, we prove for this transversality an analogue of the Thom's transversality theorem.

We introduce some notations. Let X and X′ be C^p ($p \in N \cup \{\infty\}$) paracompact manifolds with corners modeled on Banach spaces, and $k \in N \cup \{0\}$ with $k \leqslant p$. We consider the set $G^p(X,X') = \{(x,f)/x \in X$ and f is a C^p map defined on a neighborhood of x and with values into $X'\}$. Then the relation $(x,f)\ R_k\ (x',f')$ if and only if $x = x'$ and localitations of f and f′ have the same derivatives up to order k in x, is a relation of equivalence and the quotient set $G^p(X,X')/R_k$ is denoted by $J^k(X,X')$. For each $(x,f) \in G^p(X,X')$ the correspondent equivalence class is denoted by $[(x,f)]$ or $J^k_x f$ ($j^k_x f$ is the k–jet of f at x).

There are well defined source and target maps:
s: $J^k(X,X') \to X$, $s(j^k_x f) = x$, b: $J^k(X,X') \to X'$, $b(j^k_x f) = f(x)$. We denote by $T^{(k)}$ the unique topology in $J^k(X,X')$ such that for all $c = (U, \psi, (E,\Lambda))$, a chart of X, and all $c' = (U', \psi', (E',\Lambda'))$, a chart of X′, $J^k(U, U') \in T^{(k)}$, and the map $\pi^{c'}_c: J^k(U,U') \to E \times E' \times \mathcal{L}(E,E') \times \dots \times \mathcal{L}^k_s(E,E')$, defined by
$\pi^{c'}_c(j^k_x f) = (\psi(x), \psi' f \psi(x), D(\psi' f \psi)(x), \dots, D^k(\psi' f \psi)(x))$, is a homeomorphism of $J^k(U,U')$ on their image, when considering in $J^k(U,U')$ the topology $T^{(k)}|_{J^k(U,U')}$. ($\mathcal{L}^k_s(E,E')$ denote the set of k-linear continuous symmetric maps from E into E′).

(If $\delta X' = \Phi$, $\pi^{c'}_c(J^k(U,U')) = E \times E' \times \mathcal{L}(E,E') \times \dots \mathcal{L}^k_s(E,E')$).

If j^k: $C^p(X,X') \to c(X,J^k(X,X'))$ is the map defined by $j^k(f)(x) = j^k_x f$, we denote by $T_w(p,k)$ the topology in $C^p(X,X')$ induced by j^k from the strong topology $T^{(k)}_s$ on $c(X,J^k(X,X'))$ defined by the topology of X and the topology $T^{(k)}$ of $J^k(X,X')$ ($T_w(p,k)$ is Whitney's topology on $C^p(X,X')$ of order k). The topology $T_w(p,k)$ has a basis formed by the set $\{M(U)/U \in T^{(k)}_s\}$, where $M(U) = \{f \in C^p(X,X') \,/\, j^k f(X) \subset U\}$. Finally, if $A,B \subset R^n$, $D(A,B)$ means $\inf\{d(x,y) \,/\, x \in A, y \in B\}$.

Definition 1. Let f be a map of class p ($p \in NU\{\infty\}$) from X to X′ and let X″ be a submanifold of X′ (X and X′ are manifolds modeled on Banach spaces).

a) We say that f is transversal to X″ in $x \in X$ and we put $f \pitchfork_x X''$ if $f(x) \notin X''$ or, if $f(x) \in X''$, there exit $(U', \psi', (E', \Lambda'))$ chart of X′ adapted to X″ in $f(x) \in X''$ by (E'', Λ''), F′ topological supplementary of E″ in E′ and U open set in X such that $x \in U, f(U) \subset U'$ and the map

$$h: U \xrightarrow{f|_U} U' \xrightarrow{\psi'} \psi'(U') \xrightarrow{\theta^{-1}} E'' \times F' \xrightarrow{P_2} F'$$

is a submersion in $x \in U$, where $\theta: E'' \times F' \to E'$ is defined by $\theta(a,b) = a + b$ and $p_2(a,b) = b$.

b) If A is a subset of X, we say that f is transversal to X" over A, and we put $f \pitchfork_A X''$, if $f \pitchfork_x X''$ for every $x \in A$. If f is transversal to X″ over X, we will only write $f \pitchfork X''$.

