INFINITE-FACTORABLE HOLOMORPHIC MAPPINGS ON LOCALLY CONVEX SPACES

by

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ABSTRACT:

We introduce the notion of holomorphic mappings of uniformly bounded A-type between locally convex spaces where A denotes any normed operator ideal in the sense of A. Pietsch. In this note we consider such holomorphic mappings for the operator ideals L_∞, S_∞ and K_∞, respectively, of all ∞-factorable, strongly ∞-factorable and ∞-compact operators.

INTRODUCTION

The main purpose of this note is to characterize those holomorphic mappings of uniformly bounded type $f:E\to F$ (in the sense of J.F. Colombeau, J. Mujica [3]) between locally convex spaces E,F which possess an extension $\tilde{f}$: $G\to F$ of the same kind for all locally convex spaces G containing E as a topological subspace. We shall introduce the notion of holomorphic mappings of uniformly bounded A-type where A stands for any operator ideal in the sense of A. Pietsch [16], and we shall prove that the above holomorphic extension problem can be solved for such holomorphic mappings with respect to the operator ideal S_∞ of all strongly ∞-factorable operators. In this note we shall consider also holomorphic mappings of uniformly bounded L_∞- and K_∞-type where L_∞ (resp. K_∞) denotes the operator ideal of all ∞-factorable (resp. ∞-compact) operators.

In the first section we introduce the notion of an ideal of polynomial operators which is a natural generalization of the concept of an operator ideal in the sense of A. Pietsch.

In section 2 we define the polynomial ideal of ∞-factorable, strongly ∞-factorable and ∞-compact polynomials, respectively. We shall show that the ∞-compact m-homogeneous polynomials can be represented as a certain infinite series of one-dimensional m-homogeneous polynomials. From this it follows that each continuous polynomial defined on an ϵ-(S)-space has an infinite series representation analogous to continuous polynomials on nuclear spaces.

In the third section it will be shown that the polynomial ideal of ∞-factorable (resp. strongly ∞-factorable, ∞-compact) polynomials form a holomorphy type Θ in the sense of L. Nachbin [15] which is a consequence of a general result.

In section 4 we introduce the holomorphic mappings of uniformly bounded $\mathcal{A}$-type and give conditions on the domain space under which a holomorphic mapping of uniformly bounded type is of L_∞-(resp. S_∞-, K_∞-) type.

In section 5 we shall prove that each holomorphic mapping of uniformly bounded L_∞-type $f:E\to F$ has an extensión $\widetilde{f}:G\to F_n''$ of the same kind for all locally convex spaces $G\supset E$ where F_n'' denotes the bidual space of F equipped with the natural topology, and that a holomorphic mapping of uniformly bounded type $f:E\to F$ is holomorphic of uniformly bounded S_∞-type if and only if f can be extended to a holomorphic mapping of uniformly bounded type $\widetilde{f}:G\to F$ for very $G\supset E$.

NOTATIONS AND TERMINOLOGY

In this note all locally convex spaces are assumed to be complex vector spaces. If E is a locally convex space (shortly l.c.s.), then $\mathcal{U}(E)$ denotes a O-neighbourhood basis of closed, absolutely convex O-neighbourhoods in E and $\mathcal{B}(E)$ a fundamental system of closed, absolutely convex, bounded subsets of E. For $U\epsilon\mathcal{U}(E)$ and $B\epsilon\mathcal{B}(E)$ we denote by E_U and E_B the associated normed spaces and we shall write $K_U:E\to E_U$ and $J_B:E_B\to E$ for the canonical mappings. We shall consider K_U also as a mapping from E into the completion $\widetilde{E}_U$ of E_U. If U, $V\epsilon\mathcal{U}(E)$ with $V\subset U$ then $K_{UV}:E_V\to E_U$ denotes the canonical mapping and $\widetilde{K}_{UV}:\widetilde{E}_V\to\widetilde{E}_U$ the continuous extension of K_{UV}. For l.c.s. E and F, $\mathcal{L}(E;F)$ denotes the vector space of all continuous linear mappings from E into F.

Let E,F be Banach spaces. We shall write $I_E:E\to E''$ for the evaluation mapping. An operator $A\epsilon\mathcal{L}(E;F)$ is called ∞-factorable if there exist a measure space (Ω,μ) and mappings $R\epsilon\mathcal{L}(E;L_\infty(\Omega,\mu))$ and $S\epsilon\mathcal{L}(L_\infty(\Omega,\mu);F'')$ such that $I_F\circ A=S\circ R$. $A\epsilon\mathcal{L}(E;F)$ is said to be strongly ∞-factorable if A factors through $L_\infty(\Omega,\mu)$ for a suitable measure space (Ω,μ). An operator $A\epsilon\mathcal{L}(E;F)$ is called ∞-compact if A has a factorization $A=S\circ R$ where $R:E\to c_o$ and $S:c_o\to F$ are compact operators (cf. A. Pietsch [17], 19.3 and 18.3). The operator ideal of all ∞-factorable, strongly ∞-factorable and ∞-compact operators, respectively, is denoted by L_∞, S_∞ and K_∞.

A l.c.s E is called ϵ-space if for every $U\epsilon\mathcal{U}(E)$ there exists a $V\epsilon\mathcal{U}(E)$ contained in U such that $\widetilde{K}_{UV}:\widetilde{E}_V\to\widetilde{E}_U$ is ∞-factorable. Let us remark that a Banach space E is an ϵ-space if and only if E is an $\mathcal{L}_\infty$-space in the sense of J. Lindenstrauss, A. Pelczynski [13]. For the general theory and examples of ϵ-spaces we refer to R. Hollstein [7], [8], and [9]. A l.c.s. E is said to be an S_∞-space if for every $U \epsilon \mathcal{U}(E)$ the canonical mapping $\widetilde{K}_{UV}:\widetilde{E}_V \to \widetilde{E}_U$ is strongly ∞-factorable for a suitable O-neighbourhood $V \epsilon \mathcal{U}(E)$ with $V \subset U$.