Here submanifold means that for every $x'' \in X''$ there exist a chart $c' = (U', \psi', (E', \Lambda'))$ of X′ with $x'' \in U'$, E″ closed linear subspace of E′ having topological supplementary in E′ and $\Lambda'' \subset \mathcal{L}(E'',R)$ finite and linearly independent set such that $\psi'(x'') = 0$, $\psi'(U' \cap X'') = \psi'(U') \cap E''^{+}_{\Lambda''}$ and $E''^{+}_{\Lambda''} \subset E'^{+}_{\Lambda'}$ (shortly we say that c′ is a chart of X′ adapted to X″ in x″ by (E″, Λ″)).

h is submersion in x means that there exist $V^{h(x)}$ open neighborhood of h(x) and s: $V^{h(x)} \to \text{dom}(h)$ map of class p such that $sh(x) = x$ and $hs(x') = x'$ for every $x' \in V^{h(x)}$.

Theorem 2. Let f be a map of class p from X to X′, let X″ be a submanifold of X′ and $x \in f^{-1}(x'')$ with $\text{ind}(x) = k \geq 0$. Then the following statements are equivalent:

1) $T_{f(x)}X' = \text{im}\, T_x(f|_{B_k X}) + T_{f(x)}j(T_{f(x)}X'')$ and $(T_x(f|_{B_k X}))^{-1}(T_{f(x)}j(T_{f(x)}X''))$ has a topological supplementary in $T_x B_k X$, where j: $X'' \hookrightarrow X'$.

2) $f \pitchfork_x X''$

3) $(f|_{B_k X}) \pitchfork_x X''$.

Furthermore if $f \pitchfork_x X''$ we have that:

a) $T_{f(x)}X' = im(T_x f) + T_{f(x)}j\,(T_{f(x)}X'')$

b) $(T_x f)^{-1}\,(T_{f(x)}j\,(T_{f(x)}X''))$ has a topological supplementary in T_xX. These conditions form the classical definition of transversality. (X and X′ are manifolds modeled on Banach spaces).

Finally $x \epsilon$ int X, a) and b) imply that $f \pitchfork_x X''$. But the two preceding definitions of transversality are not equivalent. Consider for instance $\psi(t) = (t,t)$ from $\{ t \epsilon R / t \geqslant 0 \}$ to $\{ (x,y) \epsilon R^2 / x \geqslant 0 \}$ and the submanifold $x \geqslant 0, y = 0$.

Lemma 3. Let X be a manifold of class $p \epsilon N \cup \{ \infty \}$, Hausdorff, second countable with dimension $m \epsilon N$ and $\delta X = \emptyset$. Then $X = \bigsqcup_{i \epsilon N} X_i$, where X_i is a compact submanifold of X with $\delta X_i \neq \emptyset$, $\delta^2 X_i = \emptyset$ and int X_i is an open set of X and $X_i \subset X_{i+1}$ for every $i \epsilon N$.

Proof. We can suppose that $p = \infty$. It is well-known that there is $f: X \to R^+ \cup \{0\}$ proper map of class p. Then from Sard's theorem we have that $f(C(f))$ has measure zero in $R^+ \cup \{0\}$ and hence there is a sequence $\{ a_n \epsilon R^+ \cup \{0\} / n \epsilon N \}$ such that $a_n < a_{n+1}$ and a_n is a regular value of f for every $n \epsilon N$. Finally we take $X_n = f^{-1}([0,a_n]) = f^{-1}((\leftarrow, a_n])$.

Lemma 4. Let X, X′ be manifolds of class $p \epsilon N \cup \{ \infty \}$, Hausdorff, second countable with dim X = m and dim X′ = m′ ($m, m' \epsilon N$).

Let Y be a submanifold of X′, let Y_o be an open set of Y such that $\bar{Y}_o \subset Y$ and $B_k X = \bigcup_{i \epsilon N} X_i^k$ for all $k \epsilon \{ 0, \ldots, m \}$, following the preceding lemma and let $r \epsilon N$ with $r \leqslant p$. Then for each $i \epsilon N$ and for each $k \epsilon \{0, \ldots m\}$,

$$G_i^k = \{ g \epsilon C^p(X,X') / \quad x \epsilon X_i^k$$

with $g(x) \epsilon \bar{Y}_o$ we have that $g_{|B_k X} \pitchfork_x Y \}$ is an open set of $C^p(X,X')$ with the Whitney's topology $T_W(p,r)$.