For l.c.s E,F and $m \epsilon N$ we shall denote by $L(^m E;F)$ the space of all continuous m-linear mappings from $E^m = Ex...xE$ (m times) into F and by $L_s(^m E;F)$ the subspace of all symmetric continuous m-linear mappings. If $A \epsilon L(^m E;F)$, $m \epsilon \mathbb{N}$ and $x \epsilon E$, we shall write $Ax^m = A(x,...,x)$ (m times) and denote by $\hat{A}:E \to F$ the m-homogeneous polynomial given by $\hat{A}(x) = Ax^m$.

1. Polynomial ideals

A. Pietsch [17] has recently introduced the concept of ideals of multilinear functionals. In this note we shall consider ideals of polynomial operators.

Let $P^{(m)}$ be the class of all continuous m-homogeneous polynomials between arbitrary Banach spaces. For Banach spaces E,F we shall then write $P^{(m)}(E;F)$ for the vector space of all continuous m-homogeneous polynomials from E into F in place of the usual notation $P(^m E;F)$. $P^{(m)}(E;F)$ will be equipped with the norm $\|P\| = \sup \{ \|P(x)\| : \|x\| \leqslant 1 \}$ which is called the current norm.

1.1 Definition. Let $m \epsilon \mathbb{N}$. An ideal $Q^{(m)}$ of m-homogeneous polynomials (shortly m-polynomial ideal) is a subclass of $P^{(m)}$ such that the components $Q^{(m)}(E,F)$ satisfy the following conditions

(1) $\psi^m \otimes y \in Q^{(m)}(E;F)$ for all $\psi \epsilon E'$ and $y \epsilon F$.

(2) If $P_1, P_2 \in Q^{(m)}(E;F)$, then $P_1 + P_2 \in Q^{(m)}(E;F)$.

(3) If $R \in L(E_0;E)$, $P \in Q^{(m)}(E;F)$ and $S \in L(F;F_0)$, then $S \circ P \circ R \in Q^{(m)}(E_0,F_0)$.

We remark that for m=1 in 1.1 we obtain the definition of an operator ideal. It is obvious that the components $Q^{(m)}(E;F)$ of a given m-polynomial ideal $Q^{(m)}$ are linear subspaces of $P^{(m)}(E;F)$.

1.2 Definition. Let $Q^{(m)}$ be an m-polynomial ideal. A mapping γ from $Q^{(m)}$ into $\mathbb{R}^+$ is called m-polynomial ideal norm if, for arbitrary Banach spaces E,E_0,F and F_0, the following conditions are satisfied

(1) $\gamma(\psi^m \otimes y) = \|\psi\|^m \|y\|$ for all $\psi \epsilon E'$ and $y \epsilon F$.

(2) $\gamma(P_1 + P_2) \leqslant \gamma(P_1) + \gamma(P_2)$ for all $P_1, P_2 \epsilon Q^{(m)}(E;F)$.

(3) For $R \epsilon L(E_0,F)$, $P \epsilon Q^{(m)}(E;F)$ and $S \epsilon L(F,F_0)$ one has

$$\gamma(S \circ P \circ R) \leqslant \|S\| \gamma(P) \|R\|^m.$$

The couple $[Q^{(m)},\gamma]$ is then called a normed m-polynomial ideal.

It is easy to verify that for $P \epsilon Q^{(m)}(E;F)$ one has $\|P\| \leqslant \gamma(P)$ if $[Q^{(m)},\gamma]$ is a normed polynomial ideal. Next we give some examples.

1.3 Examples. The class $P^{(m)}$ of all continuous m-homogeneous polynomials is a normed m-polynomial ideal with respect to the current norm. This holds also true for the class $F^{(m)}$ of all m-homogeneous polynomials of finite type.

The Banach space $K^{(m)}(E;F)$ of m-homogeneous compact polynomials from E into F is the closure of $F^{(m)}(E;F)$ in $P^{(m)}(E;F)$ with the topology induced from $P^{(m)}(E;F)$. The class $K^{(m)}$ of all m-homogeneous compact polynomials is a normed m-polynomial ideal with respect to current norm.

A polynomial $P \epsilon P^{(m)}(E;F)$ is called nuclear (see e.g. [5]) if P can be represented by a series $P(x) = \sum_{j=1}^{\infty} \psi_j^m(x)\, y_j$ where $\psi_j \epsilon E'$, $y_j \epsilon F$ and $\sum_{j=1}^{\infty} \|\psi_j\|^m \|y_j\| < \infty$.

$P_N^{(m)}(E;F)$ will denote the vector space of all nuclear m-homogeneous polynomials from E into F equipped with the nuclear norm

$$\|P\|_N = \inf \sum_{j=1}^{\infty} \|\psi_j\|^m \|y_j\|$$

where the infimum is taken over all possible representations of P. The class $P_N^{(m)}$ of all nuclear m-homogeneous polynomials is a normed m-polynomial ideal with respect to the nuclear norm.

For each normed operator ideal $[A,\alpha]$ one can construct an m-polynomial ideal $P^{(m)} \circ A$ in the following way (for the compose of an m-functional ideal and operator ideals $A_1,\dots,A_m$ we refer to A. Pietsch [17]): Let E,F be Banach spaces. A polynomial P belongs to $P^{(m)} \circ A(E;F)$ if there exist a Banach space G and mappings $A \epsilon A(E;G)$, $Q \epsilon P^{(m)}(G;F)$ such that $P = Q \circ A$. The vector space $P^{(m)} \circ A(E;F)$ will be equipped with the norm

$$\alpha^{(m)}(P) = \inf \|Q\| (\alpha(A))^m$$

where the infimum is taken over all possible decompositions. It is not difficult to show that $[P^{(m)} \circ A, \alpha^{(m)}]$ is a normed m-polynomial ideal.