Proof. We take $U_i^k = \{ j_x^r f \epsilon J^r(X_i^k, X') / f(x) \notin \bar{Y}_o$ or $f(x) \epsilon \bar{Y}_o$ and

$$T_{f(x)}X' = T_x f(T_x X_i^k) + T_{j(x)}j'\,(T_{f(x)}Y) \} \text{ where } j': Y \hookrightarrow X'.$$

Let $\{ j_n / n \epsilon N \}$ be a sequence of elements of $J^r(X_i^k, X') - U_i^k = C_i^k$ which converges to j_o in $J^r(X_i^k, X')$ with the topology $T^{(r)}$.

Suppose that $x_o = s(j_o)$, $x_o' = b(j_o)$, $x_n = s(j_n)$ and $x_n' = b(j_n)$ for each $n \epsilon N$. By continuity $x_o = \lim x_n$ and $x_o' = \lim x_n'$. On the other hand for every $n \epsilon N$,

$x'_n \in \bar{Y}_o$ and $T_{x'_n} X' \neq T_{x_n}(j_n)(T_{x_n} X_i^k) + T_{x'_n} j'(T_{x'_n} Y)$. Then $x'_o \in \bar{Y}_o$. Suppose that $j_o = [(x_o, f_o)]$, let $c = (U, \psi, (R^s, \Lambda))$ be a chart of X_i^k with $\psi(x_o) = 0$, $U \subset \mathrm{dom}(f_o)$ and let $c'(U, \psi', (R^{m'}, M))$ be a chart of X' adapted to Y in x'_o by (E', Λ') such that $f_o(U) \subset U'$. We take the map h_o defined by:

$$h_o : \psi(U) \xrightarrow{\psi-1} U \xrightarrow{f_o} U' \xrightarrow{\psi'} \psi'(U') \xrightarrow{j''} R^{m'} \xrightarrow{\theta^{-1}} E' \times$$

$$\times E'' \xrightarrow{p_2} E'',$$

where E'' is a supplementary of E' in $R^{m'}$, $\theta(x,y) = x + y$ and $p_2(x,y) = y$.

Let $j_n = [(x_n, f_n)]$ for every $n \in N$. Since $\lim x_n = x_o$ and $\lim x'_n = x'_o$, there is $n_o \in N$ such that for every $n \geqslant n_o$, $x_n \in U$ and $x'_n \in U'$. Then for every $n \geqslant n_o$ there is V_n open set of $\psi(U)$ with $\psi(x_n) \in V_n, f_n \psi^{-1}(V_n) \subset U'$ and $\psi^{-1}(V_n) \subset \mathrm{dom}(f_n)$ and so we can define h_n as follows:

$$h_n : V_n \xrightarrow{\psi-1} \psi^{-1}(V_n) \xrightarrow{f_n} U' \xrightarrow{\psi'} \psi'(U') \xrightarrow{j''} R^{m'} \xrightarrow{\theta^{-1}} E' \times$$

$$\times E'' \xrightarrow{p_2} E''.$$

The condition $T_{x'_n} X' \neq T_{x_n}(j_n)(T_{x_n} X_i^k) + T_{x'_n} j'(T_{x'_n} Y)$ implies that $Dh_n(\psi(x_n))$ is not surjective.

Since $\{h: R^s \to E''/h$ is linear and surjective$\}$ is an open set of $\mathcal{L}(R^s, E'')$, we have that $Dh_o(\psi(x_o)) = \lim Dh_n(\psi(x_n))$ is not surjective and we can prove that $T_{x'_o} X' \neq T_{x_o}(j_o)(T_{x_o} X_i^k) + T_{x'_o} j'(T_{x'_o} Y)$ and $j_o \in C_i^k$. Then C_i^k is closed and U_i^k is open of $J^r(X_i^k, X')$ and $M(U_i^k) = \{f \in C^p(X_i^k, X')/j^r f(X_i^k) \subset U_i^k\}$ is open of $C^p(X_i^k, X')$ with the topology $T_W(p,r)$. On the other hand, since $j: X_i^k \hookrightarrow X$ is proper, the map $j^*: C^p(X,X') \to C^p(X_i^k, X')$ defined by $j^*(f) = f_{|X_i^k}$ is continuous with the Whitney's topology. Then $A = (j^*)^{-1}(M(U_i^k))$ is open in $C^p(X,X')$ with the topology $T_W(p,r)$ and $A = G_i^k$.

Lemma 5. Let X,X′ be like in Lemma 4. Suppose that $\delta X' = \emptyset$ and let $r,k \in N \cup \{0\}$ with $k \leqslant p - r$. Then the map $J^r: (C^p(X,X'), T_W(p,k+r)) \to (C^{p-r}(X, J^r(X,X')), T_W(p-r,k))$ is continuous.

Proof. Let U be an open set of $J^k(X, J^r(X,X'))$ with the topology $T^{(k)}$. Then $M(U) = \{f \in C^{p-r}(X, J^r(X,X'))/j^k f(X) \subset U\}$ is an element of the base of the topology $T_W(p-r,k)$ of $C^p(X, J^r(X,X'))$.

The map $\alpha: J^{k+r}(X,X') \to J^k(X, J^r(X,X'))$ defined by $\alpha(j_x^{k+r} f) = [(x, j^r f)]$ is continuous and $M(\alpha^{-1}(U)) = (j^r)^{-1}(M(U))$ is an open set in $T_W(p,k+r)$.

Theorem 6. Let X,X' be manifolds of class $p\epsilon N\cup\{\infty\}$, Hausdorff, second countable with $\dim X = n$, $\dim X' = n'$ and $\delta X' = \emptyset$ $(n,n'\epsilon N)$.

Let $r\epsilon N\cup\{0\}$ with $r < p$ and let Y be a submanifold of class $p-r$ of $J^r(X,X')$ with fixed codimension and strongly well situated (that is $\delta Y = \delta J^r(X,X') \cap Y$ and if $x\epsilon Y \cap B_k J^r(X,X')$, $T_x Y + T_x B_k J^r(X,X') = T_x J^r(X,X')$).

Let $s\epsilon N$ with $s \leqslant p-r$ and $p-r-1 \geqslant n - \text{codin } Y$. then

$$T_Y^r = \{ g\epsilon C^p(X,X')/j^r g \pitchfork Y \}$$

is residual, and hence dense, in $C^p(X,X')$ with the topology $T_W(p,r+s)$.

Proof. Since Y is a submanifold of class $p-r$ of $J^r(X,X')$, $Y = G \cap C$ where G is an open set of $J^r(X,X')$ and C is a closed set of $J^r(X,X')$ with the topology $T^{(r)}$. For each $y\epsilon Y$ there is V^y open set of $J^r(X,X')$ such that $y\epsilon V^y \subset \overline{V}^y \subset G$.

Then $V^y \cap Y = Y_y$ is an open set of Y such that $y\epsilon Y_y$, $\overline{Y}_y \subset G \cap C = Y$, where $\overline{Y}_y$ is the closure of Y_y in $J^r(X,X')$. Furthermore if we take V^y conveniently small, we can suppose that:

a) $\overline{Y}_y$ is compact.

b) There is $C_y = (U_y, \psi_y, (R^n, \Lambda_y)$ chart of X with $\psi_y(s(y)) = 0$, $\overline{U}_y$ compact and $\psi_y(U_y)$ bounded and there is $C'_y = (U'_y, \psi'_y, R^{n'})$ chart of X' with $\psi'_y(b(y)) = 0$ and $\psi'_y(U'_y)$ bounded such that $(s,b)(\overline{Y}_y) \subset U_y \times U'_y$.

Since $J^r(X,X')$ is second countable, we have that $Y = \bigcup_{m\epsilon N} Y_{y_m}$ and we will write $Y_{y_m} = Y_m$ for every $m\epsilon N$.

Let $B_k X = \bigcup_{i\epsilon N} X_i^k$, like in Lemma 3, for each $k\epsilon \{0,\dots,n\}$.