2. INFINITE-FACTORABE, STRONGLY INFINITE-FACTORABLE AND INFINITE-COMPACT POLYNIMIALS

We start with the following definition

2.1 Definition. Let E,F be Banach spaces. An m-homogeneous polynomial $P \epsilon P^{(m)}(E;F)$ is called

(1) ∞-factorable if there exists a measure space (Ω,μ) such that $I_F \circ P$ has a factorization $I_F \circ P = Q_1 \circ S_1$ where $S_1 \epsilon L(E;L_\infty(\Omega,\mu))$ and $Q_1 \epsilon P^{(m)}(L_\infty(\Omega,\mu);F'')$

(2) strongly ∞-factorable if $P = Q_2 \circ S_2$ for $S_2 \epsilon L(E;L_\infty(\Omega,\mu))$ and $Q_2 \epsilon P^{(m)}(L_\infty(\Omega,\mu);F)$.
(3) ∞-compact if there exist a compact operator $S_3 \epsilon L(E;c_o)$ and a polynomial $Q_3 \epsilon P^{(m)}(c_o;F)$ such that $P = Q_3 \circ S_3$.

We denote by $L_\infty{}^{(m)}(E;F)$ (resp. $S_\infty^{(m)}(E;F)$, $K_\infty^{(m)}(E;F)$) the vector space of all ∞-factorable (resp. strongly ∞-factorable, ∞-compact) m-homogeneous polynomials equipped with the norm.

$$\lambda_\infty^{(m)}(P) = \inf \|Q_1\| \, \|S_1\|^m$$

(resp. $\sigma_\infty^{(m)}(P) = \inf \|Q_2\| \, \|S_2\|^m$, $k_\infty^{(m)}(P) = \inf \|Q_3\| \, \|S_3\|^m$)

the infimum taken over all possible factorizations. Taking m=1 we obtain the normed operator ideal L_∞(resp. S_∞, K_∞) of all ∞-factorable (resp. strongly ∞-factorable, ∞-compact) operators.

By definition, one has for all Banach spaces E,F

$$P_N^{(m)}(E;F) \subset K_\infty^{(m)}(E;F) \subset S_\infty^{(m)}(E;F) \subset L_\infty^{(m)}(E;F) \subset P^{(m)}(E;F)$$

where the canonical embedding are continuous.

The following proposition can easily be verified.

2.2 Proposition. *The classes $L_\infty^{(m)}$, $S_\infty^{(m)}$, and $K_\infty^{(m)}$, respectively, of ∞-factorable, strongly ∞-factorable and ∞-compact m-homogeneous polynomials are normed m-polynomial ideals with respect to the norms $\lambda_\infty^{(m)}$, $\sigma_\infty^{(m)}$ and $k_\infty^{(m)}$. Furthermore, one has $L_\infty^{(m)} = P^{(m)} \circ L_\infty$, $S_\infty^{(m)} = P^{(m)} \circ S_\infty$ and $K_\infty^{(m)} = P^{(m)} \circ K_\infty$ as normed polynomial ideals.*

Next we show that each ∞-compact polynomial has an infinite series representation. First we need some notations.

2.3 Definition. Let F be a Banach space and let $m \epsilon \mathbb{N}$. A family $(a_{k_1,\ldots,k_m})_{k_1,\ldots,k_m=1}^{\infty}$ in F is said to be w_m-summable if

$$\sum_{k_1,\ldots,k_m=1}^{\infty} a_{k_1,\ldots,k_m} \xi_{k_1}^{(1)} \cdots \xi_{k_m}^{(m)} = \lim_{N\to\infty} \sum_{k_1,\ldots,k_m=1}^{N} a_{k_1,\ldots,k_m} \xi_{k_1}^{(1)} \cdots \xi_{k_m}^{(m)}$$

exists for all sequences $\xi^{(i)} = (\xi_j^{(i)})_{j=1}^{\infty} \epsilon c_0$ and all i=1,...,m.

We remark that for m=1 we obtain the definition of a weakly summable sequence in F.

We denote by $l_1({}^m F)$ the vector space of all w_m-summable families in F equipped with the norm

$$\epsilon((a_{k_1,\ldots,k_m})) = \sup \| \sum_{k_1,\ldots,k_m=1}^{\infty} a_{k_1,\ldots,k_m} \xi^{(1)}_{k_1} \cdots \xi^{(m)}_{k_m} \|$$

the supremum taken over all sequences $\xi^{(i)} = (\xi^{(i)}_j)_j \in B_0$ and all i=1,...,m where B_0 denotes the unit ball in c_0. The supremum exists since the m-linear mapping D: $(c_0)^m \to F$ defined by $D(\xi^{(1)}, \ldots, \xi^{(m)}) = \sum_{k_1,\ldots,k_m=1}^{\infty} a_{k_1,\ldots,k_m} \xi^{(1)}_{k_1} \cdots \xi^{(m)}_{k_m}$ is separately continuous and hence continuous.

Now we show that a w_2-summable double sequence (a_{ik}) in $\mathbb{C}$ need not be (absolutely) summable.

Let $A: c_0 \to l_1$ be a continuous linear mapping which is not nuclear. Define the double sequence (a_{ik}) by $Ae_i = (a_{ik})_{k=1}^{\infty}$, i=1,...,m, where e_i denotes the i th unit vector in c_0. Then for each $(\xi_i) \in c_0$ the sequence $(\sum_{i=1}^{\infty} \xi_i a_{ik})_k$ lies in l_1, hence $(a_{ik})_{i,k=1}^{\infty}$ is w_2-summable. On the other hand, (a_{ik}) cannot be absolutely summable since by assumption A is not nuclear.

For a multi-index $k = (k_1, \ldots, k_m) \in \mathbb{N}^m$ and a sequence $(\psi_i)_{i=1}^{\infty}$ in the dual E′ of a locally convex space E we set $\psi_k(x) := \psi_{k_1}(x) \cdots \psi_{k_m}(x)$.

Now we prove

2.4 Proposition. *Let* E,F *be Banach spaces. A polynominal* $P \in P^{(m)}(E;F)$ *is* ∞*-compact if and only if* P *can be represented by a series*

$$P(x) = \sum_{k \in \mathbb{N}^m} a_k \psi_k(x), \qquad x \in E$$

for a suitable 0-sequence (ψ_j) *in* E′ *and a* w_m*-summable family* $(a_k)_{k \in \mathbb{N}^m}$ *in* F. *Furthermore, one has*

$$k^{(m)}_{\infty}(P) \leqslant \tau^{(m)}(P) \leqslant \frac{m^m}{m!} k^{(m)}_{\infty}(P)$$

where the norm $\tau^{(m)}$ *in* $K^{(m)}_{\infty}(E;F)$ *is defined by*

$$\tau^{(m)}(P) := \inf \epsilon((a_k)) \sup \left\{ \|\psi_j\|^m : j \in \mathbb{N} \right\}$$

the infimum taken over all possible representations.