We take $T(r,m,k,i) = \{g\epsilon C^p(X,X')/ \forall x\epsilon X_i^k$ with $j_x^r g\epsilon \overline{Y}_m$, it happens that $(j_r g)_{|B_k X} \pitchfork_x Y\}$. It is clear that $T_Y^r = \{g\epsilon C^p(X,X')/j^r g \pitchfork Y\} = \cap\{T(r,m,k,i)/m,k,i$ vary$\}$. We shall see that $T(r,m,k,i)$ is an open set of $C^p(X,X')$ with the topology $T_W(p,r+s)$.

By Lemma 4, $H(i,m,k) = \{h\epsilon C^{p-r}(X,J^r(X,X'))/ \forall x\epsilon X_i^k$ with $h(x)\epsilon\overline{Y}_m$ it happens that $h_{|B_k X} \pitchfork_x Y\}$ is an open set in $C^{p-r}(X,J^r(X,X'))$ with the topology $T_W(p-r,s)$. By Lemma 5, $(j^r)^{-1}(H(i,m,k))$ is open set in $C^p(X,X')$ with the topology $T_W(p,r+s)$ and $(j^r)^{-1}(H(i,m,k)) = T(r,m,k,i)$.

We shall see that $T(r,m,k,i)$ is dense in $C^p(X,X')$ with the topology $T_W(p,r+s)$.

By condition b); putting $\psi_{y_m} = \psi_m$, $U_{y_m} = U_m$ and so on, we have that $\psi_m(U_m) \supset \psi_m\, s(\overline{Y}_m)$ and then there are open sets, A_m and B_m, of R^n such that $\psi_m s(\overline{Y}_m) \subset A_m \subset \overline{A}_m \subset B_m \subset \overline{B}_m \subset \psi_m(U_m)$. Since $\overline{B}_m$ is compact, it follows that $D(\overline{B}_m, R^n - \psi_m(U_m)) > 0$ and there is α_m: $R^n \to [0,1]$ of class ∞

such that $\alpha_m(y)=1$ if $y\epsilon\overline{A}_m$, $\alpha_m(y)=0$ if $y\notin B_m$. Similarly $\psi'_m b(\overline{Y}_m) \subset \psi'_m(U'_m)$ and there are open sets A'_m, B'_m of $R^{n'}$ such that $\psi'_m b(\overline{Y}_m) \subset A'_m \subset \overline{A}'_m \subset B'_m \subset \overline{B}'_m \subset \psi'_m(U'_m)$ and there is $\alpha'_m: R^{n'} \to [0,1]$ of class ∞ such that $\alpha'_m(y')=1$ if $y'\epsilon\overline{A}'_m$, $\alpha'_m(y')=0$ if $y'\notin B'_m$.

Let $f\epsilon C^p(X,X')$, $M' = R^{n'} \times \mathcal{L}(R^n,R^{n'}) \times \ldots \times \mathcal{L}^r_s(R^n,R^{n'})$ and let $\overline{Y}^*_m$ be an open set of Y with $\overline{Y}_m \subset \overline{Y}^*_m$ such that $\psi_m s(\overline{Y}^*_m) \subset A_m$ and $\overline{\psi'_m b(\overline{Y}^*_m)} \subset A'_m$.

By continuity it is clear that there is an open set, M_1, of M' such that $0\epsilon M_1$ and if $x\epsilon U_m$ and $f(x)\epsilon U'_m$, then

$$\lambda(m',x) = (\alpha_m\psi_m(x)\,.\,\alpha'_m\,\psi'_m\,f(x)\,m'\,\psi_m(x) + \psi'_m\,f(x))\,\epsilon\,\psi'_m(U'_m)$$

for every $m'\epsilon M_1$, where $m'(y) = m'_0 + m'_1(y) + \ldots + m'_r(y,\overset{r)}{\ldots},y)$ for every $y\epsilon R^n$, $m' = (m'_0,m'_1,\ldots,m'_r)\,\epsilon\,M_1$. Indeed: Since $\psi_m(U_m)$ is bounded there is an open set, M_1, of M' such that $0\epsilon M_1$ and for every $m'\epsilon M_1$ and every $x\epsilon U_m$, $\|m'\psi_m(x)\| < D(\overline{B}'_m, R^{n'} - \psi'_m(U'_m))$, and M_1 is in fact the solution.

We take ρ: $M_1 \to C^p(X,X')$ defined by $\rho(m')(x) = f(x)$ if $x\notin U_m$ or $f(x)\notin U'_m$, $\rho(m')(x) = (\psi'_m)^{-1}\lambda(m',x)$ if $x\epsilon U_m$ and $f(x)\epsilon U'_m$. It is clear that $\rho(m')$ is a map of class p. Furthermore, from the fact that e_ρ: $M_1 \times X \to X'$ is a map of class p it follows that ρ is a C^p-representation.

Then the map $j^r\rho$: $M_1 \to C^{p-r}(X,J^r(X,X'))$ defined by $(j^r\rho)(m') = j^r\rho(m')$, is a C^{p-r}-representation, which follows by localization.

Let $\xi = \frac{1}{2}\min\{D(\text{sop}\,\alpha'_m, R^{n'} - \psi'_m(U'_m)), D(\overline{\psi'_m\,b\,\overline{Y}^*_m}, R^{n'} - A'_m)\}$ and $M = \{m'\epsilon M_1/\|m'\psi_m(x)\| < \xi, \forall\, x\epsilon U_m$ such that $\psi_m(x)\epsilon\,\text{sop}\,\alpha_m\}$ which is an open set of M_1 with $0\epsilon M_1$.

Let $(m',x)\,\epsilon\,M \times X$ such that $j^r_x(\rho(m'))\,\epsilon\,\overline{Y}^*_m$. Then $x\epsilon s(\overline{Y}^*_m) \subset U_m$ and hence $\psi_m(x)\epsilon\,\text{sop}\,\alpha_m$, in fact $\alpha_m\psi_m(x) = 1$, and $\rho(m')(x)\,\epsilon\,b(\overline{Y}^*_m) \subset U'_m$. Then $(x,\rho(m')(x))\,\epsilon\,U_m \times U'_m$. On the other hand $f(x)\,\epsilon\,U'_m$ and we have that

$$z = d(\psi'_m f(x), \psi'_m\rho(m')(x)) = d(\psi'_m\,f(x), \lambda(m',x)) \leq \|m'\psi_m(x)\| < \xi.$$

Since $\psi'_m\rho(m')(x)\epsilon\psi'_m b\overline{Y}^*_m$ and $\xi < D(\overline{\psi'_m\,b\,(\overline{Y}^*_m)}, R^{n'} - A'_m)$ we have that $\psi'_m f(x)\epsilon A'_m \subset (\alpha'_m)^{-1}(1)$.

Furthermore for every $y\epsilon A_m \supset \psi_m\,s(\overline{Y}^*_m)$ we have that $\alpha_m(y) = 1$ and hence $\psi'_m\rho(m')(x) = m'\psi_m(x) + \psi'_m f(x)\,\epsilon\,\psi'_m(U'_m)$.

Since $\psi'_m f(x)\,\epsilon\,A'_m$ and $\psi_m(x)\,\epsilon\,A_m$, we have that there is an open set, V^x, in $\psi_m^{-1}(A_m) \cap f^{-1}(U'_m) \subset U_m$ such that $x\epsilon V^x$ and for every $y\epsilon V^x$, $\psi'_m f(y)\epsilon A'_m$.

Then for every $(m'',y)\epsilon M \times V^x$ we have that $\psi'_m\rho(m'')(y) = m''\psi_m(y) + \psi'_m f(y)$ and $e_{j^r\rho}(m'',y) = j^r_y\,\rho(m'')\,\epsilon\,J^r(V^x, U'_m)$.

We consider the following commutative diagram

$$\begin{array}{ccc} M \times V^x & \xrightarrow{\;e_{j^r\rho}\;} & J^r(V^x, U'_m) \\ \Big\downarrow{\scriptstyle 1 \times \psi_m} & & \Big\downarrow{\scriptstyle \pi^{c'_m}_{c_m}} \\ M \times \psi_m(V^x) & \xrightarrow{\;\;h\;\;} & \psi_m(V^x) \times \psi'_m(U'_m) \times \mathcal{L}(R^n,R^{n'}) \times \ldots \times \mathcal{L}^r_s(R^n,R^{n'}) \end{array}$$

where $h(m'',z) = (z, m''(z) + \psi'_m f \psi_m^{-1}(z), Dt_{m''}(z), \ldots, D^r t_{m''}(z))$, $t_{m''}(y) = m''(y) + \psi'_m\, f\, \psi_m^{-1}(y) = \psi'_m\, \rho\,(m'')\, \psi_m^{-1}(y)$.