Proof. Let $P \in K^{(m)}_{\infty}(E;F)$ and $\epsilon > 0$. By definition, there exist a compact operator $S \in L(E;c_0)$ and a polynomial $Q \in P^{(m)}(c_0;F)$ such that $P = Q \circ S$ and $k^{(m)}_{\infty}(P) + \epsilon >$

$\|Q\| \|S\|^m$. The operator S can be represented by a series

$$S(x) = \sum_{j=1}^{\infty} \psi_j(x)e_j, \quad x\epsilon E$$

where e_j is the j-th unit vector in c_0 and (ψ_j) is a sequence in E′ with $\|\psi_j\| \to 0$ (cf. G. Köthe [11], p. 226). For $A\epsilon L_s({}^m c_0;F)$ with $\hat{A} = Q$ one has

$$P(x) = Q\circ S(x) = A(\lim_{N\to\infty} \sum_{j=1}^{N} \psi_j(x)e_j)^m = \sum_{k_1,\ldots,k_m=1}^{\infty} a_{k_1,\ldots,k_m} \psi_{k_1}(x)\cdots\psi_{k_m}(x)$$

where $a_{k_1,\ldots,k_m} := A(e_{k_1},\ldots,e_{k_m})$. The family $(a_k)_{k\epsilon \mathbb{N}^m}$ is obviously w_m-summable, and we have

$$\|A\| = \sup\left\{\| \sum_{k_1,\ldots,k_m=1}^{\infty} a_{k_1,\ldots,k_m} \xi^{(1)}_{k_1}\cdots\xi^{(m)}_{k_m} \| : \xi^{(1)},\ldots,\xi^{(m)} \epsilon B_0\right\} =$$

$$= \epsilon((a_{k_1,\ldots,k_m}))$$

Because of $\|A\| \leqslant m^m (m!)^{-1} \|Q\|$ (cf. [5], p.5) and $\|S\| = \sup\left\{\|\psi_j\| : j\epsilon \mathbb{N}\right\}$ we get

$$k_\infty^{(m)}(P) + \epsilon \geqslant (m!)m^{-m} \epsilon((a_{k_1,\ldots,k_m})) \sup\left\{\|\psi_j\|^m : j\epsilon\mathbb{N}\right\}$$

and hence $k_\infty^{(m)}(P) \geqslant (m!)^{-m} \tau^{(m)}(P)$.

Conversely, suppose that $P\epsilon P^{(m)}(E;F)$ admits a representation $P(x) = \sum_{k\epsilon\mathbb{N}^m} a_k\psi_k$ where (a_k) belongs to $l^1({}^m F)$, (ψ_j) is a 0-sequence in E′ and $\tau^{(m)}(P) + \epsilon > \epsilon((a_k)) \sup\{\|\psi_j\|^m : j\epsilon\mathbb{N}\}$ for a given positive number $\epsilon>0$. Define the operator $S\epsilon L(E;c_0)$ by $Sx = (\psi_j(x))_j$ which is compact since $\psi_j \to 0$ (cf. G. Köthe [11]. p. 226). Let $Q\epsilon P^{(m)}(c_0;F)$ be defined by $Q((\xi_j)_{j=1}^{\infty}) = \sum_{k\epsilon\mathbb{N}^m} a_k\xi_k$ and let $A\epsilon L_s({}^m c_0;F)$ with $\hat{A}=Q$. Then we get $P=Q\circ S$ and

$$k_\infty^{(m)}(P) \leqslant \|Q\| \|S\|^m \leqslant \|A\| \|S\|^m = \epsilon((a_k)_{k\epsilon\mathbb{N}^m}) \sup\left\{\|\psi_j\|^m : j\epsilon\mathbb{N}\right\} <$$

$$< \tau^{(m)}(P) + \epsilon$$

This completes the proof.

Next we show that each continuous polynomial defined on an ϵ-(S)-space has an infinite series representation analogous to continuous polynomials on nuclear spaces. We remark that an ϵ-(S)-space need not be nuclear. Consider e.g.

any non nuclear echelon space λ_0 of order 0 which is an (S)-space (cf. R. Hollstein [10])

2.5 Proposition. *Let* E *be an* ϵ*-(S)-space and let* F *be a Banach space. Then each m-homogeneous polynomial* $P \in P^{(m)}(E;F)$ *has a series representation*

$$P(x) = \sum_{k \in \mathbb{N}^m} a_k \psi_k(x)$$

where $(a_k)_{k \in \mathbb{N}^m} \in l_1({}^m F)$ *and* $(\psi_j)_{j=1}^{\infty}$ *is a 0-sequence in the strong dual* E'_b *of* E.

Proof. Let $P \in P^{(m)}(E;F)$. Since P is continuous, there is an $U \in U(E)$ such that $\|P(x)\| \leq (q_U(x))^m$ for any $x \in E$ where q_U denotes the Minkowski functional of U. The mapping $P_U: E_U \to F$ defined by $P_U(K_U(x)) = P(x)$ is well defined and lies in $P^{(m)}(E_U;F)$. P_U has a continuous extension $\widetilde{P}_U \in P^{(m)}(\widetilde{E}_U;F)$. Since E is an ϵ-(S)-space one can find a $V \in U(E)$ with $V \subset U$ such that $\widetilde{K}_{UV}$ $\widetilde{E}_V \to \widetilde{E}_U$ is ∞-compact, i.e. $\widetilde{K}_{UV}$ is the compose $\widetilde{K}_{UV} = R \circ S$ of two compact operators $S \in L(\widetilde{E}_V, c_0)$ and $R \in L(c_0; \widetilde{E}_U)$. By definition, the mapping $P_V := \widetilde{P}_U \circ \widetilde{K}_{UV} = \widetilde{P}_U \circ R \circ S \in P^{(m)}(\widetilde{E}_V;F)$ is ∞-compact and admits, by 2.4, a representation

$$P_V(K_V(x)) = \sum_{k \in \mathbb{N}^m} a_k \varphi_k (K_V(x)), \quad x \in E$$

where $(a_k) \in l_1({}^m F)$ and $\lim_{j \to \infty} \|\varphi_j\| = 0$ in E'_V. Setting $\psi_j := \varphi_j \circ K_V$ the sequence $(\psi_j)_{j=1}^{\infty}$ converges to 0 in E'_b and one has

$$P(x) = P_V\, K_V(x) = \sum_{k \in \mathbb{N}^m} a_k \psi_k(x)$$

for any $x \in E$. This completes the proof.

3. HOLOMORPHY TYPES

L. Nachbin [15] introduced the concept of holomorphy type Θ. In this section we shall show that the polynomial ideals of ∞-factorable, strongly ∞-factorable and ∞-compact polynomials, respectively, form a holomorphy type Θ.

A holomorphy type Θ from a Banach space E into a Banach space F is a sequence of Banach spaces $P_\Theta^{(m)}(E;F)$, $m \in \mathbb{N}_0$, such that the following conditions hold true.

(1) $P_\Theta^{(m)}(E;F)$ is a vector subspace of $P^{(m)}(E;F)$ for each $m \in \mathbb{N}$.

(2) $P_\Theta^{(0)}(E;F)$ is isometrically isomorphic to $P^{(0)}(E;F)$.

(3) There exists a real number $\sigma \geqslant 1$ such that for each $k \in N_0$, $m \in N_0$, $k \leqslant m$, $x \in E$, and $P \in P_\Theta^{(m)}(E;F)$ one has

$$\hat{d}^k P(x) \in P_\Theta^{(k)}(E;F)$$

$$\| \frac{1}{k!} \hat{d}^k P(x) \|_\Theta \leqslant \sigma^m \|P\|_\Theta \|x\|^{m-k}$$

where $\|P\|_\Theta$ denotes the norm in $P_\Theta^{(m)}(E;F)$.

We now show that each normed operator ideal $[A,\alpha]$ assigns a holomorphy type Θ.

3.1. Proposition. *Let* $[A,\alpha]$ *be a normed operator ideal. Then for each Banach space* E *and* F *the sequence* $P^{(m)} \circ A(E;F)$, $m \in \mathbb{N}_0$, *is a holomorphy type* Θ.

Proof. Given any $m \in \mathbb{N}$. We prove that for each $k \in \mathbb{N}$, $k \leqslant m$, $x \in E$ and $P \in P^{(m)} \circ A(E;F)$ one has $\hat{d}^k P(x) \in P^{(k)} \circ A(E;F)$ and

$$\alpha^{(k)}\left(\frac{1}{k!} \hat{d}^k P(x)\right) \leqslant 2^m \alpha^{(m)}(P) \|x\|^{m-k}$$

Let $x \in E$, $x \neq 0$, and let $P \in P^{(m)} \circ A(E;F)$. For a given $\epsilon > 0$ there exist a Banach space G and mappings $S \in A(E;G)$ and $Q \in P^{(m)}(G;F)$ such that $P = Q \circ S$ and

$$\|Q\| (\alpha(S))^m \leqslant \alpha^{(m)}(P) + \epsilon\, 2^{-m} \|x\|^{k-m}.$$

The polynomial $\hat{d}^k P(x)$ lies in $P^{(k)} \circ A(E;F)$ since $\hat{d}^k P(x) = \hat{d}^k Q(Sx) \circ S$ where $\hat{d}^k Q(Sx) \in P^{(k)}(G;F)$. If $Sx \neq 0$, then we set $y := (1/\|Sx\|)Sx$ and we obtain

$$\alpha^{(k)}\left(\frac{1}{k!} \hat{d}^k P(x)\right) \leqslant \| \frac{1}{k!} \hat{d}^k Q(Sx) \| (\alpha(S))^k \leqslant \|Sx\|^{m-k} \| \frac{1}{k!} \hat{d}^k Q(y) \| (\alpha(S))^k$$

$$\leqslant \| \frac{1}{k!} \hat{d}^k Q(y) \| (\alpha(S))^m \|x\|^{m-k}$$

By the Cauchy inequalities we get

$$\| \frac{1}{k!} \hat{d}^k Q(y) \| \leqslant \sup \left\{ \|Q(z)\| : \|y-z\| = 1 \right\} \leqslant 2^m \|Q\|.$$

It follows

$$\alpha^{(k)}\left(\frac{1}{k!} \hat{d}^k P(x)\right) \leqslant 2^m \|Q\| (\alpha(S))^m \|x\|^{m-k} \leqslant 2^m \alpha^{(m)}(P) \|x\|^{m-k} + \epsilon$$

wich completes the proof.

From 2.2 and 3.1 follows that the sequences $\mathcal{P}_{L_\infty}^{(m)}(E;F)$, $\mathcal{P}_{S_\infty}^{(m)}(E;F)$ and $\mathcal{P}_{K_\infty}^{(m)}(E;F)$, $m\in\mathbb{N}_0$, are holomorphy types Θ.

4. Holomorphy mappings of uniformly infinite-factorable bounded type

A continuous mapping $f:E\to F$ between locally convex spaces is called holomorphic in E if for each $y'\in F'$ the function $z\to y'\circ f(a+zb)$ is holomorphic in $\mathbb{C}$ for all $a,b\in E$. The vector space of all holomorphic mappings from E into F is denoted by $H(E;F)$.

If E and F are normed spaces, then a mapping $f\in H(E;F)$ is said to be of bounded type if it is bounded on all bounded subsets of E. We denote by $H^b(E;F)$ the vector space of all holomorphic mappings of bounded type from E into F equipped with the topology of uniform convergence on bounded sets.

4.1 Definition. Let $[A,\alpha]$ be a normed operator ideal and let E, F be Banach spaces. A mapping $f\in H(E;F)$ is said to be holomorphic of bounded A-type if f can be represented by a series

$$f(x) = \sum_{m=0}^{\infty} P_m(x), \quad x\in E$$

where $P_m\in\mathcal{P}^{(m)}\circ A(E;F)$ and $\limsup_{m\in\mathbb{N}}(\alpha^{(m)}(P_m)) < \infty$.