Then $h : M' \times \psi_m(v^x) \to \psi_m(V^x) \times R^{n'} \times \mathcal{L}(R^n,R^{n'}) \times \ldots \times \mathcal{L}^r_s(R^n,R^{n'})$, defined like h, is a diffeomorphism of class p–r and hence $h{:}M \times \psi_m(V^x) \to \mathrm{im}(h)$ is a diffeomorphism and $e_{j^r\rho}: M \times V^x \to \mathrm{im}(e_{j^r\rho})$ is a diffeomorphism of class p–r. Then $p = e_{j^r f}\big|_{M \times B_k X \cap V^x} : M \times B_k X \cap V^x \to B_k J^r(X,X') \cap J^r(V^x, U'_m)$ is a diffeomorphism of class p–r from $M \times B_k X \cap V^x$ to im(p), where k = ind(x). By the hypothesis over Y it follows that $e_{j^r\rho} \pitchfork_{(m',x)} \overline{Y}^*_m$ and by [1] we have that $M_{\overline{Y}^*_m} = \{ m' \epsilon M / j^r\rho(m') \pitchfork \overline{Y}^*_m \}$ is a residual set in M. Then there exists a sequence, (m'_p), of elements of $M_{\overline{Y}^*_m}$, which converges to 0 and hence $\rho(m'_p) \to f$ with the topology $T_w(p,r+s)$ in $C^p(X,X')$. Indeed: Following the properties of the Whitney's topology. It suffices to prove:

1) $\rho(m'_p)\big|_{X-\overline{U}_m} = f\big|_{X-\overline{U}_m}$

2) if α is a metric in $J^{r+s}(X,X')$,

$\{ (j^{r+s}\rho\,(m'_p))\big|_{\overline{U}_m} \;/\; p \epsilon N \}$ converges uniformly to $(j^{r+s}f)\big|_{\overline{U}_m}$

1) is obvious from the definition of ρ.

In order to prove 2), we remember that $e_{j^r\rho}: M \times X \to J^{r+s}(X,X')$ is continuous and hence $j^{r+s}\rho: M \to C^{p-(r+s)}(X, J^{r+s}(X,X'))$ is also continuous with the compact-open topology and then the statement 2) follows from the fact that the compact-open topology describes the uniform convergence over the compacts.

The H $= \{ g \epsilon C^p(X,X')/j^r g \pitchfork \overline{Y}^*_m \}$ is dense in $C^p(X,X')$ with the topology $T_w(p,r+s)$. Finally $H \subset T(r,m,k,i)$ and $T^r_Y = \{ g \epsilon C^p(X,X')/j^r g \pitchfork Y \}$ is residual in $C^p(X,X')$ with the topology $T_w(p,s+r)$.

***Corollary** 7.* Let X,X′ be Hausdorff and second countable manifolds of class $p \epsilon N \cup \{\infty\}$, with dim X = n, dim X′ = n′ and $\delta X' = \emptyset$. Let Y′ be a submanifold of X′ with $\delta Y' = \emptyset$ and cod Y′ fixed and let $s \epsilon N$ with $s \leqslant p$ and $p - 1 \geqslant n - \text{cod}(Y')$. Then $T_{Y'} = \{ g \epsilon C^p(X,X') / g \pitchfork Y' \}$ is residual in $C^p(X,X')$ with the topology $T_w(p,s)$.

BIBLIOGRAPHY

[1] MARGALEF, J. - OUTERELO, E.: "Un teorema de densidad parametrizado para variedades con borde anguloso". (Actas del Congreso de la Universidad de Murcia, enero, 1985).

C.E.C.I.M.E. del
C.S.I.C.
MADRID 28006

Facultad de Matemáticas
Universidad Complutense
MADRID 28040