The vector space $H_A^b(E;F)$ of all holomorphic mappings of bounded A-type is a metrizable locally convex space with respect to the topology defined by the norms

$$\alpha_r(f) = \sum_{m=0}^{\infty} r^m \alpha^{(m)}(P_m), \quad r\in\mathbb{R}.$$

We write $H_A^b(E)$ instead of $H_A^b(E;\mathbb{C})$.

If $H_{Nb}(E;F)$ denotes the vector space of all holomorphic mappings of nuclear bounded type in the sense of C.P. Gupta [6], then we have for all Banach spaces E,F

$$H_{Nb}(E;F) \subset H_{K_\infty}^b(E;F) \subset H_{S_\infty}^b(E;F) \subset H_{L_\infty}^b(E;F) \subset H^b(E;F)$$

Simple examples show that these inclusions are generally strict. Obviously $H_{S_\infty}^b(E;F) = H_{L_\infty}^b(E;F)$ holds if F is complemented in F″ and $H_{S_\infty}^b(E;F) = H^b(E;F)$ if E is an $L_\infty(\mu)$-space. Now we show.

4.2 Proposition. *For each infinite-dimensional Hilbert space* E *one has*

$$H^b_{L_\infty}(E) \neq H^b(E)$$

Proof. Let $I:E \to E'$ be the canonical linear norm-isomorphism. The mapping $P:E \to \mathbb{C}$ defined by $P(x) = \langle x, Ix \rangle$ belongs to $\mathcal{P}^{(2)}(E)$. We assume that $P \in H^b_{L_\infty}(E)$. Then P lies in $\mathcal{P}^{(2)}_{L_\infty}(E)$, hence there exist a measure space (Ω,μ) as well as mappings $S \in \mathcal{L}(E, L_\infty(\Omega,\mu))$ and $Q \in \mathcal{P}^{(2)}(L_\infty(\Omega,\mu))$ such that $P = Q \circ S$. Let $B \in \mathcal{L}_s({}^2 L_\infty(\Omega,\mu))$ with $Q = \hat{B}$ and let $\widetilde{B} \in \mathcal{L}(L_\infty(\Omega,\mu),(L_\infty(\Omega,\mu))')$ be the associated linear mapping of B. The continuous bilinear mapping $L:E \times E \to \mathbb{C}$ defined by $L(x,y) = \langle x, Iy \rangle$ is symmetric with $\hat{L} = P$. For all $x,y \in E$ we have

$$\langle x, Iy \rangle = L(x,y) = B(Sx,Sy) = \langle Sx, \widetilde{B} \circ Sy \rangle = \langle x, S' \circ \widetilde{B} \circ Sy \rangle$$

hence $I = S' \circ \widetilde{B} \circ S$. It follows that $I:E \to E'$ can be factored through an $L_\infty(\mu)$-space, thus I is 2-absolutely summing (cf. A. Pietsch [16], p. 307) which is the desired contradiction.

Later we need

4.3 Proposition. *Let* $[A, \alpha]$ *be a normed operator ideal, let* E,F,G *be Banach spaces and let* $A \in \mathcal{A}(E;F)$. *Then for each* $f \in H^b(F;G)$ *the compose* $f \circ A$ *lies in* $H^b_{\mathcal{A}}(E;G)$. *Moreover, the mapping.*

$$J: H^b(F;G) \to H^b_{\mathcal{A}}(E;G),\ f \to f \circ A$$

is continuous.

Proof. Let $f \in H^b(F;G)$ and let $f(x) = \sum_{m=0}^{\infty} P_m(x)$ be the Taylor series of f where $P_m \in \mathcal{P}^{(m)}(E;F)$ and $\lim_{m \to \infty} \|P_m\|^{\frac{1}{m}} = 0$ (cf. S. Dineen [5], p. 166). Setting $Q_m :=$ $= P_m \circ A$ we have $Q_m \in \mathcal{P}^{(m)} \circ \mathcal{A}(E;F)$, $f \circ A(x) = \sum_m Q_m(x)$ and $\lim_{m \to \infty} (\alpha^{(m)}(Q_m))^{\frac{1}{m}} = 0$ since $\alpha^{(m)}(Q_m) \leqslant \|P_m\| (\alpha(A))^m$. It follows that $f \circ A \in H^b_{\mathcal{A}}(E;F)$. It remains to show that J is continuous. For a given $r > 0$ we set $\rho := 2r\alpha(A)$ and $M(\rho) :=$ $= \sup \left\{ \|f(x)\| : \|x\| \leqslant \rho \right\}$. By the Cauchy inequalities we have $\|P_m\| \leqslant \rho^{-m} M(\rho)$ for all $m \in \mathbb{N}$ and hence

$$\alpha_r(f \circ A) = \sum_m r^m \alpha^{(m)}(Q_m) \leqslant \sum_m r^m \|P_m\| (\alpha(A))^m \leqslant$$

$$\leqslant \sum_m \left(\frac{r}{\rho}\right)^m (\alpha(A))^m M(\rho) = 2M(\rho),$$

thus J is continuous.

Following J.F. Colombeau, M.C. Matos [2], a mapping $f\epsilon H(E;F)$ from a l.c.s. E into a locally-complete space F is called holomorphic of uniformly bounded type if there exist $U\epsilon\mathcal{U}(E)$, $B\epsilon\mathcal{B}(E)$ and $f_0\epsilon H^b(\widetilde{E}_U,F_B)$ such that $f = J_B\circ f_0\circ K_U$. $H^{ub}(E;F)$ denotes the vector space of all holomorphic mappings of uniformly bounded type from E into F.

If $[A, \alpha]$ is any normed operator ideal, then a mapping $f\epsilon H(E;F)$ is said to be holomorphic of uniformly bounded A-type if there exist $U\epsilon\mathcal{U}(E)$, $B\epsilon\mathcal{B}(F)$ and $f_0\epsilon H^b_A(\widetilde{E}_U,F_B)$ with $f=J_B\circ f_0\circ K_U$. We denote by $H^{ub}_A(E;F)$ the vector space of all holomorphic mappings of uniformly bounded A-type.

Now we prove

4.4 Proposition. *Let* F *be any locally-complete space. The following assertions hold true*

(1) If E *is an* ϵ*-space, then* $H^{ub}_{L_\infty}(E;F) = H^{ub}(E;F)$.
(2) If E *is an* S_∞*-space, then* $H^{ub}_{S_\infty}(E;F) = H^{ub}(E;F)$.
(3) If E *is an* ϵ*-(S)-space, then* $H^{ub}_{K_\infty}(E;F) = H^{ub}(E;F)$.
(4) If E *is an* ϵ*-(DFM)-space and* F *is an (F)-space, then* $H^{ub}_{K_\infty}(E;F) = H(E:F)$.

Proof. (1) Let $f\epsilon H^{ub}(E;F)$. There exist $U\epsilon\mathcal{U}(E)$, $B\epsilon\mathcal{B}(F)$ and $f_0\epsilon H^b(\widetilde{E}_U;F_B)$ with $f=J_B\circ f_0\circ K_U$. Since E is an ϵ-space one can find a $V\epsilon\mathcal{U}(E)$ contained in U such that the canonical mapping $\widetilde{K}_{UV}:\widetilde{E}_V\to\widetilde{E}_U$ is ∞-factorable. In view of 4.3, $f_0\circ\widetilde{K}_{UV}$ belongs to $H^b_{L_\infty}(\widetilde{E}_V,F_B)$, hence $f=J_B\circ f_0\circ\widetilde{K}_{UV}\circ K_U\epsilon H^{ub}_{L_\infty}(E;F)$.

Assertion (2) can be proved in the same way. Statement (3) is a consequence of the fact that for each 0-neighbourhood $U\epsilon\mathcal{U}(E)$ of an ϵ-(S)-space E there exists a $V\epsilon\mathcal{U}(E)$ with $V\subset U$ such that $\widetilde{K}_{UV}$ is ∞-compact.

(4) If E is an ϵ-(DFM)-space, then E is an (S)-space and by (3) we have $H^{ub}_{K_\infty}(E;F) = H^{ub}(E;F)$. By a result of J.F. Colombeau, J. Mujica [3], 4.1, one has $H^{ub}(E;F) = H(E;F)$ if F moreover is an (F)-space. This completes the proof.

5. Extension of holomorphic mappings

In this section we shall characterize those holomorphic mappings of uniformly bounded type $f:E\to F$ between l.c.s. which can be extended holomorphically to each l.c.s. G containing E as a topological subspace. First we prove.

5.1 Proposition. *Let* E,F *be l.c.s. and* F *locally complete.*

(a) For each $f\epsilon H^{ub}_{L_\infty}(E;F)$ *and each l.c.s.* $G\supset E$ *there exists an* $\widetilde{f}\epsilon H^{ub}_{L_\infty}(G,F''_n)$ *with* $\widetilde{f}|_E = f$.

(b) The restriction mapping

$$R : H^{ub}_{S_\infty}(G;F) \to H^{ub}_{S_\infty}(E;F),\ f \to f|_E$$

is surjective for all l.c.s. $G \supset E$.

Proof. (a) Let $f \in H^{ub}_{L_\infty}(E;F)$ and let G be a l.c.s. containing E as a topological subspace. By definition, there exist $U \in \mathcal{U}(G)$, $B \in \mathcal{B}(F)$ and $f_0 \in H^{ub}_{L_\infty}(\widetilde{E}_W;F_B)$ with $W := U \cap E$ such that $f = J_B \circ f_0 \circ K_W$. f_0 can be represented by a series $f_0(\hat{x}) = = \sum_{m=0}^{\infty} P_m(\hat{x})$, $\hat{x} \in \widetilde{E}_U$, with $P_m \in P^{(m)}_{L_\infty}(\widetilde{E}_W;F_B)$ and $\limsup_{m \in \mathbb{N}} (\lambda^{(m)}_\infty(P_m))^{\frac{1}{m}} = 0$.

For a given $m \in \mathbb{N}$ there exist a measure space (Ω_m, μ_m) and mappings $A_m \in L(\widetilde{E}_W; L_\infty(\Omega_m, \mu_m))$ and $Q_m \in P^{(m)}(L_\infty(\Omega_m,\mu_m), F''_B)$ such that $I_{F_B} \circ P_m = Q_m \circ A_m$ and $\|A_m\|^m \|Q_m\| < \lambda^{(m)}_\infty(P_m) + m^{-m}$. Since $L_\infty(\Omega_m,\mu_m)$ has the metric extension property and since $\widetilde{E}_W$ can be considered as a normed subspace of $\widetilde{G}_U$ there exists an extension $\widetilde{A}_m \in L(\widetilde{G}_U, L_\infty(\Omega_m,\mu_m))$ of A_m with $\|\widetilde{A}_m\| = \|A_m\|$. Now the mapping $\widetilde{P}_m := Q_m \circ \widetilde{A}_m$ belongs to $P^{(m)}_{L_\infty}(\widetilde{G}_U;F''_B)$ and we have

$$\lambda^{(m)}_\infty(\widetilde{P}_m) \leqslant \|\widetilde{A}_m\|\ \|Q_m\|^m < \lambda^{(m)}_\infty(P_m) + m^{-m}.$$

Let $\widetilde{f}_0(\hat{x}) := \sum_{m=0}^{\infty} \widetilde{P}_m(\hat{x})$ for $\hat{x} \in \widetilde{G}_U$. Since

$$\limsup_m (\lambda^{(m)}_\infty(\widetilde{P}_m))^{\frac{1}{m}} \leqslant \limsup_m (\lambda^{(m)}_\infty(P_m)^{\frac{1}{m}} + m^{-1}) = 0$$

it follows that $\widetilde{f}_0$ is defined for all $\hat{x} \in \widetilde{G}_U$, that $\widetilde{f}_0 \in H^b_{L_\infty}(\widetilde{G}_U, F''_B)$ and $\widetilde{f}_0|_{\hat{E}_W} = f_0$. The mapping $\widetilde{f} = J''_B \circ \widetilde{f}_0 \circ K_U \in H^{ub}_{L_\infty}(G;F''_n)$ is then the desired extension of f.

The assertion (b) can be shown in the same way.

For any l.c.s E and F with F locally complete, let $H_{uNb}(E;F)$ be denote the vector space of all holomorphic mappings of uniformly nuclear bounded type in the sense of J.F. Colombeau, J. Mujica [3]. $H_{uNb}(E;F)$ is then a linear subspace of $H^{ub}_{K_\infty}(E;F)$, in particular of $H^{ub}_{S_\infty}(E;F)$. By [3], 7.3, the restriction mapping $R: H_{uNb}(G;F) \to H_{uNb}(E;F)$ is surjective for each l.c.s $G \supset E$. Now we shall show that $H^{ub}_{S_\infty}(E;F)$ is the largest subspace of $H^{ub}(E;F)$ such that the restriction mapping R in 5.1(b) is surjective for all l.c.s. G containing E as a topological subspace.

First we need the following notations: Let E_∞ be the topological product $\prod_{U \in U(E)} l_\infty(U^0)$ where $l_\infty(U^0)$ is the vector space of all bounded functions on the polar of U in E equipped with the supremum norm. E can be then identified with a topological subspace of E_∞. We denote by J_∞: $E \to E_\infty$ the canonical embedding.

5.2 Proposition. *Let* E,F *be l.c.s. and let* F *be locally complete. If* $f \in H^{ub}(E;F)$ *has an extension* $g \in H^{ub}(E_\infty;F)$, *then* $f \in H^{ub}_{S_\infty}(E;F)$.

Proof. Set $G := E_\infty$ and assume that $f \in H^{ub}(E;F)$ admits an extensión $g \in H^{ub}(G;F)$. g has a factorization $g = J_B \circ g_0 \circ K_V$ where $B \in \mathcal{B}(F)$, $V \in \mathcal{U}(G)$ and $g_0 \in H^b(\widetilde{G}_V;F_B)$. Since G is as a topological product of the spaces $l_\infty(U^0)$ an S_∞-space (cf. H. Junek [12], 7.2.2), there exists a $W \in \mathcal{U}(G)$ with $W \subset V$ such that $\widetilde{K}_{VW}: \widetilde{G}_W \to \widetilde{G}_V$ is strongly ∞-factorable. Thus $\widetilde{K}_{VW}$ admits a factorization $\widetilde{K}_{VW} = S \circ R$ through $L_\infty(\Omega,\mu)$ for a suitable measure space (Ω,μ) where $R \in L(\widetilde{G}_W; L_\infty(\Omega,\mu))$ and $S \in L(L_\infty(\Omega,\mu);\widetilde{G}_V)$. For $W_0 := W \cap E \in \mathcal{U}(E)$ we denote by $J_W: \widetilde{E}_{W_0} \to \widetilde{G}_W$ and $K_{W_0}: E \to \widetilde{E}_{W_0}$ the canonical mappings. Setting $g_1 := g_0 \circ S \in H^b(L_\infty(\Omega,\mu);F_B)$ and $f_0 := g_1 \circ R \circ J_W \in H^b(\widetilde{E}_{W_0};F_B)$ one has

$$f = g \circ J_\infty = J_B \circ g_0 \circ K_V \circ J_\infty = J_B \circ g_0 \circ \widetilde{K}_{VW} \circ K_W \circ J_\infty =$$

$$= J_B \circ g_0 \circ S \circ R \circ J_W \circ K_{W_0} = J_B \circ f_0 \circ K_{W_0}.$$

By 4.3. $f_0 = g_1 \circ R \circ J_W$ belongs to $H^b_{S_\infty}(\widetilde{E}_{W_0};F_B)$ since $g_1 \in H^b(L_\infty(\Omega,\mu);F_B)$ and $R \circ J_W \in S_\infty(\widetilde{E}_{W_0};L_\infty(\Omega,\mu))$. Thus f lies in $H^{ub}_{S_\infty}(E;F)$ which completes the proof.

Combining 5.1 and 5.2 we obtain.

5.3 Proposition. *Let* E,F *be a pair of l.c.s. and let* F *be locally complete. The following assertions are equivalent*

(1) $H^{ub}(E;F) = H^{ub}_{S_\infty}(E;F)$.

(2) The restriction mapping $R : H^{ub}(G;F) \to H^{ub}(E;F)$ *is surjective for each l.c.s.* $G \supset E$ *(resp. for* $G = E_\infty$*).*

From 4.2 and 5.3 it follows (cf. R. Aron, P. Berner [1], p. 21).

5.4 Corollary. *For each infinite-dimensional Hilbert space* E *there exists a Banach space* G *such that the restriction mapping* $R : H^b(G) \to H^b(E)$ *is not surjective.*

Let us remark that by a result of R. Meise, D. Vogt [14] the restriction mapping $R: H^{ub}(G;F) \to H^{ub}(E;F)$ is however surjective if G is a l.c.s. whose topology can be defined by seminorms induced by semiscalar products, E is a linear subspace of G which is a (DFM)-space in the induced topology and F is an (F)-space.

Applying 4.4 and 5.1 we further obtain the following holomorphic Hahn-Banach theorem (cf. R. Aron, P. Berner [1] and R. Hollstein [10]).

5.5 Corollary. *Let* E,F *be l.c.s. and* F *locally complete. The following assertions hold true*

(a) *If* E *is an* ϵ*-space, then every* $f \in H^{ub}(E;F)$ *has an extension* $\tilde{f} \in H^{ub}(G,F''_n)$ *for every l.c.s.* $G \supset E$.

(b) *If* E *is an* S_∞*-space, then every* $f \in H^{ub}(E;F)$ *has an extensión* $\tilde{f} \in H^{ub}(G;F)$ *for every l.c.s.* $G \supset E$.

(c) *If* E *is an* ϵ*-(DFM)-space and* F *is an (F)-space, then every* $f \in H(E;F)$ *has an extension* $\tilde{f} \in H(G;F)$ *for every l.c.s.* $G \supset E$.

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